

February 21, 2005

N15 Experiment

How To Find Average Time of Decay Within a Bin

The midpoint of the bin is $\frac{t_2+t_1}{2}$. This is the intuitive choice for where to plot the counting rate as derived from your data.

t_1 is the time at which the previous gate ends (or the dead time ends, in the case of gate # 1) and t_2 is the time at which the gate of interest ends.

So, for example, the counts you see on the scaler # 3 should be:

$$\text{counts in scalar \#2} + \text{counts between } t_1 \text{ and } t_2$$

This is because the gates overlap in the way they do, each gate being $t_2 - t_1 = \Delta t$ longer than the one before it. (The value of Δt depends on the gate. They are not all exactly equal.)

The choice of the midpoint is not, strictly speaking exactly correct, since the muon exponential decays tend to weight the decay time slightly closer to the start of the gate, t_1 than the midpoint. How large is the correction? Calculate $\bar{t} \equiv \frac{\int_{t_1}^{t_2} t e^{-\lambda t} dt}{\int_{t_1}^{t_2} e^{-\lambda t} dt}$ a weighted average. The integrals both go from t_1 to t_2 . *Poisson distrib.*

The exact answer is easy to find, but too messy to be of much practical use:

$$\bar{t} = \frac{e^{\lambda t_1}(1 + \lambda t_2) - e^{\lambda t_2}(1 + \lambda t_1)}{\lambda(e^{\lambda t_1} - e^{\lambda t_2})}$$

Since the bin width $\Delta t \equiv t_2 - t_1$ is short compared to the muon lifetime, we expect the correction to the midpoint time to be small. If the muon had a very long lifetime, $\lambda \rightarrow 0$, and we expect the correction would go to zero. Therefore, expand in a power series in λ :

$$\bar{t} = \frac{(t_1 + t_2)}{2} - \frac{\lambda}{12}(\Delta t)^2 + O[\lambda^3]$$