

G. S. Monk Mc Graw Hill 1937

having sets of values associated with the energy levels into which any no-field level is split. With the aid of selection rules which are somewhat complicated the values of these new energy levels may be calculated and an energy-level diagram prepared.

It should be pointed out that the patterns of components indicated in such a diagram and in those illustrating the Zeeman effect are only those due to the imposition of the external fields. Actually, superposed upon a complex line structure due to an external field are complex structures due to other causes, such as the presence of more than one isotope of an element in the source, the effect of the spin of the atomic nucleus on some axis, and in the case of very light atoms, the multiplets themselves. The disentanglement of the fine structure in the spectral lines due to these various causes calls for apparatus of the highest resolving power and technical skill of the highest order.

6. The Faraday Effect.—Faraday discovered in 1845 that when a transparent isotropic medium is in a strong magnetic field, there is a rotation of the plane of vibration of light which is transmitted through the medium in the direction of the field. Unlike the rotatory effect in quartz and other optically active substances, the direction of rotation depends on whether the light traverses the medium in the direction of the lines of force, i.e., from *N* to *S* pole, or oppositely. Hence on reflection back through a medium in a strong magnetic field, the effect is doubled, while the rotation due to optically active substances is canceled upon reflection of the light back on its path.

The effect may be produced with a solenoid of sufficient strength, the substance being placed along its axis as shown in Fig. 16-13. When the nicols are crossed without a field, no light will be seen, but the light will reappear when the field is on. Turning of the analyzer to the new point of extinction gives the angle through which the plane of vibration has been rotated. The direction of rotation does not depend upon whether the substance is paramagnetic or diamagnetic. A rotation is known as *positive* if it is in the same direction as the flow of current¹ producing the field, and *negative* if in the opposite direction. That is, it is positive if it appears clockwise to an observer looking in the direction of the lines of force.

The rotatory effect in naturally optically active substances such as quartz may be explained by considering the transmitted plane

vibration to be the resultant of two opposite circular vibrations, one propagated through the medium faster than the other (see Sec. 13-20). In order to apply this explanation to the Faraday effect it is necessary to assume (a) that the same sort of resolution of the plane vibration exists for an isotropic medium in a magnetic field, and (b) that the velocity of propagation of a circular vibration depends on the *direction* of the vibration, i.e., whether it is clockwise or counterclockwise, as well as upon the index of refraction of the medium. The second of these assumptions was shown to be true by Righi and Becquerel working independently. The first was proved correct by Brace.² In 1904, the velocities of the two circularly polarized components were measured by Mills,³ who found that the ray traveling faster was the one for

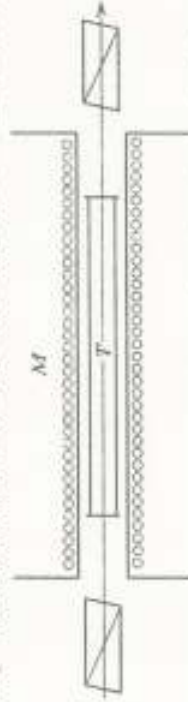


FIG. 16-13.—*T* is the tube, containing the liquid, on the axis of a solenoid magnet *M*.

which the direction of the circular vibration was the same as that of the current producing the field.

For a given temperature and wave-length the angle of rotation by a field is given in minutes of arc by

$$\theta = RHl,$$

where *l* is the light path in centimeters in the medium and *H* is the magnetic field strength in gauss. The constant *R* is known as Verdet's constant. If its value is known accurately for specified conditions, the Faraday effect may be used to calibrate a magnetic field in terms of current strength.

The magnitude of the Faraday effect for different substances is shown in Table 16-1, the values being given for the sodium lines and for room temperature. The dependence upon temperature is slight, being given for carbon disulphide as

$$R = 0.04347(1 - 1.69 \times 10^{-3} \cdot t).$$

¹ Flow of current is here used in the older Ausperian sense, and not in the sense of direction of electron flow.

² *Philosophical Magazine*, 1, 464, 1901.

³ *Physical Review*, 18, 65, 1904.

