

of all photographic processes. In certain cases the decomposition may be only started, a molecular instability being imparted by the action of light, the continuation of the process being effected by reducing agents (developers). Another interesting example of photo-chemical decomposition has been pointed out by Tyndall. The vapor of amyl nitrite in a glass flask is colorless and transparent until illuminated by a powerful beam of sun- or arc-light, when a dense white cloud at once forms, the products of decomposition condensing to liquid drops which scatter the light.

A remarkable transformation of luminous into chemical energy on a vast scale is the breaking up of the carbonic acid of the atmosphere, which takes place in the leaves of plants. Some of the oxygen is liberated in the free state, and some enters into the organic compounds which the plant forms, by the subsequent combustion of which we may recover the original energy of the light in the forms of both heat and light.

If ultra-violet rays are greatly in excess other very remarkable transformations are produced. Oxygen is changed into ozone, as we have seen. Permanganate of potash solutions are almost instantly bleached, and many other chemical substances break down, which under ordinary conditions are stable.

Chemical Combination.—Examples of chemical combination resulting from the action of light are not as common. The best-known case is the union of chlorine and hydrogen to form hydrochloric acid. The mixture of the two gases is best obtained by the decomposition of strong hydrochloric acid by electricity, the operation being conducted by feeble lamplight. The gas which comes off first contains an excess of hydrogen, owing to a solution of a part of the chlorine in the acid. Subsequent portions may be collected in small glass bulbs, blown in strings and separated by fine thin-walled tubes, which are broken and closed with warm sealing wax, care being taken not to have the wax on fire. The bulbs should be kept in a dark box until wanted. On exposing one to sunlight or the light of burning magnesium, or the electric arc, a violent explosion is said to occur. Precautions should of course be taken to avoid injury from the flying fragments of glass.

Transformation of Luminous into Electrical Energy.—Cases of this sort are worthy of mention, though in none of them is the transformation direct. In the thermopile, when illuminated by light, we have a liberation of electrical energy, which is, however, due only to the heating action of the radiation. In the photo-electric cell, which consists of two silver plates coated with silver chlorid and immersed in dilute acid, a feeble current flows through a wire joining them, when one of them is illuminated by light. In this case the action of the light is primarily chemical, the current being the result of the chemical decomposition. The case is somewhat analogous to phosphorescence, the difference being that here the reversion of the process set up by the light liberates electricity instead of light.

CHAPTER XXI

LAWS OF RADIATION

In the present chapter we shall discuss the laws which govern the emission of light by bodies in virtue of their temperature. Of the physical processes which are at work we know but little. In the heated body we believe that the molecules are in a rapid vibratory motion, which increases in violence as the temperature is raised, but the precise mechanism by which this energy is transformed into radiant energy is but imperfectly understood. The simplest assumption appears to be that the molecular collisions in some way throw the electrons into vibration, or increase the amplitude of their vibrations, and that these radiate energy into space. The same thing does not, however, occur in the case of a gas, for the same molecules can be heated to a much higher temperature—several thousand degrees even—without emitting light. Mercury may be heated "white-hot" in a sealed quartz tube, but mercury vapor can be heated to the highest temperatures at our command without emitting any visible light. It would be extremely interesting to study the emission of light by some fluid which had a critical temperature of about 700°, observing the luminosity as the liquid passed over into the gaseous state. The phenomenon might be studied in a quartz tube (which emits but little light, for reasons which will appear presently) if a suitable substance could be found. Strutt has experimented with mercury with a view of measuring the electrical conductivity of the vapor at the critical temperature, but even thick-walled capillaries of fused quartz exploded, or yielded to the pressure, before any evidence of the approach of the critical state appeared.

Nothing is to be gained by increasing the thickness of the walls beyond a certain point, since the inner layers give way or "tear" before the strain reaches the outer layer. Possibly the thing could be accomplished by enclosing the tube in an outer tube containing air at a pressure of several hundred atmospheres. This principle is the one used in the wire-wound guns.

The electrons of the gas molecules can be made to emit light by the stimulus of electrical discharges, or chemical changes, and in one or two cases (iodine and sodium) by virtue of temperature alone.

All substances in the *solid* or *liquid* state emit light as soon as their temperature is raised above 500° C. The intensity of the light varies, however, with the nature of the substance, being greatest for substances which absorb light strongly. A *perfectly* transparent solid or liquid would not emit light even at the highest temperature. No such substance is known, however, though a beal of microcosmic

salt, heated in a loop of platinum wire by means of a blast lamp, comes pretty near to fulfilling the required conditions. We shall presently investigate the laws which govern the emission of light by various substances, and in particular the emission of light by which is perfectly black, *i.e.* perfectly absorbing. No substance has this property, though by an experimental artifice we can produce a radiator which will give out radiation identical in every respect with the radiation which would be given out by a perfectly black body at the same temperature.

The Relation between Emission and Absorption.—We will now investigate the very intimate relation which exists between the emission of heat or light waves by a substance, and its power of absorbing the same waves.

That some relation existed between the emissive and absorbing power in the case of radiant heat was indicated by the experiments of Lœlie, Melloni, Provostaye and Desains, and others. Their measurements were, however, made for the most part with apparatus of insufficient sensitiveness, the spectrum regions being only roughly determined by means of absorbing screens. It was determined, however, without question, that bodies which possessed a strong emissivity acted also as powerful absorbers of the radiant heat which they emitted, and the approximate equality of the emitting and absorbing powers was recognized.

Ritchie's Experiment.—This relation was shown by a very simple and ingenious experiment devised by Ritchie (*Pogg. Ann.*, 28, p. 378, 1833). Two air-tight metal chambers were connected by a glass tube containing a drop of fluid, the whole forming an air thermometer. Between them a third metal chamber of the same size was mounted, which could be heated by filling it with boiling water. One surface of this heat radiator was covered with lampblack, the other with the substance under investigation, for example powdered cinnabar. (See Fig. 360.) The surface of the air thermometer which faced the radiating lampblack surface was coated with cinnabar, while the surface which faced the radiator was coated with lampblack. With the apparatus arranged in this way no movement of the fluid drop occurred when boiling water was poured into the radiator, which established the fact that the emitting and absorbing powers were equal. Let us assume that the lampblack radiates powerfully, the cinnabar feebly. The powerful radiations coming from the former are but slightly absorbed by the latter, while the feeble radiations from the latter are strongly absorbed by the former, the heating of the two chambers of the thermometer being equal. Calling E the amount of heat emitted by the cinnabar and A its absorbing power, e and a the corresponding expressions for the lampblack, the lampblack surface emits an amount of radiant heat e , of which the cinnabar surface absorbs the amount eA . The cinnabar surface emits an

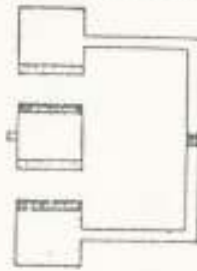


Fig. 360.

cinnabar radiator was coated with lampblack. With the apparatus arranged in this way no movement of the fluid drop occurred when boiling water was poured into the radiator, which established the fact that the emitting and absorbing powers were equal. Let us assume that the lampblack radiates powerfully, the cinnabar feebly. The powerful radiations coming from the former are but slightly absorbed by the latter, while the feeble radiations from the latter are strongly absorbed by the former, the heating of the two chambers of the thermometer being equal. Calling E the amount of heat emitted by the cinnabar and A its absorbing power, e and a the corresponding expressions for the lampblack, the lampblack surface emits an amount of radiant heat e , of which the cinnabar surface absorbs the amount eA . The cinnabar surface emits an

amount E , which the lampblack completely absorbs (since its absorbing power = 1). The equality of temperature indicated by the thermometer shows us that $eA = E$ or $\frac{E}{e} = A$. Now $\frac{E}{e}$ is the ratio of the amount of heat emitted by cinnabar to the amount emitted by lampblack at the same temperature. This ratio we will call the emissivity of the cinnabar, and our equation shows us that it is equal to the absorbing power.

Kirchhoff's Law.—This relation was reduced to a more definite form in 1859 by Kirchhoff, and independently by Ralfour Stewart, who showed that it must be true for each wave-length in the emitted spectrum, and formulated the law which has since gone by his name. "At a given temperature the ratio between the emissive and absorptive power for a given wave-length is the same for all bodies." The theoretical considerations from which Kirchhoff's law is deduced will be discussed later on in the chapter. It may be remarked that we frequently meet with the statement that the absorption of light by flames which contain the vapors of metals is a necessary consequence of this law. This is by no means the case, for Kirchhoff's law is only to be applied to radiation which results from temperature. In the case of the emission by flames, the phenomena are probably connected with chemical changes which are taking place.

Cotton has called attention to the fact that there are two distinct relations which are almost invariably confused: a qualitative rule, which connects the absorption and emission for a given substance, and a quantitative rule, which establishes relations between different bodies. From the former we can only draw the conclusion that if a body emits certain radiations it absorbs them when they come from without. It may, however, absorb other radiations, as is illustrated by the selective absorption of colored substances. For these radiations $\frac{E}{A} = 0$, since at the temperature in question there is no emission of visible radiations.

Absorption by Flames.—The absorption by flames and the reversal of spectral lines are special cases of the qualitative rule. These cases have been studied by Gouy, who sought to determine by experiment whether flames were transparent to the radiations which they emitted. His method consisted in comparing photometrically the light of different thicknesses of radiating gas. If no absorption occurred, doubling the thickness should double the intensity of the illumination. He found, in the case of every line examined, that after the line had attained a certain brightness absorption manifested itself. If E and A be the emissive and absorptive powers of unit thickness of the flame and we increase the thickness, we shall find that the emissive power (*i.e.* the intensity sent out by the thick layer) approaches $\frac{E}{A}$ as a limit.

This can be readily seen by calculating the amount of light sent out in the direction B by a flame of thickness 6 (Fig. 361).

Assume unit thickness to emit light of intensity 100 ($E = 100$) and to have the power of absorbing one-half, of the light of the same wave-length, which traverses it. The intensity of the light

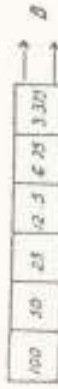


FIG. 361.

from the element 1 is reduced to 50 by the absorption of element 2, and on emergence from element 6 has an intensity of only 3.37. The total amount of light emerging from element 6 is obviously the sum of the numbers in the different elements, which in this case is 197.62. If the number of elements were increased indefinitely this sum would be 200, the limiting intensity which is equal to $\frac{E}{A}$, as defined above.

It should be possible therefore to reverse all lines by a sufficient increase in the thickness of the flame, for the line will appear darker than the background of the continuous spectrum when the intensity of the light of the given wave-length emitted by the flame is less than the intensity of the continuous spectrum at the point in question. By increasing the thickness we progressively increase the absorption, while the intensity of the flame remains practically constant after a certain thickness is reached.

