Notes on Cornell's Physics 510 Lab C-9
RF Transmission Line Measurements

Origin unknown, revised by Robert Sieman (1980s) and by Georg Hoffstaetter (2009)

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Abstract

This experiment makes use of a parallel-wire transmission line to investigate radio frequency (RF) circuits, wave propagation, resonant excitation of standing waves, quality factors, and impedance matching. One of the features of this system is that it allows one to see both the magnetic and electric fields of the wave on a transmission line.

The following topics should be addressed:

1. the impedance and mode converter that couples the coaxial cable into the parallel-wire transmission line.
2. the loaded quality factor Q_L and the natural quality factor Q_0 of the parallel-wire transmission line.
3. standing wave patterns in voltage and current for a resonantly excited transmission line.
4. standing wave patterns in voltage and current on a mismatched transmission line.
5. stub tuners for impedance matching.

Experimental setup: Radio frequency waves produced by a frequency generator are coupled to the transmission line which can be terminated by a short or by a resistor.

The output of the frequency generator is amplified and then attenuated to isolate the signal source from the line, i.e. to have reflected power from the transmission line be attenuated again before it returns to the amplifier. Before the amplifier, a fraction of the power is diverted to measure the frequency. This can be done with a frequency meter or with the oscilloscope. The RF is then run through two directional couplers; one diverts a fraction of the wave that travels forward to the transmission line. The power of this fraction is thus proportional to the forward power. The other diverts a fraction of the reflected wave to measure the reflected power. These powers can be measured with an oscilloscope or with a crystal-power meter. The directional couplers and power measurements should be cross calibrated by reflecting all power. This can be achieved by disconnecting the coaxial cable from the transmission line and leaving the cable either open, or terminating it by a short. In both cases, all power is reflected, except for a very small absorption in the cable. The output of the second directional coupler is connected to the parallel-wire transmission line by a coaxial cable and a mode and impedance converter.

All mechanical parts of this setup can (and some should) be taken apart for detailed investigation. The oscilloscope is an excellent tool to measure the function of directional couplers, impedance converters, coaxial cables, phase velocities in cables, etc.

Power measurements: The oscilloscope can display the oscillating RF voltage and therefore simplifies measurements greatly. To measure power with a crystal-power meter, the RF amplitude has to be modulated at a much lower frequency. For this purpose, one output of the frequency generator goes to a frequency meter, the other output goes to a diode switch controlled by a Wavetek running at 500Hz. This modulates the RF amplitude at 1kHz (twice the

Wavetek frequency). Since the RF is modulated at 1kHz, the outputs of the crystal diode detectors (which are used in several places) are also modulated at 1kHz. These signals are measured with narrow bandwidth amplifiers tuned to 1kHz. These amplifiers are designed for use with the crystal detectors, and are calibrated with the “square law” response in mind that relates the diode signal and the RF voltage. Part of the preparation for serious measurements will be to cross calibrate meters and to be sure they are tuned to the Wavetek frequency.

Transmission line essentials: A transmission line, whatever its form, has an impedance at any location z, i.e. a ratio between the voltage to ground at z and the current along the line at that place. This ratio can in general depend on the frequency and on the location. But when one considers only a forward traveling wave, it is independent of these and is referred to as “characteristic impedance” Z_0. As shown in Fig. 1, a voltage change dV over a section of length

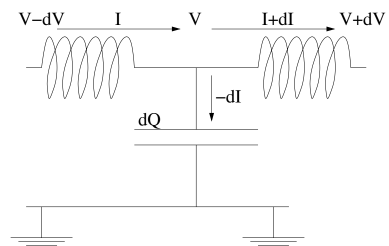

Figure 1: Current, voltage, and impedance of a transmission line.