Substance	Verdet's Constant, Minutes of Arc for the D-lines	
	D-lines	
Crown glass.....	0.0203	
Heavy flint glass.....	0.0647	
Ethyl alcohol.....	0.0107	
Benzene.....	0.0297	
Carbon disulphide (liquid).....	0.0441	
Water.....	0.0259	
Atmosphere.....	6.83×10^{-4}	
Carbon dioxide.....	13.00×10^{-4}	

Kundt¹ found that very large rotations were produced by thin semitransparent films of ferrous metals deposited on glass. These films, only a fraction of a wave-length of light in thickness, were deposited electrolytically on platinized glass plates. Kundt found that the rotation in an iron film 5.5×10^{-6} cm. thick was as much as $1^\circ 48'$, over 30,000 times that for an equal thickness of glass.

The Faraday effect in solutions is not proportional to the concentration, since the interactions of the molecules must be taken into account.

The rotation due to a magnetic field may be expressed in terms of e/m , the ratio of the charge on the electron to its mass, in a relationship originally deduced empirically by Becquerel. The existence of two velocities, and hence of two indices of refraction, for the oppositely rotating circular components is thought of as being due to the addition or subtraction of a small precession angular velocity ω_p to the large angular velocity ω of the components. This precession is due to the field's effect on the orbital motion of the electrons. Since $\omega = 2\pi\nu$, it follows from eq. 16-1 that

$$\omega_p = \frac{e}{2mc} \cdot H \quad (\text{radians per sec.}) \quad (16-6)$$

where e is in e.s.u. Also

$$\omega = \frac{2\pi c}{\lambda} \quad (16-7)$$

The time required for the faster component of velocity ω_+ to transverse a centimeter of the medium is $1/\omega_+$, while that for the

¹ *Philosophical Magazine*, 18, 308, 1884.

slower component is $1/\omega_-$. Hence the angle in radians through which the plane of vibration is turned is

$$\theta = \frac{1}{2}\omega \left(\frac{1}{\omega_+} - \frac{1}{\omega_-} \right).$$

But

$$\omega_+ = \omega + \omega_p \frac{dv}{d\omega} \quad \text{and} \quad \omega_- = \omega - \omega_p \frac{dv}{d\omega}$$

so that

$$\theta = \frac{1}{2}\omega \left(\frac{1}{\omega + \omega_p \frac{dv}{d\omega}} - \frac{1}{\omega - \omega_p \frac{dv}{d\omega}} \right),$$

which reduces to

$$\theta = \frac{1}{2}\omega \left(\frac{-2\omega_p \frac{dv}{d\omega}}{\omega^2 - \omega_p^2 (dv/d\omega)^2} \right).$$

Since both ω and $dv/d\omega$ are very small compared to v , very approximately

$$\theta = -\frac{\omega\omega_p}{v^2} \frac{dv}{d\omega} \quad (16-8)$$

Substituting in this the values of ω , ω_p , and $dv/d\omega$ obtained from eqs. 16-6 and 16-7, and the values of v and dv from $n = c/v$, where n is the index of refraction, it follows that

$$\theta = -\frac{e}{2mc^2} \cdot H \cdot \frac{\lambda dn}{d\lambda} \quad (16-9)$$

radians per centimeter of thickness.¹

7. The Kerr Magneto-optical Effect.—In Sec. 13-14 it was stated that in ordinary metallic reflection incident plane-polarized light becomes elliptically polarized, except (a) at normal incidence, and (b) when the plane of vibration of the incident light is in or perpendicular to the plane of incidence. Kerr² found that if the reflection takes place at the polished pole face of a strong electromagnet, elliptical polarization results even

¹ For this derivation see Larmor, "Aether and Matter," Cambridge University Press; also Campbell, "Modern Electrical Theory," Cambridge Physical Series.

² *Philosophical Magazine*, 3, 321, 1877; 5, 161, 1878.

in cases (a) and (b) above. He found no effect when the wave front of the incident light is parallel to the direction of the magnetic field. The ellipticity is small, so that the effect may be considered for simplicity merely as a rotation of the plane of vibration. The amount of this rotation is found to be proportional to the intensity of magnetization \mathcal{M} , that is,

$$\theta = K\mathcal{M},$$

where K is the *magnetic Kerr constant*.

