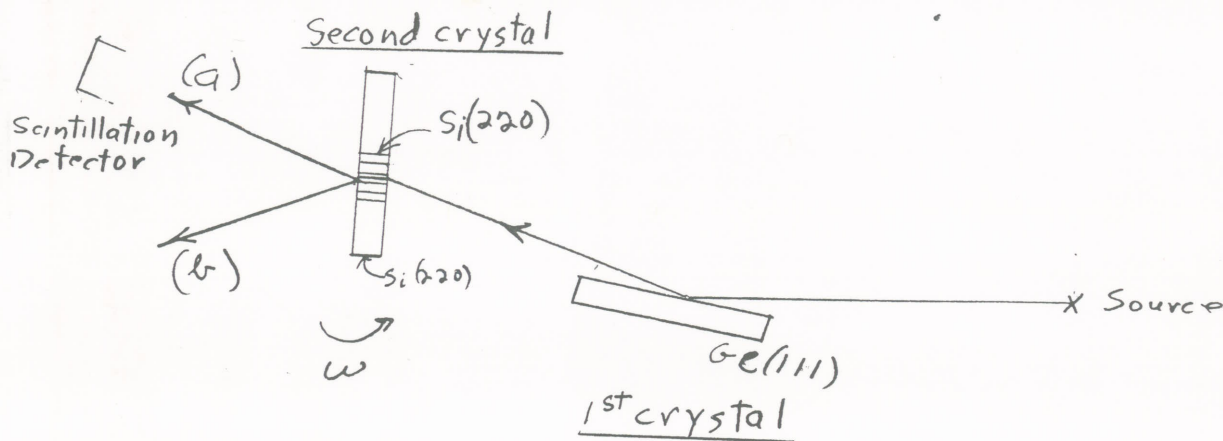# Anomalous Transmission of X-Rays
The Borrmann Effect

Using CuK$_\alpha$ radiation and a germanium (111) reflection as a primary beam, find the (220) reflection through a silicon slab in symmetric (220) reflection as indicated in the figure. Find both the forward diffracted beam (a) and the diffracted (220) beam (b).

To facilitate the search for these reflections, you may first want to reflect the primary beam from the edge of the crystal picking up the strong (220) reflection Bragg peak from the crystal edge.

Record the diffraction curves of (a) and (b). Explain the doublet structure. (It is important to have both the Si(220) and the germanium (111) reflection parallel to the same vertical line).

Adjust tube voltage (below 20keV) and current (not to exceed 20 ma) for clean signals. Using x-ray film, record the diffracted and forward diffracted beam simultaneously. Translate the crystal out of the beam and take a fractional second exposure to locate the primary beam on the film (film must be unperturbed from the previous exposure). Get a diffracted beam and measure the attenuation factor of a couple of .004" foils of Al. Use these foils to measure the intensity of the primary beam hitting the silicon crystal.

Results

   a) Use a planimeter to measure the relative integrated intensities of the forward (a) and the diffraction beam (b) - $\int(\theta)d\theta$

   b) Does the film support the dynamical theory prediction that the Poynting vector is along the (220) planes in the Borrmann effect?

   c) Make an estimate of the effective anomalous absorption coefficient I/I_0 for the peak intensity of the Laue diffracted beam (see equ. 93 B&C). Compare this with the normal value $e^{-\mu_0 t}$ of the Si crystal (t is the thickness along the incident beam).

   d) In our experiment, the incident beam from the Ge (111) is nearly completely unpolarized. Equation 93 is for a polarized beam. Explain the factor of 1/8 in Eqn. 93.

Density of Si=2.3 g/cc, e^{-M} = .969 for Si 220 at room temp; mass absorption coefficient of Si (λ=1.54) = 60.3cm^2/g.

References

B.W. Batterman and H. Cole, Rev. of Mod. Physics 36, 681 (1964).
B.W. Batterman, Phys. Rev. 126, 1461 (1962), see Table IV.