It by no means follows, however, that all vapors which emit radiations of definite wave-length will show the corresponding wavelengths reversed, even when a very thick layer is used. The above argument is based on the fact that there is a certain absorbing power. If $A = 0$ reversal can never occur. Now it is a noteworthy fact that reversals are only obtained in the case of the bright lines in metallic spectra. The lines in the spectra of the metalloids have never been reversed. This shows that Kirchhoff's law does not even apply qualitatively in these cases; in other words, there is powerful selective emission without any corresponding selective absorption. In such cases $\frac{E}{A}$ is infinite. Summing up, we find that for colored substances, absorbing without a corresponding emission, $\frac{E}{A} = 0$, for flames which show reversal of spectrum lines

$\frac{E}{A}$ is finite, for luminous gases which show no absorption $\frac{E}{A} = \infty$.

Temperature Radiation of Gases. — Kirchhoff's law states that in the case of radiation which results solely from temperature, $\frac{E}{A} = \text{const.}$ for all bodies at the same temperature. The value of the constant is a function both of the temperature and the wave-length, and is equal to the emissive power of a perfectly absorbing body, that is, a body which at the temperature in question completely absorbs, without reflection, all radiation falling up, no matter what its wave-length. This amounts to saying that at a given temperature no substance can emit more light of a given wave-length than a perfectly black body. Paschen has compared the intensity of

the light of the two D lines in the sodium flame with the total intensity of a region, completely enclosing the D lines, in the continuous spectrum of a black substance heated in the same flame. The total intensity of the D radiation was more than twice as great as that of the region of the continuous spectrum which enclosed them, from which the inference can be drawn that something other than temperature is concerned with the emission of light by the sodium flame. The same thing was found by Kayser and Paschen in the case of the ultra-violet bands of the arc, which were much brighter than a corresponding region of the spectrum of the positive crater, notwithstanding the fact that the temperature of the latter is higher than that of the arc proper.

The experiments of Pflüger and of Ladenburg on the emission and absorption of ionized hydrogen in vacuum tubes described in the chapter on absorption have been considered by Ladenburg to prove that the quotient E/A is not a constant, and that the emission is therefore not a true temperature effect. Pflüger, however (*Verh. der d. phys. Ges.*, 12, 208, 1910), and Koenen (*Phys. Zeit.*, August 1910) claim that this is by no means the case. The source of light in these experiments did not emit a continuous spectrum, for there were strong maxima at the positions occupied by the hydrogen lines. Koenen shows that all of the double reversal effects, and the behavior of the absorption lines, when the intensity of the source is diminished by rotating one of the Nicols, can be explained, if we have emission and absorption curves of certain types. His treatment is too long to be included here, and the reader is referred to the original paper. Ladenburg has apparently defended his position successfully, however, and in the opinion of the author it seems very improbable that the emission of hydrogen is a pure temperature effect. The question cannot, however, be regarded as quite settled at the present time.

There are certain cases in which we have a true temperature emission of a gas. Carbon dioxide, when heated, emits an infrared radiation, the spectrum showing a very sharp band at $\lambda = 4.3 \mu$. Paschen (*Wied. Ann.*, 51, p. 1, 1894) found that a layer of the gas 7 cms. thick emitted and absorbed as strongly as a layer 33 cms. thick. This indicated that the radiation from a layer could be regarded as the equivalent of the radiation from a layer of infinite thickness, or in other words, the radiation from a black substance at the same temperature. By heating the gas in a tube, and measuring the intensity of the emitted radiation with a spectro-bolometer, Paschen found that for all temperatures between 150° and 500° the intensity of the radiation of wave-length 5.12 μ was only a little below that of a black body at the same temperature. As a black body he used a smoked strip of platinum heated by an electric current. The proof of the law follows from the fact that for a layer

¹This holds, however, only for the absorption of a narrow range of wave-lengths. Rubens and E. Ladenburg (*Verh. der Deutsch. Phys. Ges.*, 1905) have made a study of the absorption of CO₂ for the long waves isolated by a prism of sylvite, and also for the total radiation. In the latter case the absorption increased with the logarithm of the thickness of the gas layer. In the case of homogeneous radiation the logarithm of the absorption is proportional to the thickness.

from which the radiation is the equivalent of that from an infinitely thick one, we have the relation $\frac{E}{A} \rightarrow c$, the emissivity of a black body, as we can at once see by comparing the equation $cA = E$, given at the beginning of the chapter with the relation previously deduced, namely, that as the thickness of an emitting absorbing layer increases, the intensity of the radiation leaving it approaches the value $\frac{E}{A}$ as a limit. An interesting conclusion has been drawn by

Kayser from Paschen's experiment. The thickness of the layer necessary in order that the radiation may equal that of a black body will vary for the different lines in the spectrum, the greatest thickness being necessary for the wave-lengths for which the absorption is a minimum. If we employed a layer so thick that the conditions were realized for all of the lines, and plotted the intensities of the lines as ordinates, and the wave-lengths as abscissae, the curve joining the points should be the emission curve of a black body at the same temperature as shown in Fig. 362.

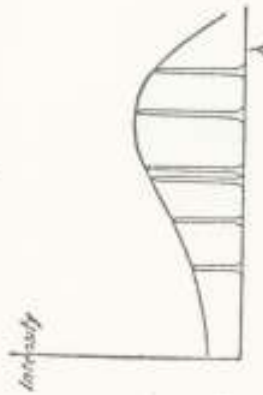


FIG. 362.

We may perhaps apply the same reasoning to the case of the bright lines of flames. The sodium flame, for example, exhibits in addition to the *D* lines a very faint line in the green. By increasing the thickness of the flame the ratio of the intensity of the *D* lines to that of the green line should become less. Multiplying the flame by repeated reflections between two parallel mirrors amounts to the same thing as increasing its thickness, and Warner (*Wied. Ann.*, 68, p. 143) found that a sodium flame placed between two concave silvered mirrors showed the green line with great distinctness. The *D* lines were not increased in brilliancy in anything like the same proportion, and appeared much broadened, a faint continuous spectrum appearing in addition. Attempts made by the author to repeat this experiment both by the use of mirrors and a sodium flame, over a metre in length, gave negative results.

The most complete investigation of the subject of the relation between the emission and the absorption of gases has been made by H. Schmidt (*Ann. der Phys.*, 29, 1909). He studied the absorption and emission of the flame of a Bunsen burner, measured the temperature of the flame most carefully with a thermo-element, and found that the laws of black-body radiation could be applied to it, at least within the region of the strong absorption and emission bands at 2.8 and 4.3 μ . By comparing the radiation of the flame with that emitted by a black body at a temperature of 980° Schmidt was able to calculate the temperature of the flame; the value found was 1670°, in close agreement with the observed value of 1640°.

Very few cases are known in which visible radiations can be obtained by merely heating a gas or vapor. An immense amount of work has been done by Pringsheim and others in endeavors to obtain a luminous emission from gases as a result of high temperature alone. Efforts in this direction have been, almost without exception, in vain, and Pringsheim came to the conclusion that, at least for temperatures which could be commanded in the laboratory, gases remained dark.

There are, however, one or two exceptions to this rule, which were studied by Salet and Evershed. Iodine vapor when heated to a temperature of only six or seven hundred degrees gives off a reddish orange light. The experiment is easily performed by arranging a small spiral of platinum wire, which can be heated by arranging a test-tube in which a little iodine is vaporized by means of a Bunsen burner. An orange-colored flame is seen to rise from the hot wire. A still better method is to enclose a few crystals of iodine in an exhausted bulb of fused quartz. The body of the bulb is heated to a high temperature by the flame of a blast lamp, and the iodine in the narrow neck then vaporized by the application of a small flame as shown in Fig. 363. The quartz bulb emits little or no light until the vapor enters it, when it immediately glows with an orange-



FIG. 363.

red light. A similar phenomenon has been observed in the case of sodium vapor, but is not as easily reproduced as the iodine emission. The best arrangement is a long steel tube containing metallic sodium, and highly exhausted, heated by a row of burners, or better in one of the electrical ovens made by Heraeus of Hanau. Evershed was of the opinion that the spectrum was continuous, but by employing a vapor of small density Koenen (*Wied. Ann.*, 65, p. 256) succeeded in resolving it into bands, which corresponded to the bands seen in the absorption spectrum. It is not difficult to see how a continuous spectrum for we have only to apply the principles involved in the case suggested by Kayser to a spectrum consisting of bands, the intensity having a finite, though different value, for each wave-length.

Temperature Radiation of Solids and Liquids. — The radiation of solids and liquids is especially adapted to the proof of Kirchhoff's law, since in these cases we can be sure that it is the result of temperature alone.

That the emission of light by heated substances is proportional to the absorption can be easily shown by heating a fragment of a piece of decorated china in a blast-lamp. The design emits much more light than the white background, owing to its stronger absorbing power.

The more powerfully a body absorbs, the more powerfully will

it emit when heated, this relation holding for every individual wave-length. Black bodies then give out the most light when heated. The fact that a white block of lime is far more luminous than a carbon rod when heated in the oxy-hydrogen flame is not usually cited in support of this law, while the fact that the most luminous body of all, the Welsbach mantle, is also quite white is equally unsatisfactory as an illustration, for white bodies are in reality transparent, that is, they are made up of masses of small transparent particles, and transparent bodies ought not to emit at all. It is of course necessary to define just what we mean by transparency in this case, and it may be well to consider first a somewhat analogous case. The absorption which is accompanied by high emissivity is true absorption, and not selective reflection — which is sometimes confused with absorption. A highly reflecting polished metal surface is a poor radiator, but by properly constructing its surface we may give it the power to absorb and emit. A bundle of polished steel needles with their points all turned towards the source of light reflects scarcely any light at all, the rays undergoing multiple reflections between the conical ends of the needles. Such a bundle of needles should emit much more powerfully than a polished steel surface, and it is easy to see just why it should do so. Each needle, seen end on, sends not only emitted light to the eye, but reflects rays coming from its neighbors. The surface formed by the points of the needles can be regarded as an absorbing surface, which absorbs in virtue of its structure: it is analogous to the hollow "black bodies" with which we are now familiar.

The point to be emphasized is that such a surface, which absorbs but little in virtue of its molecular nature, may be also a powerful radiator, the mechanism by which its radiating power has been increased being as indicated above. Suppose now we take a perfectly transparent body, which like a perfect reflector has no emitting power. A bead of microcosmic salt (sodium pyro-phosphate) heated in a blast lamp is a good example. Though the platinum wire which supports it glows with vivid incandescence, the bead remains perfectly dark. A glass bead, however, emits a good deal of light, doubtless from the fact that its transparency is much less at high temperatures, a very common behavior of transparent substances. The microcosmic salt on cooling becomes traversed by hundreds of cleavage planes, which give it a milky appearance. On reheating it it emits light, strongly until it finally fuses into a transparent drop, when it instantly becomes dark again. The reason for this behavior is not quite as apparent as in the case of the needles. In fact I am not quite sure that I understand it at all. Quartz behaves in the same way. A drop of clear fused quartz, heated in the blast, emits little or no light, but if it contains spots made up of an emission of quartz and air, these spots emit strongly. In other words an opacity resulting from a pulverization of the transparent medium seems to be accompanied with a strong emitting power. Apparently we cannot apply the same reasoning as in the case of the needles, and it looks rather as if the radiation was largely a surface effect.