A method, due to duBois, combining the Faraday and Kerr effects, makes possible an accurate measurement of strong mag-

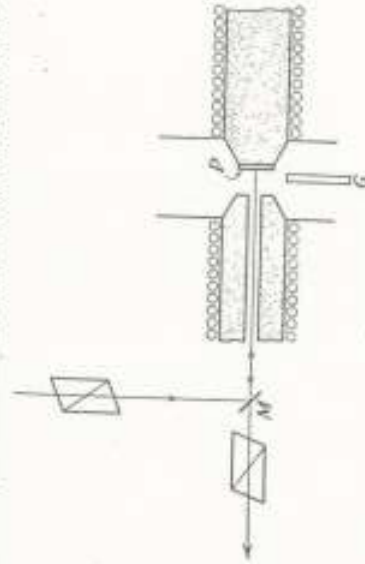


FIG. 16-14.

netic fields and great magnetizations. The arrangement of the apparatus is shown in Fig. 16-14, in which P is the polished pole piece at which the Kerr effect is measured, giving the magnetization \mathcal{M} . To obtain the field strength, a plate G of glass for which the Verdet constant is known is then inserted. This plate is silvered on the side farthest from the source so that the light is reflected back, doubling the rotation of the plane of vibration. The angle of rotation is measured, and H is calculated.

8. **The Kerr Electro-optical Effect.**—Faraday long sought for an electrical rotatory effect corresponding to the Faraday effect due to magnetic fields, but with no success. It was first observed by Kerr,¹ who discovered that an isotropic dielectric medium placed in a strong electric field acquires the properties of double refraction of a uniaxial crystal with its optic axis in the direction of the lines of force. Some substances behave like positive and

¹ *Philosophical Magazine*, 4, 337, 1875, also papers in subsequent issues.

others like negative crystals. Kerr first obtained results with a block of glass into which two holes were drilled from opposite sides, ending about $\frac{1}{4}$ in. apart. The terminals of a powerful induction coil were placed in these. Before the coil was put in action, the light transmitted at right angles through the gap between the terminals was extinguished with a pair of crossed nicols, one on either side of the block. After the switch was closed, the light reappeared. It could not be entirely extinguished by rotating the analyzer and hence was elliptically polarized.

In this experiment the light does not appear as soon as the exciting electrical field is turned on, but takes about 30 sec. to reach its maximum intensity; similarly, it takes about the same time to fade when the field is removed. The effect is greatest when the principal section of the polarizer makes an angle of 45 deg. with the direction of the field, and zero when it is parallel or perpendicular. Care must be taken in drilling the holes and fitting the terminals to avoid any mechanical strains in the glass, as these restrict the region in which the effect is observed, and may even mask it altogether.

It is not clear from this experiment alone that the effect is the production of an anisotropic condition by the electrical stress, since the lag in the appearance of the light might also indicate a temperature effect because of the slight conductivity of the glass. Kerr believed that it was really due to a rearrangement of the molecules by the electrical stress, the delay being because of opposing frictional force. That the effect may be compensated by placing in the light path between block and analyzer a thin strip of glass, compressed or stretched in the direction of the field, is taken as an indication that the glass truly takes on the characteristics of a uniaxial crystal. The double refraction exhibited by glass is like that of a *negative* crystal and is the same whether alternating or direct current is used, the effect being proportional to the *square* of the field strength.

In liquids the rearrangement of the molecules is more rapid, permitting the use of the Kerr effect for many important experiments, some of which will be discussed later. The liquid is placed in a glass vessel with flat ends and a pair of metal plates, arranged like parallel condenser plates, as shown in Fig. 16-15. It is extremely important that the windows, or ends,

through which the light passes should be of optical glass free from strains and with flat surfaces. The vessel with liquid and plates installed is called a *Kerr cell*. When an electric field is applied to the plates, the liquid becomes double-refracting. Since the light transmitted is elliptically polarized, it may be analyzed by means of a Babinet compensator (see Sec. 13-12), placed between the cell and the analyzer. Or instead, a Wollaston prism may be used (see Sec. 13-10), in which case the analyzer should be oriented so that the two images are of equal intensity.

