Accelerator Physics for an ERL x-Ray Source
Homework 7

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Exercise 1:
a) Use the transport matrix from s_0 to s written in terms of Twiss parameters at s_0 and s to show that the matrix of a periodic section of an accelerator s can be written as

$$\mathbf{M} = \mathbf{1}\cos\mu + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \sin\mu \tag{1}$$

when α, β, and γ are the Twiss parameters that are periodic with the length L of the periodic section $\mu = \Psi(L) - \Psi(0)$ is the phase advance for that section.
b) Show that the matrix before $\sin\mu$ in this equation has a characteristic of the complex i in that squaring it leads to $-\mathbf{1}$.
c) Use this to compute $\mathbf{M}^n$.

Exercise 2:
If the matrix of a peridic section

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{2}$$

is known, specify how the periodic Twiss parameters and the phase advance can be computed. Under what conditions is the phase advance real? What does this mean for the motion through many periodic cells which is described by $\mathbf{M}^n \vec{z}_0$ for large n?

Exercise 3:
Characterize Twiss parameters by $\{\beta(s), \alpha(s), \psi(s)\}$. Imagine two sections of a beam line where the first section transports Twiss parameters $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_1, \alpha_1, \psi_1\}$ and the second transports $\{\beta_1, \alpha_1, 0\}$ to $\{\beta_2, \alpha_2, \psi_2\}$. Show that the total beam-line transports $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_2, \alpha_2, \psi_1 + \psi_2\}$.