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Special Notes for X-8

Anomalous Transmission of X-Rays
The Borrmann Effect

I. Background

These notes will give an introduction to the physical aspects of anomalous transmission. It is not intended to be a rigorous treatment, but the physical aspects are correct.

To start off, consider a simple plane wave of wave vector $\vec{k}_0, |k_0| = 2\pi / \lambda$, (Fig. 1) traveling in space. It can be represented as $\vec{\varepsilon}_0 = E_0\, e^{-i\vec{k}_0 \cdot \vec{r}}\, e^{i\omega t}$

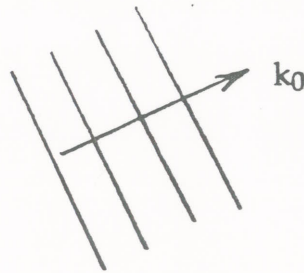Figure 1

We add to this another plane wave of wave vector $\vec{k}_H$ and amplitude E_H so that the total amplitude is $\varepsilon = e^{i\omega t}\left(E_0\, e^{-i\vec{k}_0 \cdot \vec{r}} + E_H\, e^{-i\vec{k}_H \cdot \vec{r}}\right)$

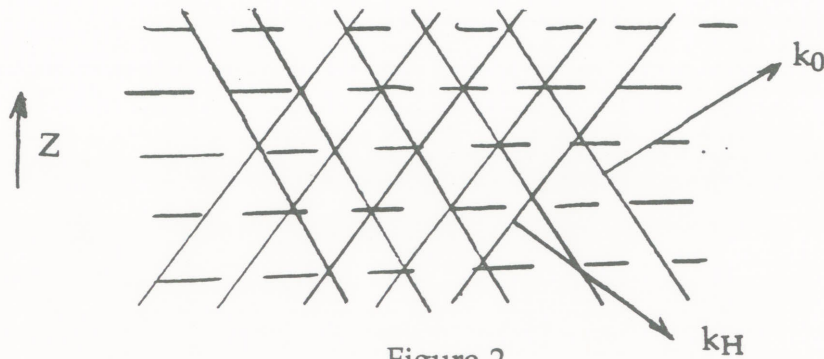
Figure 2

1

In Fig. 2 we've drawn horizontal planes through intersections of the nodes which are bisectors of the angle between $\bar{k}_0$ and $\bar{k}_H$. These planes represent regions for which the intensity is zero for all space in the plane and all time, if the amplitude of the separate plane waves are equal. To see this analytically the total amplitude is $\left(\text{for}\quad |E_0| = |E_H|\right)$.

$$\varepsilon = E_0\, e^{i\omega t}\, e^{i\bar{k}_0\cdot\bar{r}}\ \left(1 + e^{i\left(\bar{k}_H - \bar{k}_0\right)\cdot\bar{r}}\right)$$
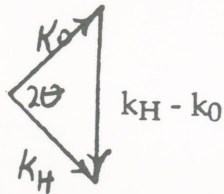
$$\left|k_H - K_0\right| = (2\sin\theta)\frac{2\pi}{\lambda} = 4\pi\sin\theta/\lambda$$

The vector $\left(\bar{k}_H - \bar{k}_0\right)$ has a magnitude $4\pi\sin\theta/\lambda$ and is in the z direction which is perpendicular to the angle bisector of $\bar{k}_0$ and $\bar{k}_H$. Thus, the dot product $\left(\bar{k}_H - \bar{k}_0\right)\cdot\bar{r} = \dfrac{4\pi\sin\theta}{\lambda}|r|\cos\phi$ where ϕ is the angle between a position vector and the z axis, so that $r\cos\phi = z$, a distance in the z direction with some suitably chosen origin. The intensity is proportional to $\varepsilon\varepsilon^*$ and therefore

$$I \approx E_0^2\left(2 + e^{i\left(\bar{k}_H - \bar{k}_0\right)\cdot\bar{r}} + e^{-i\left(\bar{k}_H - \bar{k}_0\right)\cdot\bar{r}}\right) = 2E_0^2\left(1 + \cos\left(\frac{4\pi\sin\theta}{\lambda}\right)Z\right)$$

The function is constant in the planes Z=constant. In particular it is periodic in Z with period $\lambda/2\sin\theta$ such that for certain planes spaced $\dfrac{\lambda}{2\sin\theta}$ apart I=0.

