Accelerator Physics for an ERL x-Ray Source
Homework 8

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Exercise 1 (Space Charge):
Assume that a cylindrically symmetric bunch of electrons has a Gaussian density distribution in the longitudinal and transverse direction, i.e.

$$\rho(x,y,z) = \frac{1}{2\pi\sigma_x^2} e^{-\frac{x^2+y^2}{2\sigma_x^2}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{z^2}{2\sigma_z^2}} \ . \tag{1}$$

Assume the bunch with a charge of 80pC was created at the cathode with a thermal distribution at room temperature (300K) with an rms width of 2mm. Later, when each particle has an energy of 750keV, the bunch has a length of 0.6mm and is again focused to an rms width of 2mm. Compute the following quantities:
a) the plasma frequency in the center of the accelerated bunch.
b) the Deby length at the center of that bunch.
c) Are space-charge forces important for the dynamics of this bunch at that energy?
d) Are the fields of individual particles important or can one calculate the motion of each particle in the collective field of the smooth distribution of all other particles?

Exercise 2 (RF cavities):
Compute the transfer matrix of a linac cavity that accelerates a 10MeV beam to 30MeV in one meter. Assume the standing wave in that cavity has the following form: $\vec{E}(z,t) = E_0 \sin(\omega t)(2-\cos(kz)^2)\cos(kz)$.
a) Where does a particle that initially travels parallel to the central axis cross this axis?
b) What are the Twiss parameters at the exit of this cavity, when at the entrance $beta_0 = 1m$, $\alpha_0 = 0$?