We have perhaps a better illustration, and one which is easier to handle, in the case of mica.

A thin film of mica, when heated, becomes silvery white and almost opaque. The change appears to be due to the fact that the mica splits into a multitude of parallel laminae, in other words its opacity is due to the presence of a large number of reflecting planes. If we hold it at the polarizing angle it becomes fairly transparent again. The mica in this state glows brilliantly when heated in a flame, while ordinary mica shows little or no luminosity. Calcined mica films are useful for demonstrating polarization by a pile of plates. It is best to place the film in a piece of thin platinum foil, folded once, and heated in a Bunsen burner. At normal incidence, a film of suitable thickness is opaque as a result of the reflecting planes. Turned to the polarizing angle, it becomes partially transparent, as the planes refuse to reflect the perpendicular components of the vibrations at this angle. The mica shows the same peculiarity as the microcosmic bead: an omission of light resulting from cleavage planes.

There is, of course, the possibility that the roasting of the mica has increased its power of absorption, but of this there appears to be no evidence, for the reflected + the transmitted light appears to equal the incident light in intensity. A flake placed between two equally illuminated white planes disappears, i.e. has the same intensity as the background. Such a flake if placed within a hollow body heated to a uniform temperature should disappear for the same reason. It is difficult to see how the reflecting planes can increase the emitting power, for if we consider that the elements within the mass radiate with equal intensity, the introduction of reflecting laminae cannot increase the total output, unless we assume that the surface has a higher radiating power than the interior. Professor Jeans suggested to me that this might well be the case, since the electrons will have their largest accelerations at the free surface. If this is the case, any circumstance which increases the amount of surface should increase the emitting power. Returning now to the mica with its numerous parallel cleavage planes and the enclosed air films, — assume it introduced into the interior of a hollow white-hot body. It should disappear when viewed through a small aperture, since the reflected + the transmitted light equals the incident light in intensity, even before its temperature is raised in the slightest degree. Now let its temperature go up until it reaches that of the hollow body. We have now, in addition to the reflected and transmitted light, the emitted light, and the mica should appear brighter than the background. This would seem to be a test of a surface emission. The experiment was tried and the mica remained invisible; that is, it behaved like any other substance placed within a uniformly heated enclosure. The microcosmic salt bead behaved in the same way. The action of the cleavage planes in the microcosmic salt remains therefore unexplained, and requires further investigation.

Returning now to the subject of the powerful emission of light by the lime block and the feeble glow of the carbon, we may find

the following analogy helpful in explaining some of the apparent anomalies.

Hydraulic Analogy of a Radiating Body.—The radiator is represented by a tall hollow cylinder, open at the top and closed at the bottom, provided with a number of outflow pipes of different sizes. Water flows into the cylinder at a certain definite rate from a horizontal pipe or flume, the height of which (T_1) above the base of the cylinder represents the temperature of the flame. Obviously the level of the water in the cylinder will rise until the rate at which the water flows out exactly equals the rate at which it flows in. This height (T_2) is the temperature which the radiator acquires in the flame. The jets of water which issue from the tubes represent radiation of different wave-lengths, the small jets representing the short waves. Their velocity corresponds to intensity of radiation. We will first suppose our hydraulic radiator to represent a black body, say a lump of carbon. In this case all of the pipes at the bottom are wide open and we have the maximum outflow of all wave-lengths for any given temperature, i.e. for any given height of the fluid within the cylinder. (If we take the cylinder empty, and plunge it into water, jets will squirt into it through the pipes; that is, it is a perfect absorber for all wave-lengths.) With all of the pipes open, however, the level of the water within the cylinder will not rise to any great height, owing to the limited rate at which water flows in from the horizontal pipe. This means that the lump of carbon in the flame does not rise to a very high temperature because it radiates energy at a high rate. At the low temperature there is comparatively little visible light in the radiation, for the shorter waves only appear in quantity at high temperatures. We can imitate this condition in our hydraulic model if we choose by putting valves on the inside of the tubes, those on the small tubes opening only at high pressures.

To make our model imitate the head of microcosmic salt we plug up all of the pipes. The cylinder now represents a transparent body. If immersed in water it absorbs nothing through the pipes, and no matter how high the level of the water rises in it when water is poured in there is no emission of fluid, in other words no radiation. The body rises in temperature until the temperature is equal to that of the flame, but there is no radiation. Take next the case of the lime in the oxy-hydrogen flame. It is a partially transparent substance, and we can imitate it by plugging the tubes with glass beads or cotton. Owing to the lesser rate at which the water now flows out through the tubes, the level rises much higher than when the tubes are all open, and owing to the greater pressure (temperature) we have liquid jets through the small tubes (short wave-length radiation). The inferiority in the emissivity is more than made up for by the higher temperature which the body can acquire. We are now ready for the Welsbach mantle. It has been conclusively shown by Rubens that the peculiar brilliancy of the thorium mantles, caused by a small trace of cerium, is due to the fact that the cerium makes the thorium selectively absorbing for the short waves at high temperatures. If we have a Bunsen flame over a

mantle in a brilliantly lighted room, it will be seen to turn yellow at a temperature a little below a red heat. In other words it becomes a strong absorber for the short waves. It is, however, transparent for the long waves, consequently it does not emit energy at anything like the rate at which a black body does, and in consequence can rise to a high temperature in the flame, exactly as a pure thorium mantle does. Its band of absorption in the blue region enables it to pour out visible radiations nearly as powerfully as those which a black body at the same temperature would emit, hence its enormous brilliancy. Our hydraulic model with all of its tubes plugged with cotton represents the mantle of pure thorium, while to transform it into the Welsbach mantle we have only to pull out the porous plugs from some of the smaller tubes. In this condition, owing to the impeded flow in the large tubes, the water will rise in the cylinder to a great height, and we get very powerful jets from the small tubes which we have opened, much more powerful than in either of the previous cases considered. Of course with all of the tubes open we could get equally intense small jets if we poured the water in at the top at a sufficient rate. There is a limit to this rate, however, for it is obvious that the rate at which the water is poured in at the top corresponds to the rate at which the flame can pour energy into the radiating body, a circumstance which depends on the conductivity of the body for heat and other circumstances.

Emission of Substances having Absorption Bands.—It is frequently stated that ruby glass when heated emits an excess of green light, but the phenomenon is not very striking, if it exists at all. Cobalt glass was investigated by Rizzo (*Ann. Chim. Torino*, 29, 424, 1894), who was unable to establish any relation between its emitting and absorbing power. His apparatus was not very sensitive, however, and it is difficult to draw very definite conclusions from his observations.

The emission and absorption of rock-salt has been studied by Abramczyk (*Wied. Ann.*, 64, p. 625, 1898). Unfortunately he made use of absorbing screens, instead of spectroscopic dispersion, and his results cannot on this account be regarded as wholly trustworthy. He found, however, that the heat emission consisted of two parts, one of which was stopped by a salt plate, while the other was freely transmitted. He found that 40 per cent of the radiation from the salt was reflected by a polished plate of the same material, from which he inferred that there was a selective emission at a region in the spectrum corresponding to the position of the band of metallic reflection observed by Rubens and Nichols. Rubens and Aeschkinas have pointed out, however, that a strong emission is not necessarily to be expected at a band of metallic reflection. Though this region is one of relatively strong absorption, the absorbed portion may be only a small fraction of the amount reflected, and a high value of the emissivity is not to be expected, as it is the absorption proper, and not reflection, that is related to the emission. In fact high reflecting power is usually associated with low emissivity, as is shown by the small ratio of the emissivity of a polished

metal surface to a surface of the same metal brought into a spongy condition, *i.e.* into the state of platinum black. A hint has been given in the Chapter on Absorption as to the physical explanation of the increased absorption in this case, and it is instructive to reverse the reasoning and apply it to emission. As we shall see later the case is not unlike that of a hollow body, the emission from the interior of which we shall show is equal in intensity to that of the radiation of a perfectly black body at the same temperature.

In the case of substances which are not transparent, as we have seen, a portion of the incident energy is reflected and a portion absorbed. If the intensity of the light is 1, the absorbed fractional part A , and the reflected part R , we have $1 = R + A$ or $A = 1 - R$. Substituting this value in our formula $\frac{E}{A} = e$ gives us an expression for Kirchhoff's law, in which the relation between emissivity and reflecting power is established:

$$\frac{E}{1 - R} = e, \text{ the emissivity of a black substance.}$$

This formula has been verified by Rosenthal (*Wied. Ann.*, 68, p. 783), who investigated the emission and reflection of quartz, mica, and glass, with a spectrometer and thermo-element, and compared the results with the values calculated from the above formula. The low emissivity at regions of the spectrum corresponding to those of the maxima of the reflection curve is clearly shown in Fig. 364, and the close agreement between the observed

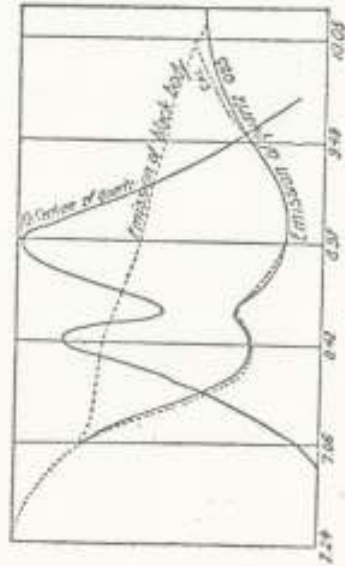


FIG. 364.

values and those calculated from the above formula may be regarded as a most excellent proof of the law. As will be seen, the law has been proven quantitatively for but few substances. That there is a relation between absorption and emission in a large number of cases, where Kirchhoff's law cannot be expected to hold, is evident. Some of these cases we have already considered. A noteworthy example often quoted as a proof of the law is the remarkable relation between the emission of heated oxides of erbium and didymium,