If we now plot the intensity in the Z direction we have

$$\lambda/2\sin\theta = Z_p$$
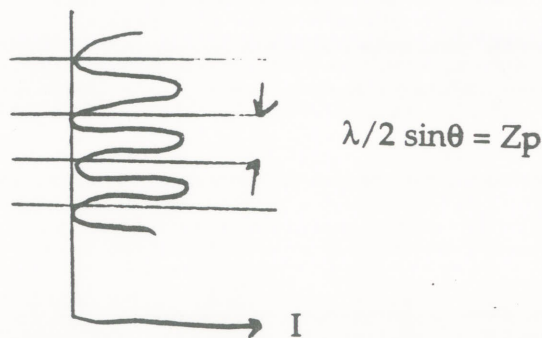
so that the intensity has nodal planes as in the earlier diagram (note that the analytic treatment could be carried out with the wave amplitudes as real function

$\varepsilon_0 = E_0 \cos(\omega wt + \vec{k} \cdot \vec{r})$, etc. This would give the same results as the complex treatment but with more arithmetical difficulty).

Now if we consider the case of a Bragg reflection in transmission (this geometry is called the Laue Case) the
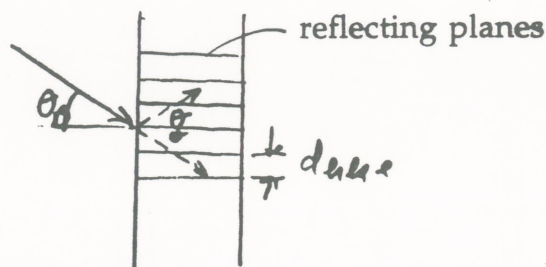
initial beam incident at the Bragg angle scatters at the angle θ from the planes. Those scattered rays make the Bragg angle with the bottom side of the plane above and can therefore be diffracted a second time back into the initial beam direction. It's not hard to believe, from these considerations, that in a very short time the internal wave picture is composed of a superposition of two plane waves one in the incident direction and one in the diffracted direction in a similar manner to our initial plane wave superposition.

Since $\vec{k}_0$ and $\vec{k}_H$ are at the Bragg angle with the planes, $\left| k_H - k_0 \right| = \dfrac{4\pi \sin\theta_B}{\lambda}$ and the period between nodes is $Z_p = \dfrac{\lambda}{2\sin\theta_B} = d_{hkl}$. The last relationship following from Bragg's law, $\lambda = 2d_{hkl}\sin\theta_B$ where d_{hkl} is the lattice spacing of the Bragg plane involved. Thus the nodal planes of the wave filed are spaced the same as the interplanar spacing of the Bragg reflection.

Now consider what would happen if the nodal planes of the wave field coincided with atoms situated in the Bragg planes. The intensity of x-rays that each atom saw would be very much smaller than if there were a single plane wave (average intensity $\approx 1/2\, E_0^2$). If in fact an atom (or more exactly the absorbing electrons were very small compared to d_{hkl}, the atom would see almost no wave field, and hence, it would absorb no x-rays.

In this case, the wave field would propagate very deeply and could be transmitted through crystals which would ordinarily be opaque to x-rays. As an example, a crystal whose transmission ratio is $I/I_0 = e^{-20}$ could, when the exact Bragg condition is satisfied have a much smaller attenuation so that $I/I_0 \approx e^{-0.4}$. Here a

crystal, thick enough to be opaque to x-rays would suddenly open up and let a strong beam through when the Bragg angle was satisfied by the incident beam. This in fact is the origin of the anomalous transmission of x-rays called the Borrmann effect.