Oils in general show negative, while carbon disulphide, molten sulphur, bromine, and many other substances show positive

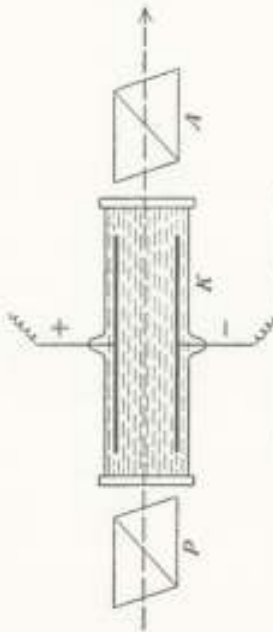


FIG. 16-15.—The Kerr cell.

double refraction. The strength of the effect in any substance is given by its *electrical Kerr constant* C_e . This is expressed in terms of the relative retardation in optical path produced between the components of the elliptical vibration parallel and perpendicular to the field, for unit thickness of the dielectric. Let δ be this retardation, then

$$\frac{\delta}{\lambda} = \frac{l(n_e - n_o)}{\lambda} = C_e E^2, \quad (16-10)$$

where l is the thickness in centimeters and E is the field strength in e.s.u. The effect also depends upon the temperature of the substance. The value of C_e for carbon bisulphide at 20°C. for $\lambda = 5.893 \times 10^{-8}$ cm. is 3.22×10^{-7} . Values for several other substances are given in Table 16-2 at the end of the following section.

As the table shows, gases exhibit a Kerr electric effect which is about $1/1000$ that for ordinary liquids. In gases the value of C_e ,

which depends upon the temperature and pressure, can be represented as the sum of a number of terms

$$C_e = k_1 + k_2 + k_3 \dots,$$

in which k_1 is proportional to $1/RT$, k_2 to $1/RT^2$, etc., where R is the gas constant and T is the absolute temperature. The values of k are positive or negative for different gases, as in the case of liquids, but the sign is sometimes a function of the temperature.

9. The Cotton-Mouton Effect.—Cotton and Mouton found that the same type of double refraction observed in the Kerr electro-optical effect could be produced in a dielectric when it was placed in a *magnetic field*. By an equation analogous to eq. 16-10, the retardation in path, δ , is given by

$$\frac{\delta}{\lambda} = \frac{l(n_e - n_o)}{\lambda} = C_m H^2, \quad (16-11)$$

in which C_m is the Cotton-Mouton magnetic birefringence constant, and H is the magnetic field strength in gauss. This effect should be distinguished from the magnetic Kerr effect discussed in Sec. 16-7, which is a rotation of the plane of polarization analogous to the Faraday effect. The Cotton-Mouton

The electric birefringence $\gamma_e = 100(C/C_{CS_2})_e$ (5893Å)

The magnetic birefringence $\gamma_m = 100(C/C_{C_6H_6})_m$ (5780Å)

Substance	Symbol	γ_e	γ_m
Liquids:			
Carbon disulphide.....	CS ₂	+ 100.0	- 19.6
Carbon tetrachloride.....	CCl ₄	+ 2.3	- 2.8
Chloroform.....	CHCl ₃	- 100.2	+ 23.3
Benzene.....	C ₆ H ₆	+ 12.0	0
Ethyl alcohol.....	C ₂ H ₅ OH	+ 23.8	+ 1.6
Acetone.....	C ₃ H ₆ O	+ 505.0	+ 100.0
Nitrobenzene.....	C ₆ H ₅ NO ₂	+ 10,670.0	+ 77.0
Nitrotoluene.....	C ₇ H ₇ NO ₂	+ 5,500.0	+ 2.5
Nitric acid.....	HNO ₃	-	-
Water.....	H ₂ O	+ 123.0	-
Gases:			
Sulphur dioxide.....	SO ₂	+ 0.051	-
Ammonia.....	NH ₃	+ 0.018	-
Carbon dioxide.....	CO ₂	+ 0.007	-

effect is analogous to the Kerr electro-optical effect in the sense that both are the production of artificial double refraction in media. The former is sometimes called *magnetic birefringence* and the latter *electric birefringence*. In Table 16-2 are given the values of the electric and magnetic birefringences for several substances. In accordance with custom, the electric effect is given in terms of carbon disulphide (CS_2) and the magnetic effect in terms of nitrobenzene ($\text{C}_6\text{H}_5\text{NO}_2$). The value of C_K for nitrobenzene at 20°C . for $\lambda = 5.780 \times 10^{-5}$ cm. is 2.46×10^{-12} .