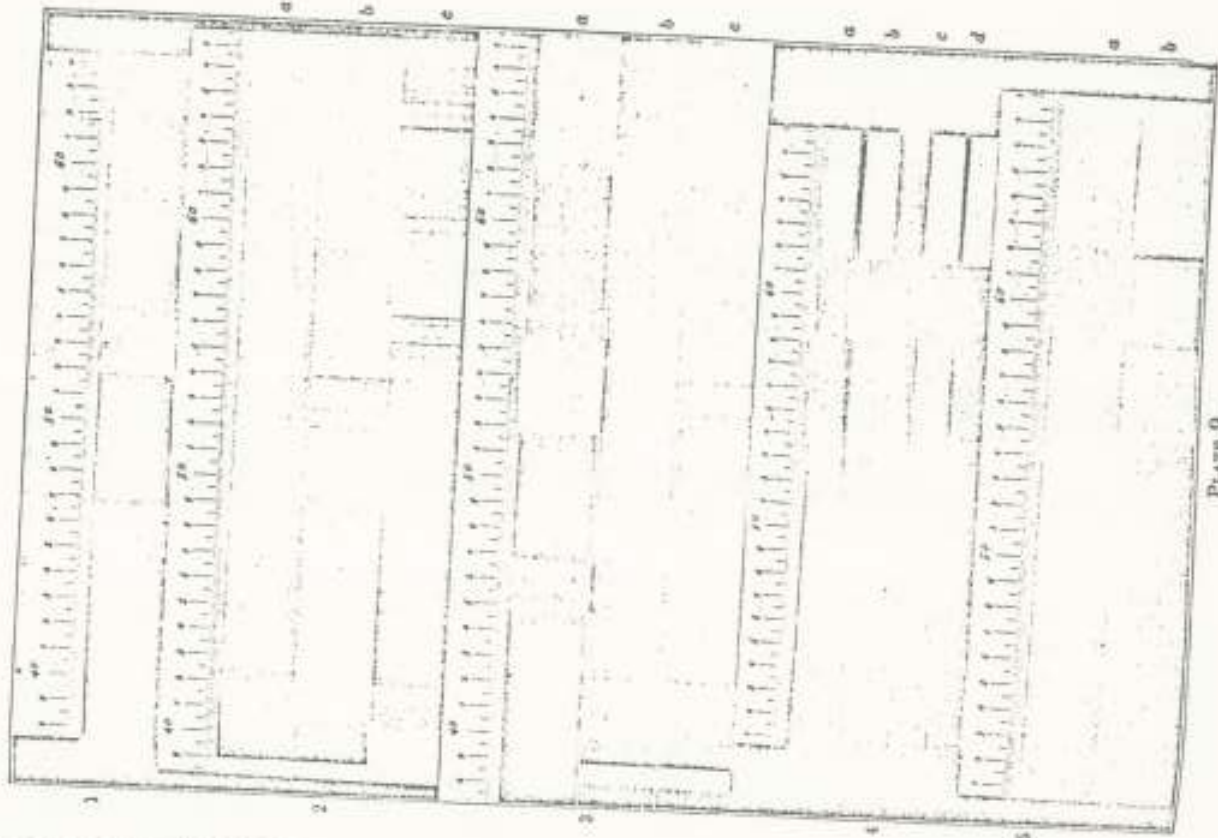


PLATE 9.

and the absorption spectra of the same oxides and solutions of the salts of the metals. The oxides, unlike all other solid substances, when heated to incandescence, show in addition to a continuous spectrum a number of bright bands, which correspond in position to the absorption bands, at least approximately. This can be easily shown by dipping a platinum wire in a concentrated solution of cerium chloride and heating it in a Bunsen burner, the continuous spectrum of the white-hot oxide being crossed by a number of bright bands.

A study of the absorption and emission spectra of the oxides of some of the rare earths has been made by Anderson (*Astrophys. J.*, XXVI, No. 2, 1907), who found that the absorption spectrum, obtained by the "body-color" method, that is by illuminating the powdered or fused substance with white light, was very different at different temperatures. The emission spectrum of heated cerium oxide showed a very broad and hazy band at $\lambda = 58-60$, the spectrum being almost continuous. If the oxide existed as an impurity in cerium oxide the bands were much narrower, as shown on Plate 9, Fig. 1. When mixed with calcium oxide, however, the bands were similar to those obtained with the pure oxide. The absorption spectrum obtained by illuminating the substance with white light varied with the nature of the surface, as is shown on Plate 9, Fig. 2, in which *a* is the spectrum obtained with the powder; *b*, with a rod of the substance heated for 100 hours in a Bunsen flame; and *c*, with a rod the surface of which was fused with the oxy-hydrogen flame. The bands are much blacker and more numerous in the case of the fused oxide, a circumstance which may be ascribed to the penetration of a greater thickness of the substance by the light in this case. In Fig. 3 we have spectrum *a*, the emission of incandescent cerium oxide, *b* its absorption at a high temperature, and *c* its absorption at room temperature.

As will be seen the absorption spectrum at a high temperature is complementary to the emission spectrum. Fig. 4 shows the absorption of the gray oxide of neodymium at the following temperatures, *a* 600°, *b* 400°, *c* 200°, *d* below 100°. The absorption spectrum depends upon the treatment of the oxide, which when first formed by careful ignition of the oxalate is pink, becoming bluish gray by prolonged heating. In Fig. 5 *a* and *b* we have the absorption spectra of the pink and gray forms respectively. It is not known positively whether they have the same chemical composition.

SELECTIVE EMISSION AND ABSORPTION OF INCANDESCENT SALTS

Some extremely interesting experiments have been recently made by Lenard (*Annalen der Physik*, 17, p. 197, 1905) with beads of the fused salts of the alkali metals supported upon platinum wires in the Bunsen flame. The emitted light was found to be strongly colored, the color depending upon the metal. The sulphates of K, Rb, and Cs emitted green light, while sodium sulphate shone with a bluish tinge. An examination of the absorption

spectra of the fused salts showed that the color of the transmitted light was complementary to that of the emitted light, as should follow from Kirchhoff's law. The salts were colorless when cold, however, showing that some sort of dissociation resulted from the high temperature, metallic ions being set free which had the property of absorbing and emitting radiations of the same frequency. The failure of the ions to show color when the salts are dissolved in water is ascribed by Lenard to a loading of the ions with water. In the case of most of the salts examined the color was found to depend upon the metal, *i.e.* upon the cathions: the borates and phosphates were marked exceptions, however, the color being chiefly due to the anions, the nature of the metal being immaterial. In the same paper will be found a number of very interesting conclusions regarding the emission of colored light by flames and the probable centres from which radiate the different lines of the spectra.

Emission of Polarized Light.—Certain crystals, tourmaline for example, have an absorbing power which differs according to the plane in which the vibrations are taking place. Suppose the crystal to be so oriented that its absorptive power is greatest for horizontal vibrations. We might expect, on heating the crystal, to find a preponderance of horizontal vibrations in the emitted light. This was found to be the case by Kirchhoff, who heated a crystal in a Bunsen flame and found that, on viewing it through a double-image polarizing prism, one of the images was distinctly brighter than the other.

A quantitative proof of Kirchhoff's law in the case of glowing tourmaline has been made by Pfleger (*Annalen der Physik*, 7, p. 806, 1902), who measured with a spectrophotometer the absorption and emission of the crystal at the same temperature and for the same wave-length. If J is the intensity of the incident light, $J D$ that of the transmitted light, and $J R$ that of the reflected, the intensity of the absorbed light $A = J(1 - R - D)$. R was calculated from the reflection formula $R = \left(\frac{n-1}{n+1}\right)^2$, while D and E were observed with the spectrophotometer. Designating by E_e and E_r the emissivity for vibrations parallel respectively to the vibrations of the ordinary and extraordinary rays, and by A_e and A_r the corresponding absorptive powers, we have, if Kirchhoff's law can be extended so as to include the direction of the vibration,

$$\frac{E_e}{A_e} = \frac{E_r}{A_r} \text{ or } \frac{E_e}{E_r} = \frac{A_e}{A_r}.$$

After eliminating all sources of error, Pfleger obtained as final values for the two ratios,

$$\frac{A_e}{A_r} = .650, \quad \frac{E_e}{E_r} = .641,$$

a very beautiful verification of the law as applied to anisotropic media.

Deduction of Kirchhoff's Law.—The law of Kirchhoff can be deduced from purely theoretical considerations. Kirchhoff's method is free from serious objections, but assumptions are made which cannot be regarded as truths without further treatment. Moreover, his method involves the consideration of bodies which really have no existence, such as perfect reflectors and perfectly transparent substances.

The most logical and concise treatment is due to Pringsheim (*Verh. d. deutsch. physik. Ges.*, 3, pp. 81-84, 1901). Consider a ball κ composed of any material enclosed in a hollow vessel, opaque to radiation of all wave-lengths, and uniformly heated to any given temperature. The ball emits in unit time the total radiation E , while there falls upon it from the walls in the same time the amount ϵ , of which the fraction $\lambda \epsilon$ is absorbed. Since by Carnot's principle the temperature cannot change, the amount of radiation emitted by the ball must equal the amount absorbed, so that $E = \lambda \epsilon$. If the ball is made of a conglomerate of different substances, some parts of its surface may absorb more strongly than others. Suppose we rotate the ball: the amount of energy ϵ falling upon it will only be changed by an infinitely small amount, since only the part of the radiation which came originally from the body, and is reflected back from the walls, can be responsible for the change: the amount of this which falls across the body is of course very small. If, however, the radiation from the walls is not uniform, *i.e.* if it has especial states of polarization, or is more intense in certain directions than in others, the amount of heat absorbed by the body would be changed by its rotation. If, for example, the rotation brought a strongly absorbing surface element into the path of an especially intense ray coming from the wall, the absorption of heat would be increased.

In this case A is variable, and the heat absorbed is $\int A \epsilon$ taken over the body κ . Since this is equal to E , it follows that $\int A \epsilon$ is constant for all positions of the ball. If A is variable this can only be the case if ϵ is independent of direction.

The total radiation ϵ is made up of waves of all possible lengths between 0 and ∞ . Call ϵ_λ the radiant energy comprised between the limits λ and $\lambda + d\lambda$, then

$$\epsilon = \int_0^\infty \epsilon_\lambda d\lambda.$$

Further, let A_λ be the absorption coefficient of κ for waves of length λ : the total absorbed energy is then

$$A \epsilon = \int_0^\infty A_\lambda \epsilon_\lambda d\lambda,$$

or

$$E = \int_0^\infty A_\lambda \epsilon_\lambda d\lambda.$$

Now let κ be brought into another hollow vessel of different

material but at the same temperature. The emission of κ remains the same, also its absorption coefficient A_λ , for waves of the designated length.

If in the present case the radiation e_λ which falls upon κ is different from e_λ (in the previous case), we should have

$$\int_0^\infty A_\lambda e_\lambda d\lambda = \int_0^\infty A_\lambda e_{\lambda'} d\lambda.$$

Since, however, A_λ is quite independent of e_λ , the above equation can only hold if

$$e_\lambda = e_{\lambda'}.$$

By comparing this with $E = Ae$ we see at once that the radiation e is equal to that which κ would emit if it were perfectly absorbing, i.e. if $A = 1$. This shows us that the radiation within a hollow vessel heated to a uniform temperature is independent of the material and shape of the vessel, and is identical in every respect with the radiation emitted by a perfectly absorbing body at the same temperature.