dz produces an increasing current in negative direction $-\dot{I}$, the increase is slowed down by the magnetic field that it creates, and thus by the inductance dL of that line segment. Therefore $\dot{I} = -dV/dL$, or $V' = -L'\dot{I}$, where the prime indicates a derivative with respect to z. Accordingly, a charge dQ on that line segment produces a voltage against ground that is given by the capacitance dC of that section, i.e. $VdC = dQ$. With $d\dot{Q}+dI = 0$ this leads to $I' = -C'\dot{V}$. In complex notation, when the fields are excited to vary with angular frequency ω as $\exp(i\omega t)$, these equations lead to the telegraph equations, $V' = -i\omega L'I$ and $I' = -i\omega C'V$. The resulting wave equation $I'' = -\omega^2 L'C'I$ has the solution $I = I_f e^{-ikz} + I_r e^{ikz}$ with $k = \sqrt{L'C'}\omega$, one describing a forward and the other a reflected and thus backward traveling wave. The voltage is then computed to be $V = iI'/(\omega C') = \sqrt{L'/C'}(I_f e^{-ikz} - I_r e^{ikz})$. This shows that $Z_0 = \sqrt{L'/C'}$ is the impedance for a forward traveling wave, the characteristic impedance.

Reflected waves: When the line is connected to another line or to a terminating resistor that has the same impedance, no reflection of an RF wave occurs at the connection. At this location, the two lines have the same voltage. Because the impedances are the same, also the current flowing out of the transmission line is the same as that flowing into the second line, or into the termination, and no current can be reflected. For example, a coax cable with characteristic impedance of 50Ω terminated with a 50Ω resister therefore does not reflect waves at the termination. A line terminated in a short circuit, on the other hand, produces a reflected wave of opposite polarity. This reflected wave guarantees that the boundary condition of zero voltage at the short circuit termination is always satisfied. In general, the reflection that occurs when the termination is Z_{load}, can be computed by a simple wave analysis. In the transmission line there is an incoming and a reflected wave,

$$I = I_f e^{-ikz} + I_r e^{-ikz} \ , \quad V = Z_0(I_f e^{-ikz} - I_r e^{-ikz}) \ . \tag{1}$$

At the transition, located for simplicity at $z = 0$, the current flowing into the second line with impedance Z_{load} is equal to the current at the end of the line, i.e. $I(0)$.

$$V(0) = Z_{load}I(0) = Z_{load}(I_f + I_r) = Z_0(I_f - I_r) \ . \tag{2}$$

And therefore

$$I_r = I_f(1-r)/(1+r)\ , \quad r = \frac{Z_{load}}{Z_0}\ . \tag{3}$$

This describes both above mentioned cases: for $Z_{load} = Z_0$ one obtains, $I_r = 0$ and thus no reflection, and for a short with $Z_{load} = 0$ one obtains $I_r = I_f$ and therefore a negative sign for the voltage of the reflected wave.

The reflected voltage becomes $V = \frac{Z_0 I_f}{1+r}[(1+r)e^{-ikz} - (1-r)e^{ikz}] \propto r\cos(kz) - i\sin(kz)$, so that its amplitude over an oscillation period becomes $|V| \propto \sqrt{r^2\cos^2(kz) + \sin^2(kz)}$. The ratio of the lowest voltage to the highest voltage, the voltage standing wave ratio ($VSWR$), therefore becomes the smaller of r or $1/r$. The $VSWR$ therefore describes the ratio of the termination impedance to the characteristic impedance. And it can thus be found by measuring the forward and the reflected power. Because the ratio of reflected to forward power is $|I_f/I_r|^2$, Eq. (3) relates the $VSWR$ and the power ratio.

Resonant excitation: Resonant excitation of the transmission line is obtained by rolling two small wheels connected together by an axle (all of metal) along the wires. When resonance occurs, a fairly large current flows through the axle. This is detected by a loop of wire which couples to the magnetic field so produced. The loop voltage can either be measured with a crystal diode as discussed in [2] on page 108-114, or with an oscilloscope. Fine tuning around the resonance condition can be done by varying the frequency.