If the incident beam has a divergence larger than the natural width of the reflection (tens of seconds of arc), then the various diffracted rays in the crystal would have different values of E_H, and true nodes would not exist at the atoms. Only the ray exactly satisfying Bragg's law would have $E_H = E_0$ and this wave field (i.e., E_0 and E_H together) would have the least attenuation. Thus, if the crystal is thick enough, all the rays other than the one making the precise Bragg angle would be attenuated out and this one, would remain and travel parallel to the Bragg planes as shown.
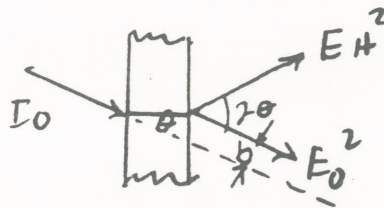
Upon leaving the crystal, the wave field will break into its two component plane waves, and the one traveling in the forward direction will be displaced from the primary ray by an amount b, depending on crystal thickness and Bragg angle θ.

II. Some Considerations of Experimental Details

The apparatus utilizes a double crystal diffractometer whose principles are described in the attached notes. The first crystal is a Germanium (111) reflection set to diffract CuK radiation ($\lambda \approx 1.54$Å). The second crystal (used to demonstrate the Borrmann Effect) has a thickness of about 3mm with a shape and crystallography as shown.

face parallel
to (110)

Adjusting the crystal for the 220 reflection from the (110) face provides a simple means of calibrating things since the intensity is very high. The counter is a 1" diameter scintillation detector.

To keep background low, use thin lead baffles to keep scattering from the slits from entering the counter.
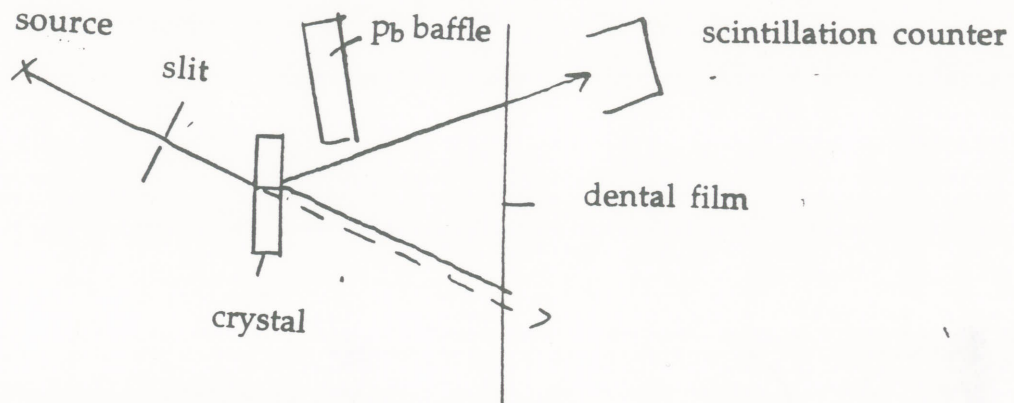

Set up the film approximately as shown to determine the energy flow. For photo, use maximum current (15 ma) and adjust Kv so that the direct beam (hard radiation) doesn't over blacken film. An exposure of 1/2 to 1 hour will probably be required.

The Double Crystal X-ray Spectrometer

is a simple and remarkable instrument. It allows one to measure intrinsic x-ray diffraction phenomena at the arc second level using rudimentary rotation stages, and relatively poorly collimated x-ray beams.

Parallel Mode
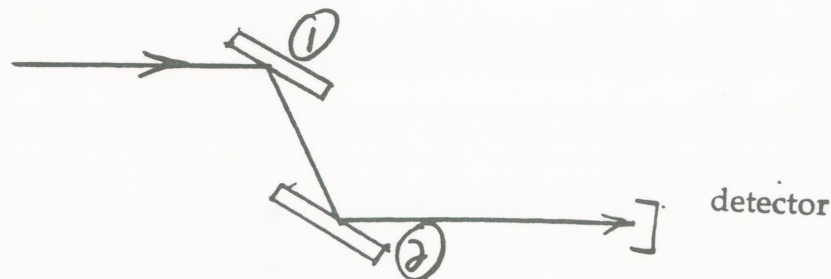
A broad beam of x-rays, with moderate divergence (degrees) and in general not monochromatic is incident on crystal (1) which is a structurally perfect crystal of silicon or germanium. (Perfect means low dislocation content, i.e., no grain boundaries, twins, impurities or point defects). Other naturally or artificially grown crystals are possible but they must be carefully selected.