Consider now the radiant energy which a surface element $d\lambda$ of the body κ sends to the distant surface element $d\lambda'$ of the vessel. We will define the emission coefficient E_λ of κ as the single radiation $E_\lambda d\lambda$, which in unit time reaches $d\lambda'$ from $d\lambda$. This radiation has a wave-length λ and any state of polarization. In a hollow vessel of uniform temperature, $d\lambda$ gives out a radiation similar to that of a black body at the same temperature. The total energy of wave-length λ and of a given state of polarization which reaches $d\lambda'$ from $d\lambda$ is therefore $e_\lambda d\lambda$, if e_λ is the emission coefficient of a black body under similar conditions.

We thus have $e_\lambda = E_\lambda + G_\lambda$, if we define $G_\lambda d\lambda$ as the energy of wave-length λ , and of a definite state of polarization, which, coming originally from the walls of the vessel, and transmitted by, or reflected from, the body κ , reaches $d\lambda'$ from $d\lambda$.

We must now determine the value of G_λ and substitute it in the above equation. The radiation from the inner surface of the vessel which reaches $d\lambda'$ by reflection or refraction from $d\lambda$ is equal to the amount which, leaving $d\lambda$, reaches the inner surface by way of $d\lambda'$. Of all the waves of length λ which leave $d\lambda$, the amount reaching $d\lambda'$ in unit time is $e_\lambda d\lambda$, of which an amount equal to $A_\lambda e_\lambda d\lambda$ is absorbed, while the remainder $(1 - A_\lambda)e_\lambda d\lambda$ are in part reflected and in part transmitted, and pass off to the inner wall again. This remainder is equal to $G_\lambda d\lambda$, and substituting the value $G_\lambda = (1 - A_\lambda)e_\lambda$ in the equation

$$e_\lambda = E_\lambda + G_\lambda,$$

$$e_\lambda = E_\lambda + (1 - A_\lambda)e_\lambda,$$

$$E_\lambda = A_\lambda e_\lambda,$$

gives us

or

an equation which expresses Kirchhoff's law, showing that the emission coefficient of any substance for any given wave-length, divided by its absorption coefficient for the same wave-length, is equal to

the emission coefficient of a perfectly black body, for, as we have seen above, e_λ represents the latter quantity.

That the intensity of the radiation from the inner surface of a hollow vessel is independent of the nature of the material can be shown by placing a fragment of decorated china in a porcelain crucible heated over a Bunsen burner. If the cover of the crucible is put on, a small opening being left through which the interior can be viewed, and the flame of a second burner be directed upon it so as to bring the whole to a nearly uniform temperature, the decorations on the china will be quite invisible, the radiations from them being equal to the radiation from the rest of the surface. As we have seen, if the china is heated in the open air the dark portions radiate more strongly, the design appearing brighter than the background. The cause of the equality in the case of an enclosed radiator can be very simply stated. The radiation is made up of two parts, the emitted and the reflected, the latter coming from the heated walls. Dark portions of the material emit more powerfully than white portions, since their power of absorption is greater; on the other hand, they reflect scarcely any of the radiation from the walls. The white portions, which emit feebly, reflect powerfully, and, owing to the proportionality between emission and absorption, a perfect balance is secured.

This principle is now made use of in experiments pertaining to radiation. In studying the nature of the radiation of perfectly absorbing bodies as a function of temperature, it was formerly the custom to make use of an electrically heated strip of platinum with a smoked surface. Such a radiator cannot, however, be brought to a high temperature, owing to the oxidation of the carbon. For high temperature work it was customary to coat the strip with platinum-black, or copper oxide.

Such radiators cannot be regarded as perfect, and at the present time the heated hollow chamber is almost exclusively used, the radiation to be examined escaping through a small hole.

The Perfect Black Radiator. — While the principle that the radiation within a closed space at a uniform temperature is identical with the radiation of a perfectly black body had been recognized for many years, Wien and Lummer (*Wied. Ann.*, 56, p. 451, 1895) were the first to actually prepare radiators acting on this principle, and make use of them in experimental work.

For studying the intensity of the radiation at low temperatures and the distribution of energy in the spectrum of the radiation, a hollow cylinder of brass, blackened on the inside, can be used. The cylinder is provided with a small aperture, and is surrounded by a steam jacket, or embedded in a mixture of sodium and potassium nitrate, and the whole packed in felt. The smaller the size of the hole in comparison to the internal capacity of the cylinder, the more nearly does the emerging radiation compare with that of an ideal black body. For high temperature work a cylinder of platinum or porcelain, electrically heated, can be employed, or even a hollow iron ball heated in a gas furnace. Kayser has proposed a very simple device, which, though superior to an electrically heated strip

of blackened platinum, is not as good as a hollow vessel. Two strips of platinum, one provided with a narrow slit, are mounted opposite to one another and heated to the same temperature by a current. The principle is of course the same as that of the device just considered.

Paschen (*Wied. Ann.*, 60, p. 719, 1897) has proposed still another device. A glowing carbon filament is mounted at the centre of a hollow silvered sphere. Assuming the silver to reflect all of the energy, it can be regarded as a hollow vessel having the same temperature as the carbon filament. The radiation escapes as before through a small hole. This same device has been applied to the bolometer, the absorbing strip being mounted at the centre of a hollow spherical chamber silvered on the inside. All radiation not absorbed at once by the bolometer is returned to it by the reflecting surface. In this way it is possible to prepare a perfectly black bolometer.

Equilibrium between Radiation and Material Bodies.—In the deduction of the remaining laws of radiation we shall employ largely a conception due to Bartoli, which, though it cannot be carried out experimentally, leads to important laws which can be verified in other ways. The idea in brief is to apply the principles of thermodynamics to radiation, performing a cyclical process similar to Carnot's cycle, employing vibrating ether instead of a gas as the working substance.

The radiation within a hollow vessel can be in equilibrium with the walls or with bodies in the interior, only when it is of the same nature as the radiation emitted by the walls or the bodies contained within the vessel.

To get an idea of exactly what we mean by equilibrium between radiation and a material body we will consider the following case: Suppose we have a hollow vessel the walls of which are perfect reflectors, which contains only ether. If we fill this cavity with monochromatic radiation, say that of the sodium flame, by opening a door in the wall and allowing the light to enter, which, of course, can be done perfectly well in theory, the radiation will, if we close the door, be reflected back and forth within the vessel forever. It will neither change in intensity nor alter its wave-length; in other words, it is in equilibrium with the reflecting walls. We shall now prove that a perfect reflector is the only body with which this radiation can be in equilibrium, with the exception of the flame which originally emitted the light. Suppose we introduce a small fragment of absorbing matter within the cavity of the reflecting vessel. It will immediately absorb the monochromatic sodium radiation as fast as this radiation falls upon it, and in a very short space of time the monochromatic waves will have vanished completely. The temperature of the absorbing body will be slightly elevated, and it will emit long heat-waves, the energy being distributed over a wide range of wave-lengths, the range and distribution depending on the temperature of the body. This radiation will now fill the cavity in place of the sodium radiation, and it will be in equilibrium with the absorbing body, i.e. a permanent state is speedily reached, after which there is no further change.

Pressure of Radiation.—The radiation within the vessel exerts a pressure upon the walls and upon the surface of the absorbing body. As we are to make use of this pressure in the derivation of laws it will be well to investigate it somewhat in detail.

Maxwell, in his electro-magnetic theory, showed that radiation must exert a pressure when it falls upon a reflecting or absorbing surface. As this pressure is the foundation upon which the laws of radiation have been built, we will briefly consider the phenomenon.

Maxwell showed that when plane electro-magnetic waves fall in a normal direction upon a perfectly absorbing surface, the pressure exerted on unit area is equal to the energy contained in unit volume of the vibrating medium.

That a pressure is exerted by heat (or light) waves may be proven by making use of the idea of Bartoli. Consider a cylinder, composed of some material which reflects perfectly, closed at the ends by black plates at temperatures $T_1 > T_2$ (Fig. 365). Introduce a screen S , made also of a reflecting material, which divides the cylinder into two compartments. The body at temperature T_1 will fill the upper compartment with radiation of energy corresponding to its temperature. T_2 (at a lower temperature) will fill the lower compartment with radiant energy of less density. Let B represent a movable reflecting diaphragm, provided with a sliding door, which, when open, allows the energy from T_2 to fill the middle compartment. Now close the door and raise the diaphragm or piston. The volume of the middle compartment is decreased, and the density of the radiant energy "trapped" within it is increased. On removing the screen S laterally, which we can do without performing work, the diaphragm will drive the radiant energy above it into the body T_1 . We have thus taken energy (or heat) from a body at low temperature and carried it to one of high temperature, which by the second principle of thermo-dynamics is impossible, unless mechanical work is done in the operation. This work can have resulted only from the overcoming of a pressure exerted upon the diaphragm, the vibrating medium resisting compression in the same way that a gas does. This pressure becomes greater as the volume is diminished owing to the increase in the energy density. In the case of the compression of a gas, the molecules rebound from the moving piston with increased velocity, consequently the force of each blow, and the number of blows per second, are increased.

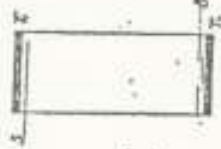


FIG. 365.

In the case of compressed radiation the mechanism is not so easy to follow; as we shall see presently, reflection from a moving diaphragm decreases the wave-length by an amount proportional to the distance through which the diaphragm moves (provided the rest of the vessel is reflecting). This means that the number of waves which strike it per second will be increased. The amplitude, as we shall see presently, remains the same, and the increase of energy density is due solely to the fact that more waves are present

in unit length of the train after the compression than existed before the motion of the diaphragm. Such a process as that described above cannot be even approximately realized experimentally. It is no less valuable, however, as our inability to carry it out is due solely to mechanical difficulties and our inability to obtain a substance which reflects perfectly.

An admirable treatment of the mechanical pressure of radiation has been given by Larmor (*Encycl. Brit.*, vol. 32, "Radiation"). Consider a wave train travelling along the x axis incident upon a perfect reflector, which is travelling in the opposite direction with a velocity v . The displacement in the incident wave train is

$$\xi = a \cos m(x + ct),$$

and in the reflected train

$$\xi' = a' \cos m'(x - ct).$$

The position of the reflector at time t is given by

$$x = vt.$$

The disturbance does not travel into the reflector, and must therefore be annulled at its surface. Thus when $x = vt$ we must have $\xi + \xi' = 0$. This gives us $a = -a'$ and $m'(c-v) = m(c+v)$.

The amplitude of the reflected disturbance is therefore equal to that of the incident one, while the wave-length is altered in the ratio $\frac{c-v}{c+v}$ or $1 - \frac{2v}{c}$, approximately, when $\frac{v}{c}$ is small.

The energy of the wave-train is half potential and half kinetic, and is given by the integration of $\rho \left(\frac{\partial \xi}{\partial t} \right)^2$ along the train, in which $\rho = \text{density}$.