10. Measurement of Time Intervals with Kerr Cells.—Abraham and Lemoine devised a method for measuring very short time intervals, of the order of magnitude of 10^{-9} sec., with Kerr cells. This is done by using as the source of the trans-

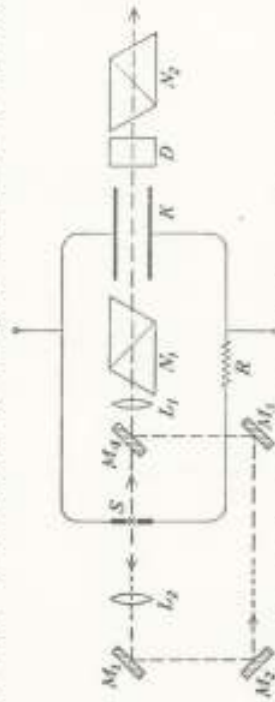


FIG. 16-16.—The apparatus of Abraham and Lemoine.

mitted light a spark discharge which is in series in the same circuit which actuates the cell. A diagram of the apparatus is shown in Fig. 16-16. The light from the spark can be passed directly into the cell through the lens L_1 , or it may be passed over a longer path by reflection at mirrors M_1 , M_2 , M_3 , and M_4 . In the latter case, one or more auxiliary lenses are used, these being indicated in the figure by L_2 , in order to procure a beam of the proper intensity and divergence at the Kerr cell K . The polarizing nicol N_1 and the analyzing nicol N_2 are at 90° deg. with each other and at 45° deg. to the field of the Kerr cell. A double-image prism D is so oriented that the two images may be made of equal intensity with their vibrations in perpendicular planes. The analyzer may be set for extinction, or, with D in place, for equality of intensity, by means of an auxiliary source placed before N_1 . This source is then removed, and the spark discharge S started. The discharge is oscillatory, giving rise to an intermittent spark. If the light from a single flash of the

spark passes through the Kerr cell while the latter is still actuated, it will be necessary to turn the analyzer through an appreciable angle to bring about equality in the intensities of the two images. If, however, the light is delayed in its passage to the cell, the field will have diminished and the angle through which the analyzer is turned will be smaller. If very long light paths are introduced, the cell will be completely restored to the isotropic state before the light reaches it, and no change in angle will be required.

It is possible to dispense with the double-image prism, and measure simply the angle through which N_2 must be rotated in order to reduce the intensity of the transmitted elliptically

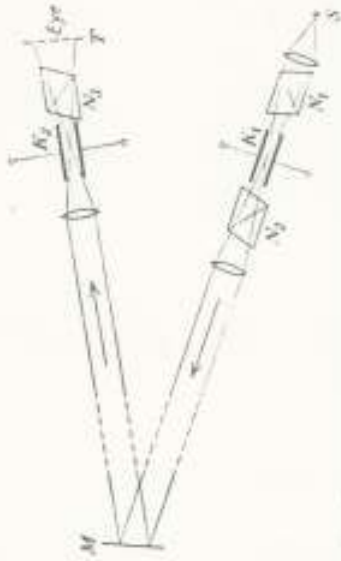


FIG. 16-17.—A schematic outline of the apparatus of Karlovic and Mittelstaedt. The letter M represents a system of mirrors.

polarized light to a minimum. This is not as exact a method as matching the intensities of the two images given by the double-image prism.

