Accelerator Physics for an ERL x-Ray Source
Takehome-Final Exam

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Exercise 1 (Simple accelerators):
Assume that the earth has an exact dipole magnetic field which is oriented parallel to the rotation axis. Assume that the field at the pole is about $2 \cdot 10^{-5}$T. Would the beam motion for protons that travel around the equator be stable or unstable in the radial and vertical direction?

Exercise 2 (RF cavities):
(a) A 1.3GHz cavity has $Q_0 = 10^{10}$ and a power loss into the cavity wall of $P = 20W$ when operated on resonance. How much energy is stored in that cavity?
(b) How small does the surface resistivity R_s have to be to obtain such a large Q_0 if the geometry factor is $G = 270\Omega$?
(c) When $R/Q = 88\Omega$, what is the accelerating voltage on resonance for that cavity?
(d) A cavity made of N identical cells has geometry factors $G = G_N$ and $R/Q = (R/Q)_N$. What geometry factors does a cavity have that consists only of one cell?
(e) How much more power is lost into the wall in a copper cavity than in an superconducting niobium cavity when producing an accelerating voltage V_a, and both cavities have the same shape. As surface resistivity use 10mΩ for copper and 10nΩ for superconducting niobium.
(f) How much HOM power can maximally be exited in a monopole mode with $R/Q = 1\Omega$ and $Q_0 = 10000$ when the beam current is 100mA?
(g) What percentage of know elements display some form of superconductivity?

Exercise 3 (Symplecticity):
A matrix $\mathbf{M}$ is symplectic if it satisfies $\mathbf{M}\mathbf{J}\mathbf{M}^T = \mathbf{J}$. Using $\mathbf{J}^{-1} = -\mathbf{J}$ and $\mathbf{J}^T = -\mathbf{J}$, show that the following properties are also satisfied:

$$\mathbf{M}^{-1} = -\mathbf{J}\mathbf{M}^T\mathbf{J}\ , \quad \mathbf{M}^T\mathbf{J}\mathbf{M} = \mathbf{J}\ . \tag{1}$$

Exercise 4 (Orbit Correction):
(a) Consider a mirror symmetric arrangement of magnets so that the beta functions are symmetric with respect to a center plane, perpendicular to the beam direction. This beam-line

section has four symmetrically arranged horizontal corrector coils, i.e. two symmetric pairs. The Twiss parameters at these coils are known, and each coil can be excited independently of the others, their strength does not have to be symmetric. Specify the relative strength of these coils so that a closed bump is created that only changes the orbit position at the location of the symmetry plane, but not the orbit angle.

(b) Specify the relative strength of these coils so that a closed bump is created that only changes the orbit slope at the symmetry point, but not the orbit position.

Exercise 5 (Complex Potentials):

When the coordinates $w = x + iy$ and $\bar{w} = x - iy$ are used, the Laplace operator has been derived to be $\vec{\nabla}^2 = 4\partial_w \partial_{\bar{w}} + \partial_z^2$.

(a) Check that this is correct.

(b) The static magnetic field in a charge free space is given by $\vec{B} = -\vec{\nabla}\psi$. Writing the magnetic field in x and y direction in complex notation as $B = B_x + iB_y$, derive a formula that expresses B and B_z in terms of $\Psi(w, \bar{w}, z)$ and only ∂_w, $\partial_{\bar{w}}$, and ∂_z.

(c) The vector potential in complex notation is $A = A_x + iA_y$ and A_z, derive a formula that expresses B and B_z given by $\vec{B} = \nabla \times \vec{A}$, again only using ∂_w, $\partial_{\bar{w}}$, and ∂_z and A, A_z.

Exercise 6 (Field Symmetries):

(a) The field in a bending magnet has usually two symmetries: Mid-plane symmetry since the upper and lower part of the magnet are built identically, and a mirror symmetry with respect to the vertical plane, since each pole is build with right/left symmetry when viewed along the beam pipe. Which multipoles, in addition to the main dipole component, satisfy this symmetry and can therefore be associated with such a bending magnet.

(b) Similarly, a focusing magnet has C_2 and midplane symmetry. Which multipoles, in addition to the main quadrupole term, satisfy this symmetry and can therefore appear when such a magnet is built.

(c) Generalize your observation to a magnet which is built with exact C_n symmetry and midplane symmetry. Which multipole terms can the field have?

Exercise 7 (Multipoles):

(a) Describe the magnetic field and the magnetic scalar potential in an duodecapole.

(b) Show what fields are created when a n pole is shifted by a distance Δ in the transverse direction. For example, show that a shifted sextupole has a quadrupole field.

Exercise 8 (Neutron Acceleration):

Dipole magnets are used to guiding charged particles in a beam line or circular accelerator. Neutrons cannot be guided by homogeneous magnetic fields since they have no charge. However, due to their magnetic dipole moment the Stern-Gerlach Force could be used to guide them.

(a) Show that the force, produced by a quadrupole on a particle with horizontal spin, corresponds to the force on a charged particle in a dipole magnet.

(b) Show that a sextupole magnet that is rotated by $\pi/6$, i.e. a skew sextupole, can be used for focusing neutral particles with horizontal spin.

(c) What would the multipole coefficient need to be to produce an instantaneous bending radius of 10m for a neutron with an energy of 1MeV?