In the reflected train it is therefore augmented, when equal lengths are compared, in the ratio $\left(\frac{c+v}{c-v} \right)^2$, but the length of the train is diminished by the reflection in the ratio $\frac{c+v}{c-v}$. This increase in energy per unit time can arise only from work done by the advancing reflector against pressure due to the radiation. The pressure per unit surface must therefore be equal to the fraction $\frac{2}{c-v}$ of the energy in the length $c+v$ of the incident wave-train; thus it is the fraction $\frac{c^2 - v^2}{c^2 + v^2}$ of the total density of energy in front of the reflector belonging to both the incident and reflected trains.

When v is small compared with c this makes the pressure equal to the density of the vibrational energy, in accordance with Maxwell's electro-dynamic formula.

The pressure due to light was, for a long time, sought for in vain. The disturbing effects of "radiometric action," or the reaction pressure of gas molecules rebounding from the surface heated by the

radiation, completely masked the very small effect which was looked for. As early as 1751 an attempt was made by DeMairan and Du Fay to detect the pressure of light. This was of course in the days of the corpuscular theory, and the looked-for pressure was that due to the arrest of the flying corpuscles. Fresnel, Zöllner, Bartoli, and Crookes also searched in vain for evidences of the pressure, the experiments of the latter, however, resulting in the discovery of the radiometer. The pressure was first observed by Lebedew in 1900 (*Rapp. près au Congrès de Phys.*, 2, 133, Paris, 1900) and by Nichols and Hull independently at about the same time. Though the latter investigators were anticipated by Lebedew by some months, their investigation was conducted with greater care, and the errors due to gas action were more carefully eliminated.

Experiments of Nichols and Hull.¹—The inability of previous observers to measure the pressure due to radiation was due to the apparent impossibility of separating the effect from the so-called "radiometer" action. Thin vanes were employed to detect the pressure, and the radiation warmed the side on which it fell. When such a condition exists the gas exerts a greater pressure on the warm than on the cold side, and in general this pressure is vastly greater than the true radiation pressure.

Nichols and Hull finally succeeded in eliminating the gas action by employing a suspended vane made of two circular disks of thin glass silvered on one side. By employing a reflecting surface the pressure is double that exerted upon a black surface, and the heating is reduced to a minimum. By measuring the deflections when the glass and silver sides were illuminated in succession the gas action could be calculated, for the silver surface is the one heated in both instances. This is due to the fact that the radiation, before its impact upon the vane, has passed through a number of lenses and plates of glass, and is consequently robbed by absorption of all rays capable of heating a glass surface. It is at once apparent that when the radiation falls upon the glass surface the gas pressure and the light pressure are opposed, while when the silver surface is illuminated they act together, *i.e.* in the same direction. Larger deflections are of course observed in the latter case than in the former. To still further eliminate gas action, the ballistic method was adopted; it had been observed that some seconds' or even minutes' exposure to the radiation were required before the gas pressure reached its maximum, while the radiation pressure is of course instantaneous. Very short exposures were consequently given, and the ballistic deflection of the vane was observed by means of a mirror and scale.

By an elaborate series of experiments the investigators determined the most suitable pressure for the air in the chamber in which the vane was suspended, the pressure, in other words, at which the gas action was at a

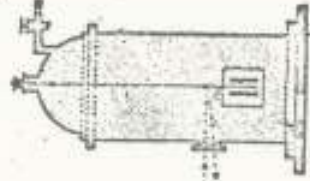


FIG. 366.

¹ *Proc. Am. Acad.*, xxxviii., April 1903; *Phys. Rev.*, xiii., 293, [1901].

minimum. This pressure proved to be about 16 mms. of mercury. After measuring the value of the radiation pressure, the energy of the radiation was determined, by allowing it to fall upon a blackened silver disk. The rise of temperature of the disk was determined by means of iron-constantan thermo-junctions imbedded in the disk. From these energy measurements the pressure to be expected was calculated.

The radiation employed was that of an arc-lamp, either with or without absorbing screens. After correcting for all possible sources of error, the following values were obtained:

ILLUMINATION	PRESSURE IN 10^{-6} DYNES OBSERVED	PRESSURE CALCULATED FROM ENERGY MEASUREMENTS
Through air only	$7.01 \pm .02$	$7.05 \pm .03$
Through red glass	$6.94 \pm .02$	$6.86 \pm .03$
Through water cell	$6.52 \pm .03$	$6.48 \pm .04$

These experiments can be regarded as establishing in a quantitative manner the existence of the Maxwell-Bartoli pressure, which measured in dynes per sq. cm. is equal to the energy contained in unit volume of the radiation. A reflecting surface doubles the energy density in the medium in front of it by superposing the reflected beam upon the incident.

The gas action was subsequently eliminated by Hull (*Phys. Rev.*, May 1905) by enclosing the reflecting and absorbing surfaces in thin glass cells, as proposed in the earlier paper. The silvered side of a thin cover glass was placed in contact with the blackened side of a similar glass, and the whole enclosed by means of two other thin glasses as shown in Fig. 367. Two cells of this description were mounted upon opposite



FIG. 367.

ends of a torsion arm suspended in a receiver from which the air could be removed. When the light falls upon the blackened surface and is absorbed, the temperature of the two outer glass surfaces of the cell are the same, since they are separated from the heated surface by equal thicknesses of glass and air. The gas action should therefore be equal on the two surfaces. Any gas action occurring within the cell will produce no effect, owing to the equality of action and reaction. Hull found that the ratio of the deflections obtained when the silvered and blackened surfaces were illuminated in succession, agreed with the calculated ratio to within 2 per cent, showing that the "radiometer" action had been practically eliminated.

Tangential Component of Radiation Pressure.—An interesting experiment was described by Poynting at the Cambridge Meeting of the British Association in 1904, in which gas action is completely eliminated.

When radiation is incident upon an absorbing surface in an oblique direction, the pressure has a component parallel to the surface. In

the case of a reflecting surface this tangential force cannot be detected, since the incident and reflected beam give rise to equal and opposite forces parallel to the surface. The magnitude of the force, when E is the energy density, μ the fraction reflected, and α the angle of incidence, is given by

$$F = \frac{E}{2}(1 - \mu) \sin 2\alpha.$$

The existence and magnitude of the force was observed with the apparatus shown in Fig. 368. Two thin glass disks were mounted on the ends of a fine glass rod, the system being suspended by a quartz fibre in a brass box provided with glass windows. One of the disks was silvered, the other blackened, and the pressure within the case was reduced to 1 cm. Sunlight, or the beam from an arc lamp, was directed against the black disk at an angle of 45° . Gas action due to heating will give rise to a pressure normal to the surface, but there will be no tendency to rotate the suspended system. The tangential component of the radiation pressure, on the other hand, will produce a deflection, the magnitude of which can be read with a mirror and scale. E was calculated from the observed deflection, and was found to be $5.8 \cdot 10^{-6}$ dynes, while a direct measurement of E , by the heating of a silver plate, gave the value $6.5 \cdot 10^{-6}$.

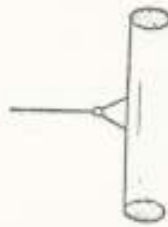


FIG. 368.

Pressure of Radiation on an Absorbing Gas.—The repulsion of the tails of comets by the sun has been explained as the result of the pressure exerted by the solar radiation. If we reduce the size of an obstacle its mass becomes less in proportion to its surface, and the pressure of the radiation may eventually become greater than the attraction of gravitation. It has been shown by Schwarzschild, however, that there is a critical size at which the ratio of pressure to gravitational attraction has its greatest value. In other words, if we make the particles too small the radiation no longer exerts any pressure on them, for they no longer act as obstacles, or diffract light. Now the spectroscopist shows us that the tail of a comet is gaseous, and the gas molecule is very much smaller than the smallest obstacle capable of feeling the pressure of radiation. There is this difference, however. The gas molecule may be capable of stopping the radiation by resonance, i.e. the gas may absorb, and it seems quite probable that radiation may exert a measurable pressure on the molecules of a gas, in spite of the fact that a cloud of material particles, each one of which is vastly larger than the molecule, experiences no pressure at all. This question has been very successfully attacked by Lebedew, who has measured the pressure which the radiation from a Nernst lamp exerts upon various absorbing gases, such as CO_2 , methane, butane, propane, etc. The gas was enclosed in a cell provided with fluorite windows, and was

set in motion by the pressure of the radiation. This motion was communicated to a very light torsion balance made of magnesium, by which it was rendered visible.

The Stefan-Boltzmann Law. — An empirical law was deduced by Stefan from observations made by other observers on the intensity of the total radiation from bodies at different temperatures. The law states that the complete emission S of a black body is proportional to the fourth power of the absolute temperature T , or

$$S = aT^4,$$

in which a is a constant.

This same law was subsequently deduced from theoretical considerations by Boltzmann (*Wied. Ann.*, 22, p. 291, 1884), who availed himself of the ingenious conception by which Bartoli proved that radiation must exert a pressure. Consider a hollow cylinder of unit cross-section, the walls of which are black, and of infinitely small heat capacity. The ends of the cylinder are also black, but of infinitely large heat capacity. Within the cylinder is a frictionless black piston, in contact with the left-hand end plate of the cylinder, which has an absolute temperature T_0 . The opposite end plate has a lower temperature T (Fig. 369). The radiant energy exerts a pressure on the piston, which in the case of plane-waves parallel to the surface is equal to the radiant energy in unit volume of the ether. Let $\psi(T)$ be the energy in unit volume. Since the energy is travelling in all possible directions, the pressure on unit surface will not be $\psi(T)$ but $\frac{1}{3}\psi(T)$. (Compare with the calculation of the pressure due to molecules moving in all directions, in the Kinetic Theory of Gases.) We have then $\frac{1}{3}\psi(T) = f(T)$, the pressure at temperature T .



FIG. 369.

Now let the piston move forward a distance a , under the influence of the pressure of the radiant energy coming from the high temperature plate T_0 . The heat energy leaving T_0 is partly spent in doing the work $af(T_0)$ on the piston, and partly in filling up the volume " a " with radiant energy. This process obviously corresponds to the isothermic expansion of the Carnot cycle, the filling of the space a with energy corresponding to the heating of the gas. The amount of heat which leaves the plate T_0 is $a[\psi(T_0) + f(T_0)]$.

We will now introduce a screen impervious to heat immediately in front of T_0 , which prevents further radiation into the space to the left of the piston. This corresponds to placing the cylinder in Carnot's cycle upon an insulating stand. The pressure to the left of the piston is greater than that to the right, owing to the higher temperature of the plate which filled this portion of the cylinder with radiation. The piston will therefore move forward until the energy per unit volume is the same on both sides. Allowing this adiabatic expansion to take place we have $a[(a+x)\psi(T)] = -\int(T)dx$, the characteristic equation for an adiabatic process (see any

Thermodynamics). In this expression T is of course variable. During this process the volume to the right of the piston has been still further diminished, and an amount of heat energy represented by $(a+x)[\psi(T) + f(T)]$ due to diminishing the volume and work done enters the plate at temperature T .