One of the factors which must be considered in experiments with Kerr cells is the delay in the establishment of the anisotropic state after the cell is charged, and the delay in the decay of the field after the charge is removed. In liquids, these times are very small, of the order of about 10^{-9} sec. The lag in the Kerr effect has been measured with considerable precision in recent years.¹

11. Velocity of Light with Kerr Cells.—It is possible to obtain light flashes of extremely short duration by the use of two Kerr cells in series. Such an apparatus has been used by Karlovic

¹ See, for instance, an article by Beams and Lawrence, *Proceedings National Academy of Sciences*, **13**, 505, 1927.

and Mittelstaedt for measuring the velocity of light.¹ The arrangement of the apparatus is shown in Fig. 16-17. The light from S was polarized by a Nicol N_1 , in a plane of vibration at 45 deg. to the field of K_1 . When K_1 was actuated, the elliptically polarized light passed through a second Nicol N_2 , whose plane of transmission was at 90 deg. to N_1 . After reflection back and forth from several mirrors, the light then passed to a second Kerr cell K_2 , and a third Nicol N_3 . The total path was 332 m. The two Kerr cells cut off the beam at intervals which could be varied. As the result of 755 measurements, a value of the velocity of light of 299,778 km./sec. \pm 20 km./sec. was reported, in good agreement with the most recent values obtained by Michelson and his associates.²

Problems

1. If the illustration of the Zeeman effect given in Fig. 16-6 has been magnified eight times from the original photograph of the spectrum, on which the dispersion was 1.1 angstrom per millimeter, what was the strength of the magnetic field used?
2. Will the normal longitudinal Zeeman effect, produced in a field of 10,000 gauss, be observable as a distinct separation of components with a spectrometer using a grating 7.5 cm. long and having 6,000 rulings per centimeter? The perpendicular effect? Assume actual resolving power about 60 per cent of theoretical, and specify the order used.
3. Other things being equal, should lines in the violet or in the red be examined in order to observe the Zeeman effect? Explain your answer.
4. The indices of refraction of a certain block of glass 10 cm. long are, for the D -lines, 1.5688; for the C -line, 1.5657; and for $H\gamma$ -5401, 1.57125. When placed in a magnetic field, the plane of vibration of a beam of plane-polarized light of the sodium lines is rotated through 10 deg. What is the strength of the field? What is the Verdet constant for the glass?

¹ *Physikalische Zeitschrift*, **29**, 698, 1928.

² Since this book went to press, an article by W. C. Anderson has appeared (*Review of Scientific Instruments*, **8**, 239, 1937) describing a modified apparatus for measuring c , using a relatively short base line, a single Kerr cell, and a photoelectric cell for the detection of the light beam modified by the Kerr cell.

CHAPTER XVII

THE EYE AND COLOR VISION

The beginner or casual worker in the field of light is likely to overlook the importance of the eye in visual observations. It is important (a) because of considerations of purely geometrical optics, including defects of image formation; (b) because it has certain characteristics which may be classified as psychophysiological, such as susceptibility to illusions, color vision, and

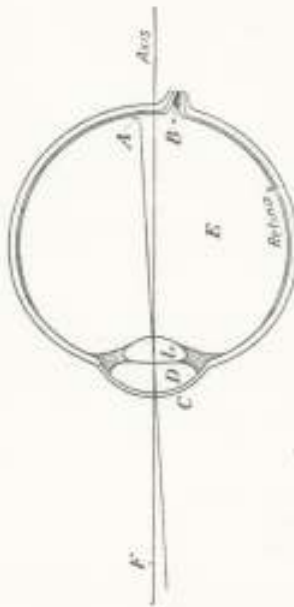


FIG. 17-1.—The schematic eye. A , fovea; B , blind spot; C , cornea; D , aqueous humor, index = 1.3365; E , crystalline lens, index = 1.4371; F , vitreous humor, index = 1.3365; G , principal focal point. Radii of curvature of cornea, 7.829 mm.; of front of lens, 10,000 mm.; of rear of lens, -6,000 mm.; distance between cornea and lens = 3.6 mm.; distance between surfaces of lens = 3.6 mm.

difference in degree of "normality." Because of these, modification of observed phenomena is possible, and ignorance of this modification may lead the observer to false conclusions. Optical experiments, especially those involving visual photometry and color, should not be undertaken without some understanding of the functions of the human eye.

1. The Optical System of the Eye.—The essential optical features are illustrated in Fig. 17-1. The meanings of the letters are given in the legend. The surfaces here represented are not such definite boundaries between media as in ordinary optical systems. Neither are the media themselves entirely homogeneous, the crystalline lens especially being composed of "shells"