1 Mode and impedance converter

Find the characteristic impedance of the coaxial line and of the transmission line and understand to which voltages and currents both refer.

For the coaxial line this is rather obvious. The outer conductor is grounded, and the inner conductor has a voltage $V(z)$ against ground that changes with the distance z along the length of the conductor. The current $I(z)$ that flows along the inner conductor and the charge per length $Q'(z)$ that accumulates on the inner conductor is accompanied by a current $-I(z)$ and $-Q'(z)$ on the outer conductor which shields the cable so that the potential on the outside of the coax is ground potential. The characteristic impedance Z_{coax} of this coaxial cable then describes the ratio between $V(z)$ and $I(z)$ for a forward traveling wave. Compare your calculation of Z_{coax} with the impedance formula for a coaxial line in [2].

For the parallel-wire transmission line, a mode should be excited where one wire has a voltage $V(z)$ against ground, and the other wire has the voltage $-V(z)$ against ground. The current and charge per length on one wire is then $I(z)$ and $Q(z)$ and that on the other wire is $-I(z)$ and $-Q(z)$. If you compare your calculation with the impedance formula for a parallel-wire transmission line in [2], you will see that by convention, $Z_{\|}$ for such a line is not the voltage against ground divided by the current, but the voltage between the lines divided by the current, i.e. $Z_{\|} = 2V(z)/I(z)$ for a forward traveling wave.

Investigate how the mode converter works, and by what fraction the impedance is converted. The last image in [1] may be helpful. Verify your conclusion by measurements. When you have understood the mode and impedance converter, you will know the frequency at which the setup should be operated. The required phase velocities can either be measured or found from cable specifications.

To simplify your effort of understanding the mode converter, consider Fig. 2. The inner conductor of the coax with voltage V is connected to one of the parallel wires, which at that location therefore also has voltage V against ground. There is a coaxial cable of length L that connects to the other wire. To have voltage $-V$ on that line as required for the desired mode, that cable must have a length that is an odd multiple of wavelengths long, e.g. $L = \lambda/2$. To find the appropriate frequency for your experiment, look up the phase velocity in the mode converter's coaxial cable, measure its length, and determine for which frequency that length is $\lambda/2$.

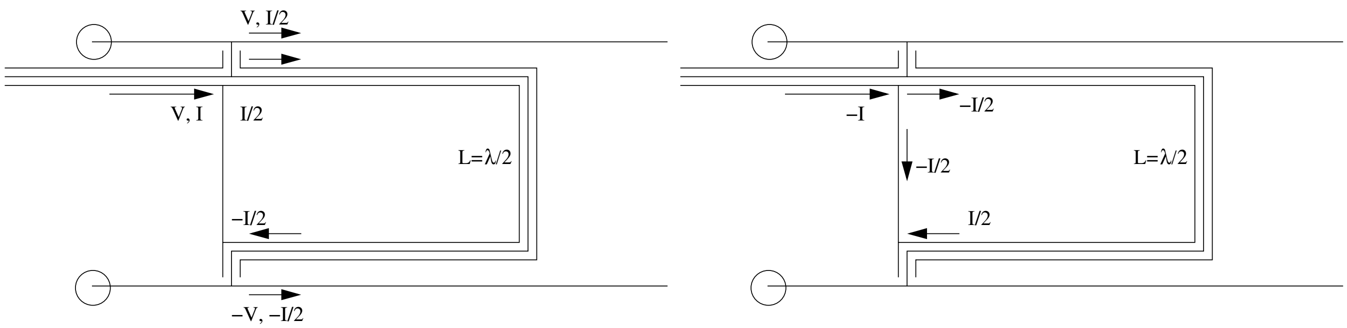

Figure 2: Right: Voltages and currents in the inner conductors of the mode and impedance converter. Left: Currents on the outer conductors of the coaxial cables of the converter.