Since the process is reversible we have, by the second law of thermodynamics,

$$\frac{(a+x)[\psi(T) + f(T)]}{T} = a \frac{[\psi(T_0) + f(T_0)]}{T_0} = C,$$

in which x and T are variables.

$$\text{Writing } (a+x)[\psi(T) + f(T)] = CT,$$

$$\text{or, for simplicity, } (a+x)(\psi + f) = CT,$$

$$(\psi + f)d(a+x) + (a+x)d(\psi + f) = \frac{(a+x)(\psi + f)}{T}dT,$$

and subtracting, $\psi d(a+x) + (a+x)d\psi = -fdx$ (adiabatic equation),

$$\text{gives us } (a+x)df = \frac{(a+x)(\psi + f)}{T}dT,$$

$$(\psi + f)dT = Tdf,$$

or, inserting the (T) which we omitted above,

$$\psi(T)dT + f(T)dT = Tdf(T).$$

Substituting for $f(T)$ its equivalent value $\frac{1}{3}\psi(T)$

$$\text{gives } \frac{4}{3}\psi(T)dT = \frac{1}{3}Td\psi(T),$$

$$\frac{d\psi(T)}{\psi(T)} = 4\frac{dT}{T},$$

$$\psi = aT^4.$$

Proof of Stefan's Law. — The law was first deduced empirically from observations made on the rate of cooling of a blackened thermometer bulb.

Lummer and Pringsheim (*Wied. Ann.*, 63, p. 395, 1897) proved the law over a range of temperatures included between 100° and 1300° C. by measuring the intensity of the radiation from a hollow chamber (black body) by means of the bolometer.

The constant a has been determined in absolute measure by Kurlbaum (*Wied. Ann.*, 65, p. 746, 1898), who heated the bolometer strip (screened from the radiation) by means of an electric current of known strength to the same temperature to which it was raised by the radiation. The radiation was thus determined in absolute units by calculating the Joule heat developed by the current. The value found was

$$a = 1.71 \cdot 10^{-5} \frac{\text{C.E.}}{\text{sec.}} = 0.408 \cdot 10^{-12} \frac{\text{E.C.}}{\text{sec.}}$$

Optical Pyrometers.—Various types of pyrometers have been designed for measuring high temperatures by optical methods.

Féry's instrument is based upon the law of total radiation. It consists of a telescope with a fluorite objective, in the focus of which is mounted a sensitive thermo-couple as shown in Fig. 370. To use the instrument one has only to point it at the object, the temperature of which is to be measured, e.g. the interior of a blast-furnace, and focus the image upon the thermo-junction by means of the eye-piece, which is moved with the latter by means of the rack and pinion wheel at *B*. The temperatures are read with a galvanometer.

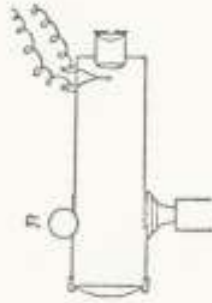


FIG. 370.

Other optical photometers have been devised, based upon the laws which we are about to study.

Temperature of the Sun.—The sun's temperature has been computed by measuring the total radiation. Assuming the solar disk to be a black body, and taking for the value of the solar constant 3 gr. cal. per minute, the computed temperature comes out a trifle over 6000°.

Change in the Spectrum of a Black Body with the Temperature.
Wien's Laws.—Making use of a conception similar to the one by means of which Boltzmann deduced Stefan's law, but extending it by the introduction of the consideration of the change in wavelength which occurs when radiation is reflected from a moving mirror, Wien (*Wied. Ann.*, 46, p. 633; 52, p. 132) arrived at a formula which expressed the change in the spectrum of a heated black body with its absolute temperature. As is well known, when a solid or liquid is heated the longer heat-waves appear first, then red light, and finally at still higher temperatures the violet and ultra-violet. If we measure the energy at different points in the spectrum with the bolometer and plot these values as ordinates, with the wavelengths as abscissae, we obtain the energy curve for the emission at the temperature in question. The maximum of this curve moves towards the region of the shorter waves as the temperature is increased, but there is an increase in the height of every ordinate; in other words, the curve does not move bodily down the spectrum. It seems very remarkable that the form and position of this curve can be determined by considering merely the motion of reflecting pistons moving in a closed cylinder, the ends of which radiate at different temperatures. This, however, is precisely what was done by Wien, whose treatment we will now consider. Consider a cylinder of unit cross-section, the walls of which reflect diffusively all of the incident energy, while the ends are composed of black material, of infinite heat capacity, at temperatures

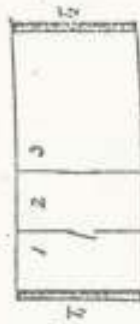


FIG. 371.

T_1 and T_2 ($T_2 > T_1$). The cylinder is divided into three compartments by means of movable pistons composed of perfectly reflecting material and furnished with openings which can be closed by means of trap-doors. At the beginning we have things arranged as shown in Fig. 371 and the perfect radiator T_1 fills up compartments 1 and 2 with radiant energy, of density $\psi(T_1)$. The density in compartment 3 is greater, namely $\psi(T_2)$.

The trap-door is now closed and the partition moved towards 3, a distance dx , such that the radiation in 2 has the same density as that in 3. The distribution of energy in the spectrum is now the same in compartments 2 and 3, since, if this were not the case, there would of necessity be rays of a certain wave-length λ in compartment 3, having a greater energy density than the waves of corresponding λ in compartment 2. This being the case, we could cover the opening in the moving plate with a screen composed of some material transparent to waves of this particular length, but reflecting all others. On opening the trap-door more energy would pass from 3 to 2 than passed back in the opposite direction, and the density in 2 would become greater. On closing the door and removing the screen, the plate would be moved to the right until the pressure became equal on both sides, furnishing an amount of work A . A small amount of heat leaves the black body at temperature T_1 , in the form of radiation of wave-length λ , to restore the original condition. This heat is the equivalent of the work A . The trap-door is now opened and the plate brought back to its original position, no work being required.

The door is now closed, and the plate which separates 1 and 2 is driven back to its original position (distance dx), by which the work is gained which was originally spent in moving the plate through the distance dx . If now we open the door in the plate, we have the original state of things, the body at temperature T_1 , neither having given up nor received heat, while the other body at temperature T_2 has given up heat corresponding to the amount which passed through the selectively transparent screen, and furnished the work A . By the second law of thermodynamics work cannot be derived by a cyclic process in the case in which a single reservoir gives up heat in such a manner that all of it is transformed into work.

We conclude therefore that when the energy density is the same in compartments 2 and 3, the distribution of energy in the spectrum is also the same.

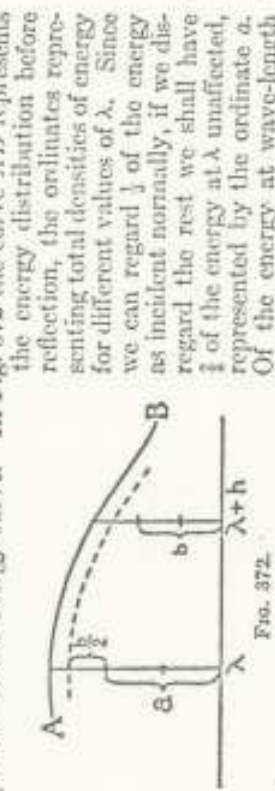
Going back now to the main part of the problem. The motion of the piston which condensed the energy in 2 until it had the same density as that in 3 is accompanied by a shortening of the wavelengths of the reflected energy, for a moving mirror will, by Doppler's principle, alter the lengths of all waves incident upon it. Though the shortening depends on the velocity of the mirror, in the present case it depends only upon the total distance through which the mirror moves. This is due to the fact that the rays are repeatedly reflected from the mirror, and if the mirror moves with slow velocity more reflections will occur from it during its movement, the increased number of reflections compensating for the

slower velocity. For normal incidence, if the velocity of the mirror is v , the wave-length after one reflection is

$$\lambda' = \frac{c-2v}{c}\lambda.$$

As we have seen, we can regard $\frac{1}{2}$ of the total radiation as incident normally, and we will assume that a single reflection shortens the waves by an amount h .

Plotting the original density distribution for wave-lengths λ and $\lambda + h$, we will determine the effect of a single reflection upon this portion of the energy curve. In Fig. 372 the curve AB represents



the energy distribution before reflection, the ordinates representing total densities of energy for different values of λ . Since we can regard $\frac{1}{2}$ of the energy as incident normally, if we disregard the rest we shall have $\frac{1}{2}$ of the energy at λ unaffected, represented by the ordinate a . Of the energy at wave-length $\lambda + h$, $\frac{1}{2}$ is unaffected (ordinate b), while $\frac{1}{2}$ is shortened in wave-length to the value λ ; consequently this portion must be added above a as indicated. If we do this for all values of λ we shall obtain the dotted curve, which represents the energy distribution after one reflection. If $f_1(\lambda)$ represents this distribution, and $\Phi(\lambda)$ the original distribution, we can express the above change as follows:

$$f_1(\lambda) = \frac{1}{2}\Phi(\lambda) + \frac{1}{2}\Phi(\lambda+h) = \Phi\left(\lambda + \frac{h}{2}\right).$$

If the radiation is reflected n times, we have

$$f_n(\lambda) = \Phi\left(\lambda + \frac{nh}{2}\right).$$

The change in the distribution of energy can thus be represented by considering $\frac{1}{2}$ of the rays as shortened by an amount nh .

If now $(a-x)$ is the distance between the pistons, we have for n , while the piston moves a distance dx , $n = \frac{dx}{2(a-x)v}$, in which c is the velocity of the radiation and v the piston's velocity. After n -fold reflection, we have

$$\lambda_n = \left(\frac{c-2v}{c}\right)^n \lambda = \left(\frac{c-2v}{c}\right)^{\frac{dx}{2(a-x)v}} \lambda = \left[\left(1 - \frac{2v}{c}\right)^{\frac{dx}{2(a-x)v}}\right]^2 \lambda.$$

For the limit $c = \infty$,

$$\lambda_n = e^{-\frac{2v}{c} \frac{dx}{2(a-x)v}} \lambda, \text{ since limit of } \left(1 + \frac{\theta}{n}\right)^n = e^\theta, \text{ when } n = \infty,$$

or writing $\lambda_n = \lambda + nh$, in which nh is infinitely small, of the order dx ,

$$nh = -\frac{dx}{a-x}\lambda.$$

Now $f(\lambda)$, the distribution of energy after n -fold reflection, is given by

$$f(\lambda) = \Phi\left(\lambda + \frac{nh}{2}\right) = \Phi(\lambda + d\lambda);$$

$$\therefore d\lambda = -\frac{dx}{3(a-x)}\lambda,$$

which gives us the change of wave-length due to the motion of the piston through a distance dx .