Crystal (1) will diffract a parallel bundle of x-rays of wavelength λ, from the source, striking the crystal planes at angle θ_1 and diffracting it at angle 2θ, according to Bragg's Law $\lambda_1 = 2d \sin\theta_1$
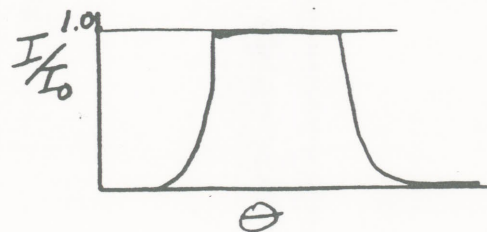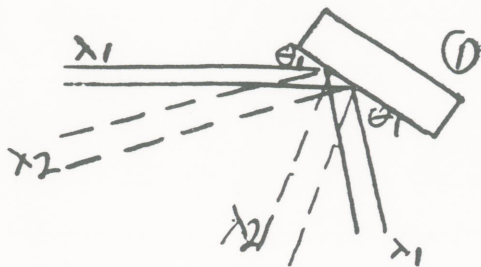
Another bundle of wavelength λ_2 will behave similarly at an angle θ_2. The beam leaves (1) at nominal angle θ and strikes crystal (2) which is parallel to crystal (1).

Under these conditions, with crystal (2) identical to (1), then all the bundles striking (2) will diffract simultaneously and send a beam out to the detector parallel to the incident beam from the source.

The instrument as described above is dispersionless, i.e., it diffracts all wavelengths at the same time. The angular width to rotate crystal (2) through Bragg reflection is determined by the intrinsic diffraction widths of (2) and (1) and is independent of extrinsic parameters such as the wavelength spread of the incident beam and its angular divergence. In fact, the angular shape of the detected beam from (2) will be the convolution of the intrinsic rocking curves of both crystals. A rocking curve is the intensity vs. angle of the Bragg reflection produced by an exactly parallel bundle of monochromatic x-rays incident at or near the Bragg angle. A typical curve for silicon 111 reflection is shown in the figure.

The double crystal diffractometer is also useful for x-ray spectroscopy when the two crystals do not use the same d-spacing reflection. There are two modes, the parallel and antiparallel which are illustrated in the figure.

parallel
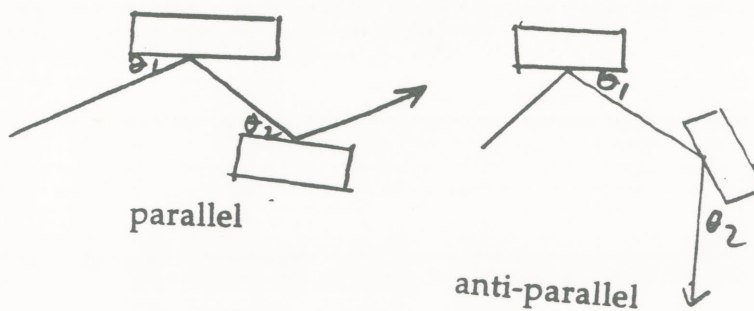
anti-parallel

If the crystals are in the parallel or antiparallel configuration, the angle $\Delta\phi$, of rotation of the 2nd crystal to diffract two wavelengths separated by $\Delta\lambda$ is

$$\Delta\phi = \left(\frac{\Delta\lambda}{\lambda}\right)\left(\tan\theta_1 \mp \tan\theta_2\right)$$

where (-) is parallel and + is antiparallel.

For the double crystal spectrometer described earlier where $\theta_1=\theta_2$, $\Delta\phi=0$ showing the device is dispersionless.

[for reference, see Compton and Allison, x-rays in Theory and Experiment, Chapter 9]