To find the characteristic impedance to which this mode converter matches, investigate which ratio of voltage to current a parallel-wire transmission line would have to have in order to transport an incoming wave without reflecting power back into the mode converter. When there is no reflection, half the current that goes out of the inner conductor of the coax should go to one transmission line, and the other half into the mode converter, so that the other transmission line is provided with the current $-I/2$. The voltage between the parallel wires is therefore $-2V$, whereas the current in each of the wires is $\pm I/2$. This produces a forward traveling wave when the characteristic impedance of the parallel-wire transmission line is $Z_{\parallel} = 2V/(I/2) = 4Z_{coax}$. This mode converter therefore converts the impedance in a ration 1:4.

2 Measuring quality factors

Preparation of the line to eliminate termination effects: Before making detailed Q measurements, the line must be appropriately prepared. The problem is that it is difficult to compute the effect of the end terminations of the line which are formed by stretching the wire through the metal end of a ceramic an then twisting the wire around itself. And it is therefore best to prepare the line such that they can be ignored. Figure 3 shows a transmission line that has 2 shorts an integral number of $\lambda/2$ apart. A standing wave could be excited in loops 2 and 3, and hence the end terminations have to be considered as part of the circuit. This can be avoided by placing additional shorts each approximately $\lambda/4$ from the original shorting bars. There can now be no standing wave in loops 2 and 3 and these sections can be ignored.

Once the effects of the end-terminations have been removed, the following two techniques can be used to measure the quality factors; [2] calls them the "transmission" method and the "impedance" method.

The "transmission method": Without disturbing the connection of the impedance converter to the line, the "transmission method" should be used to determine the quality factors. In this method, the current flow in the axle is measured vs. frequency. With one power meter measuring the current flow and the other measuring the forward power, a plot of transmitted power vs. frequency is obtained. These data can be analyzed to give the loaded quality factor Q_L of the line. The length of the line has to be adjusted quite accurately to resonantly excite a standing wave.

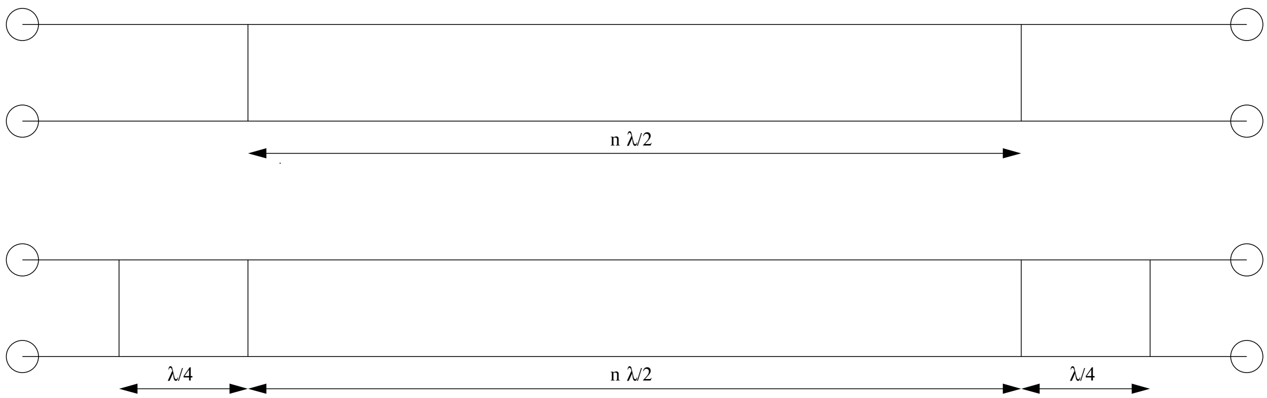

Figure 3: Top: Circuit diagram with termination on both sides of the transmission line. Standing waves can be in loops 2 and 3. Bottom: Extra shorts approximately $\lambda/4$ apart eliminate such waves so that loops 2 and 3 can be ignored.

The length should therefore be adjusted first to obtain resonant excitation, and the frequency can then be scanned in fine steps close to the resonant frequency.