Integrating the above, $\lambda = \sqrt{\frac{2}{3} \frac{a-x}{a}} \lambda_0$, in which λ_0 is the value for $x=0$, i.e. before the motion of the piston commenced.

Let E be the total energy in compartment 2, when $x=0$: its density is then

$$\Psi = \frac{E}{a-x}.$$

If x increases by dx , the energy intensity is increased by diminution of volume, and work done against the radiation pressure, by an amount

$$\frac{d\Psi}{dx} dx = \left\{ \frac{dE}{dx} \frac{1}{a-x} + \frac{E}{(a-x)^2} \right\} dx = \left\{ \frac{dE}{dx} + \Psi \right\} \frac{dx}{a-x}.$$

The pressure on the piston is $\frac{1}{3}\Psi$, therefore the work is

$$\frac{dE}{dx} dx = \frac{1}{3} \Psi dx, \text{ and } d\Psi = \frac{4}{3} \frac{\Psi}{a-x} dx,$$

$$\Psi = \left(\frac{a}{a-x}\right)^{\frac{4}{3}} \Psi_0,$$

in which Ψ_0 is written in place of the original $\Psi = \frac{E}{a-x}$, for $x=0$,

$$\frac{\Psi}{\Psi_0} = \left(\frac{a}{a-x}\right)^{\frac{4}{3}}, \text{ and as we have seen above, } \frac{\lambda}{\lambda_0} = \left(\frac{a-x}{a}\right)^{\frac{1}{3}};$$

$$\therefore \frac{\Psi}{\Psi_0} = \frac{\lambda_0^4}{\lambda^4}.$$

We can now write the expression $\frac{\Psi}{\Psi_0}$ in terms of the absolute temperatures, by applying the Stefan-Boltzmann law. If Ψ_0 is the energy at T_0 and Ψ the energy at T ,

$$\frac{\Psi}{\Psi_0} = \frac{T^4}{T_0^4},$$

and we get at once $T\lambda = T_0\lambda_0$.

This expression may be interpreted in the following way: On raising the temperature of a black body from T_1 to T_2 the ordinates of our energy curve move towards the short wave-lengths by an amount such that the product of the corresponding abscissa and the temperature remains constant for each ordinate. The maximum ordinate at say wave-length λ_m for temperature T_1 will therefore at temperature T_2 occupy the position λ'_m such that $\lambda_m T_1 = \lambda'_m T_2$; in other words, as the temperature rises, the summit of the energy curve drifts towards the region of shorter wave-lengths.

We can find, by the aid of this displacement law, the distribution of energy for any temperature, if the distribution for some given temperature is known.

Plot as before a $\phi(\lambda)$ and λ energy curve; the area of the curve is equal to the total energy Ψ .

To pass to a curve for another temperature, we take a vertical strip at λ_0 of width $d\lambda_0$, the area of which is $\phi_0 d\lambda_0$; this strip is displaced by the temperature change to, say, λ .

The quantity of energy in the strip $\phi_0 d\lambda_0$ must remain constant.

$$\therefore \phi_0 d\lambda_0 = \phi d\lambda,$$

$$\phi = \phi_0 \frac{d\lambda_0}{d\lambda} = \phi_0 \frac{T_1}{T_2}.$$

It will be observed that thus far we have neglected the circumstance that the total energy increases with the temperature, as represented by Stefan's law.

Taking this into account by itself, we have

$$\phi = \phi_0 \frac{T_1^4}{T_2^4}, \text{ and by combining this with } \phi = \phi_0 \frac{T_1}{T_2},$$

we obtain as the complete expression

$$\phi = \phi_0 \frac{T_1^5}{T_2^5},$$

and our new ordinate at λ must be equal to ordinate at λ_0 , multiplied by the ratio $\frac{T_1^5}{T_2^5}$.

Wien's two laws as applied to the wave-length at which we have the maximum energy may be written as follows:

$$\lambda_m T = A \text{ (const.),}$$

$$E_m T^3 = B \text{ (const.),}$$

in which E_m is the energy at the maximum.

Lummer and Pringsheim have tested these two laws by measuring the energy curves of a heated black body over a range 621-1650 Absolute. Their results are given in the following table:

T	λ_m	E_m	$A = \lambda_m T$	$B = E_m T^3$
1650	1.78	270	2928	2246 · 10 ⁻¹⁷
1350	2.35	69	2653	2176
1034	2.71	34	2656	2166
908	3.28	13.6	2980	2308
723	4.08	4.3	2950	2106
621	4.53	2.03	2814	2190

Neither of the two equations, however, give us any information regarding the actual distribution of energy in the spectrum of a black body.

To express this we require an expression which represents E as a function of λ and T .

A number of formulae have been developed which we will briefly discuss.

Complete Radiation Formulae. — Wien (*Wied. Ann.*, 58, p. 662), by the consideration of a peculiar type of radiator, deduced a formula connecting E with λ and T . The radiator is considered as a hollow vessel filled with a gas mixture capable of emitting waves of all lengths. He assumes that every molecule emits only a single wave-length, which depends on its velocity, the intensity of which wave is a function of this velocity. Further, the intensity $\phi(\lambda)$ of the radiation between the limits λ and $\lambda + d(\lambda)$ is proportional to the number of molecules vibrating with periods corresponding to wave-lengths within this range. From these assumptions he derived the formula

$$E = \frac{c}{\lambda^5} e^{-\frac{h}{\lambda T}}.$$

This formula represents the energy distribution very well if it is not applied to too long waves. Lord Rayleigh has pointed out that the energy at a definite wave-length, as represented by the formula, does not increase indefinitely with the temperature, but approaches a limit. For visible waves this limit would only be reached at temperatures far beyond our command, but for $\lambda = 60 \mu$ (Rest-strahlen from sylvite) the limiting value of the intensity would occur at about 1000°.

Planck has deduced a radiation formula of different form from electro-magnetic considerations:

$$E = \frac{c\lambda^{-4}}{e^{h/\lambda T} - 1}.$$

This formula has been confirmed in a remarkable manner by the work of Rubens and Kurlbaum (*Ann. der Physik*, 4, p. 519, 1901), who measured the intensity of the radiation of $\lambda = 51 \mu$ emitted by a black body over a temperature range comprised between 85 and 1773 Absolute. Their results are given in the following table,

together with values calculated from the formulae of Wien and Planck:

T	E OBS.	E CAL. (WIEN)	E CAL. (PLANCK)
85	-20.6	-107	-21.9
193	-11.8	-48	-12
293	0	0	0
523	+31	+63	+30.4
773	64.5	90	63.3
1023	98.1	118	97.2
1273	132	132	132
1523	164	141	166
1773	195	147	200
∞	—	194	∞

This table shows not only the close agreement between the observed values and those calculated from Planck's formula, but also that at a temperature of 1773 a value of E was obtained, larger than the limiting value 194 for infinite temperature, calculated from Wien's formula.

For short waves Wien's formula is perfectly satisfactory. It is questionable, however, whether it is anything more than an empirical formula, for many objections have been raised against the methods employed in its deduction. The energy curves for a black body are represented in Fig. 373 for temperatures between 750 and 1650. The shift of the maximum towards the region of shorter wavelengths with increasing temperature is clearly brought out. These curves were made from observations by Lummer and Pringsheim.

In the case of the sun's spectrum the point of maximum energy is shifted much nearer to the visible region. The solar intensity curve, as actually measured, is of course greatly modified by atmospheric absorption. Fig. 374 is from a record obtained by Langley, and shows the relative energy distribution in the solar spectrum and in the spectrum of a black body at various temperatures. The spectra were obtained by means of a prism, which by crowding the energy, so to speak, in the infra-red portion, on account of the small dispersion in this region, does not give us a true value for the position of the maximum. The deep valleys in the solar curve represent atmospheric absorption.

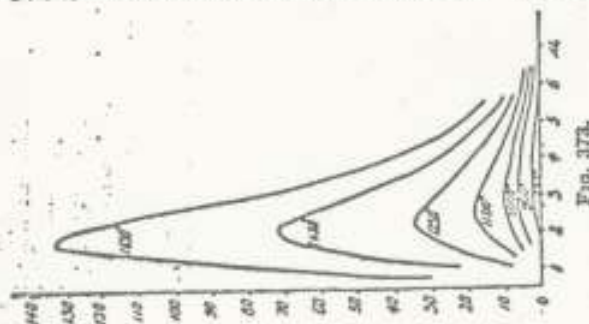


FIG. 373.

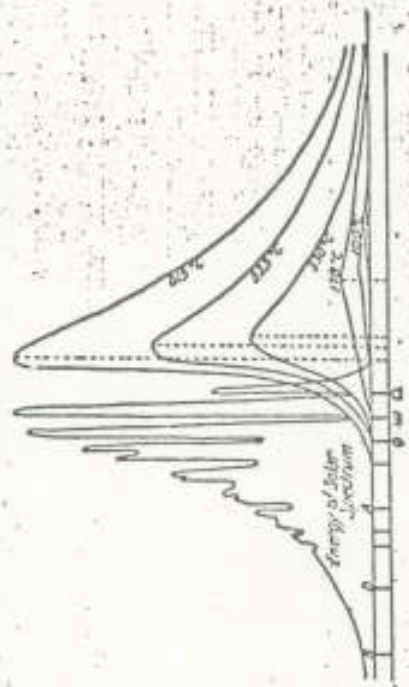


FIG. 374.

Energy Distribution in the Spark Spectrum. — As we have seen, there is a very rapid drop in the energy as we pass from the red to the violet of the spectrum of a white-hot body. Pfüger, in a series of remarkably interesting experiments, has found that in general the reverse is true in the case of the bright-line spark spectra. He investigated the energy distribution in the spark spectra of a large number of metals with a bolometer, and found that the largest deflections were obtained in the remote ultra-violet: in other words, the ultra-violet lines were "hotter" than the red or infra-red. We have

as yet no law governing the energy distribution in discontinuous spectra, but Pfüger's experiments probably may be taken as marking the beginning of a subject to which a chapter may be devoted in text-books twenty years hence. His results for the zinc spark are given in the following table, from which an energy curve can be constructed. It is instructive to compare the curve thus obtained with the curves in the last section.

WAVE-LENGTHS	GAL. DEFL.	WAVE-LENGTHS	GAL. DEFL.	WAVE-LENGTHS	GAL. DEFL.
199	50	244	35	515	80
200	195	252	85	570	45
203	225	261	20	650	50
204	205	275	25	800	80
206	280	335	60	900	85
206+	100	360	15	1.0	80
208	220	395	45	1.2	75
211	60	465	40	1.4	60

Deflections under 10
From here on, decreasing

¹ Annalen der Physik, 13, p. 800, 1904.