February 21, 2005

N15 Experiment

How To Find Average Time of Decay Within a Bin

The midpoint of the bin is $\frac{t_2+t_1}{2}$. This is the intuitive choice for where to plot the counting rate as derived from your data.

t_1 is the time at which the previous gate ends (or the dead time ends, in the case of gate # 1) and t_2 is the time at which the gate of interest ends.

So, for example, the counts you see on the scaler # 3 should be:

counts in scalar #2 + counts between t_1 and t_2

This is because the gates overlap in the way they do, each gate being $t_2 - t_1 = \Delta t$ longer than the one before it. (The value of Δt depends on the gate. They are not all exactly equal.)

The choice of the midpoint is not, strictly speaking exactly correct, since the muon exponential decays tend to weight the decay time slightly closer to the start of the gate, t_1 than the midpoint. How large is the correction? Calculate $\bar{t} \equiv \frac{\int t\, e^{-\lambda t}\, dt}{\int e^{-\lambda t}\, dt}$, a weighted average. The integrals both go from t_1 to t_2.

Poisson distrib.

The exact answer is easy to find, but too messy to be of much practical use:

$$\bar{t} = \frac{e^{\lambda t_1}(1+\lambda t_2) - e^{\lambda t_2}(1+\lambda t_1)}{\lambda(e^{\lambda t_1} - e^{\lambda t_2})}$$

Since the bin width $\Delta t \equiv t_2 - t_1$ is short compared to the muon lifetime, we expect the correction to the midpoint time to be small. If the muon had a very long lifetime, $\lambda -> 0$, and we expect the correction would go to zero. Therefore, expand in a power series in λ:

$$\bar{t} = \frac{(t_1+t_2)}{2} - \frac{\lambda}{12}(\Delta t)^2 + O[\lambda^3]$$

February 27, 2005 L.Hand

Muon Capture Correction To Lifetime in N15 and N17

Since approximately half of the muons decaying in the apparatus are negative muons, there will need to be a correction for the fact that the negative muons are disappearing slightly faster than the positive muons, hence a measured muon lifetime in which only a single exponential is fit to the data will appear slightly shorter than the published vacuum lifetimes [1], (These are equal for both types of muons–particle and anti-particle– due to a fundamental theorem of physics.)

You can use published values for the negative muon capture rate in carbon, and for the +/− ratio. Our experiments are both so accurate that a correction is necessary. A separate error should be quoted for the error in these experimentally determined quantities. It should not be combined with the statistical error from your fit to the data.

However, the method for making the correction will be somewhat different depending on whether the experiment is N15 or N17.

N-15 Experiment

You should fit the data to a single exponential (after subtracting calculated background from accidental coincidences). This will give a "$\lambda_{effective}$", an effective decay rate which includes both signs of muon charge. To make the correction, assume the experiment is averaging over two nearly equal exponentials, and that

$$\lambda_{effective} = \lambda_{measured} = \text{weighted average of } \lambda_{vacuum} \text{ and } \lambda_{vacuum} + \Lambda_{capture}$$

If the plus/minus ratio, $\frac{N_+}{N_-} \equiv r$, you should prove to yourself that the number you are trying to measure, λ_{vacuum}, is

$$\lambda_{vacuum} = \lambda_{measured} - \frac{\Lambda}{1+r}$$

you can use *all* of the information from the data to fit N_0, λ, B. (B is the background rate/bin.) You do not have to make the ad hoc assumption that the measured rate is the average of the two rates. The fit will take care of all that. And, what is more, you do not have to rely on a calculation of B, although you should calculate it as a check.

If you don't understand where the formula above comes from, or how to use Poisson statistics in the maximum likelihood calculation, ask your instructor to explain. Also, consult the Orear notes.

REFERENCES

[1] The best value of the muon lifetime from the world data (1998 Particle Data Book) is:

$$\tau = 2.19703 \pm .00004 \text{ microseconds}$$

. Thus $\lambda_{vac} = 4.55160 \pm .00008 \times 10^5 \text{ sec}^{-1}$.

[2] N.C. Mukhopadhyay, et al, Physics Letters B, v434, n 1-2, 20 Aug. 1998, p7-13

[3] Thanks to Jonathan Blender of P410 for supplying these numbers, and for a lively discussion of how to treat them in our special case.