To obtain the natural quality factor Q_0, the measurements must be repeated for several different values of the coupling β. While the absolute value of the coupling isn't known, it can be controlled by varying X, the distance between the point where the impedance converter is connected and the short circuit in Fig. 4. For X/λ small, the coupling is proportional to $(X/\lambda)^2$. Reference [3] may help you to understand this. There are several possible ways to obtain Q_0. One is to measure Q_L vs. X and extrapolate a plot of Q_L vs. X^2 to $X = 0$. Another is to adjust X until the input line is critically coupled; then $Q_0 = 2Q_L$.

The "impedance" method: In the impedance method, forward and reflected powers are measured to deuterium the $VSWR$ as a function of frequency on the coaxial input line.

The data can be analyzed as described in the section starting on page 413 of [2]. The results will be the loaded quality factor Q_L of the line. The natural quality factor Q_0 is ambiguous because you don't know whether you are over-coupled or under-coupled. Can this ambiguity be resolved?

3 Observation of standing waves:

Standing waves on the line are observed with a detector on a rolling carriage which contains crystal diodes similar to those in the rolling short. The electric field is sampled by inserting two L-shaped wires which form a small capacitance to the two sides of the transmission line. The voltage on the capacitor can be measured. To sample the magnetic field, loops are fitted into the two lugs. The line should be set to resonance and the electric and magnetic fields measured over at least one wavelength.

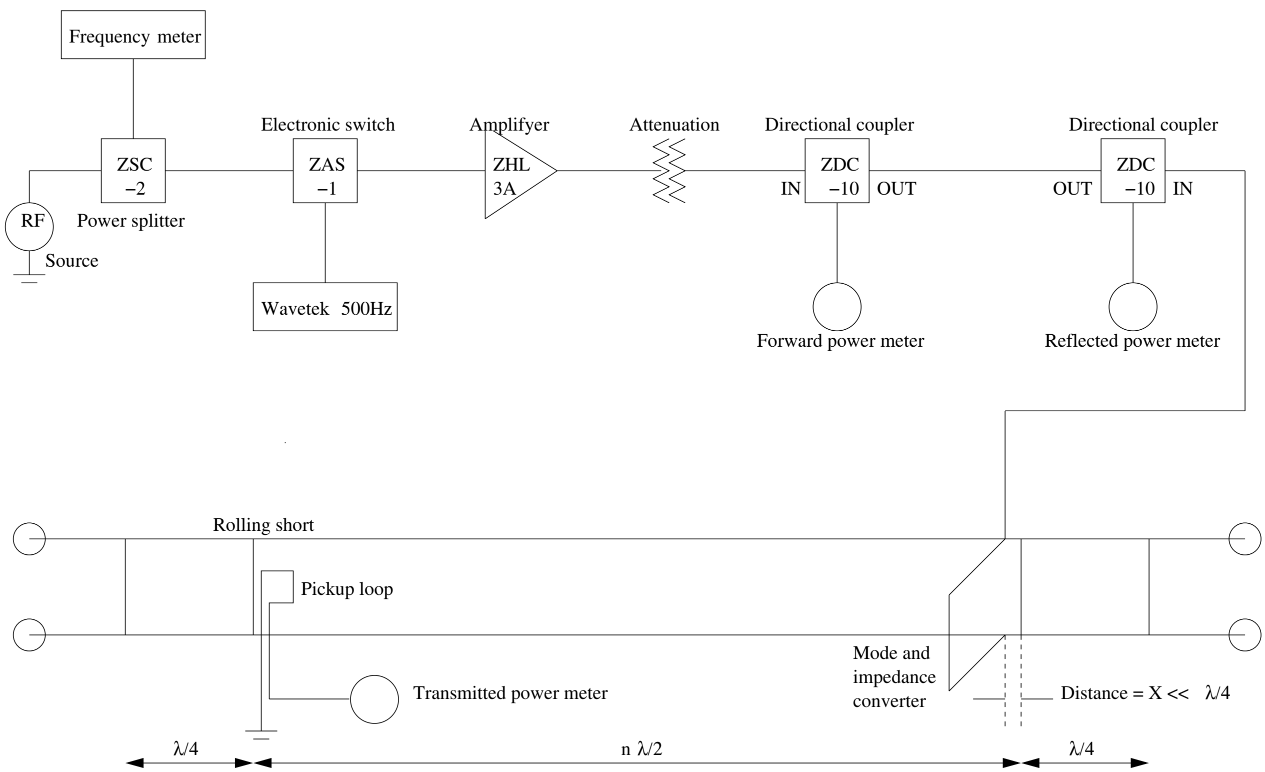

Figure 4: Overview of the laboratory setup. To determine a suitable frequency of the RF source, investigate the mode and impedance converter between coax cable and transmission line.

4 Mismatch the line:

When the wave is neither reflected on a short nor matched to a resistor to avoid reflections, the line is referred to as mismatched. To measure fields of mismatched lines, the rolling short is removed and a resistance not equal to the characteristic impedance of the transmission line is placed across the line. It is suggested that the resistance Z_{load} be in the range 1/3 to 3 times the characteristic impedance Z_0. Place the resistor $\lambda/4$ in front of the short at the end of the line. In this way, the impedance of the line just after the resistor is $Z_{eff} = \infty$ and corresponds to an open line just after the resistor, i.e. the line is terminated by the resistor. And as shown in Eq. (3), the voltage becomes $V(0) = Z_0 \frac{2I_f}{Z_0+Z_{load}}[Z_{load}\cos(kL) - iZ_0\sin(kL)]$. The derivative of $|V(0)|^2$ with respect to L is $4|I_f|^2 \frac{Z_0-Z_{load}}{Z_0+Z_{load}}\sin(2kL)$. Therefore, if the end resistance Z_{load} is greater than Z_0, there will be a voltage maximum at the end of the line; if it is less than Z_0, there will be a voltage minimum. A quarter wavelength up the line toward the power input, the situation will have exactly reversed. $\sin(2kL)$ changes sign after a quarter wavelength, and there is a minimum in voltage at that point when the load at $z = 0$ has an impedance that is larger than that of the line.

5 Stub matching:

Details about stub matching are given in section 10.10 of [4]. The stub is to be located where the real part of the admittance equals $1/Z_0$. This location is determined by the magnitude of the reflection coefficient. The length of the shorted stub is chosen to cancel the imaginary part of the admittance at the location of the stub. Its length is also determined by the reflection coefficient. After achieving a match you should replace the mismatch at the end by a resistance equal to Z_0 and then remeasure the $VSWR$. It should be what it was before any stub tuning was started except that it will be displaced along the line by 90°. It is simple interference of waves that is performed in matching a line with a stub tuner.

6 Additional questions:

You may want to investigate additional advanced questions, e.g.

- With the oscilloscope you can measure not only the amplitude but also the phase of the reflected wave. Can this information be used to measure the quality factors?
- At what frequencies would the mode converter excite an antenna mode in the transmission line? This is a mode where current and voltage on each line have the same sign for the same location z. This mode can be used for transmitting radio signals, e.g. the 1kHz of the wavetec or signals from a CD player, iPod, or cell phone. Before doing this, check if the frequency is permitted for broadcasting at the power you plan to radiate. `http://en.wikipedia.org/wiki/Part_15_(FCC_rules)` may be a good source of information.
- At what frequencies would you expect higher order modes (HOMs) on the parallel-wire transmission line? Can they be detected?

References

[1] Horowitz and Hill, The Art of Electronics, sections 13.09 and 13.10, which is attached to this manual.

[2] Ginzton, Microwave measurements, pp. 391-424, which is attached to this manual.

[3] ITT Reference Data for Radio Engineers, pp. 24-13 to 24-16, which is attached to this manual.

[4] Brownwell and Bean, which is included from chapter 8 to chapter 10.