for wat[illegible] 20°C, surface tension $T = 72.7$ dynes/cm, and the density $d = 1.00$. Plot v ag[illegible]st λ for water waves from $\lambda = 0$ to 30 cm, and graphically determine the group velocity for $\lambda = 1$ cm, 4 cm, and 25 cm.

19. Differentiate the equation for wave velocity of water waves in Prob. 18 and calculate the group velocity for $\lambda = 25$ cm, 50 cm, and 75 cm.

20. Two sources A and B emit waves of equal amplitude with phase angles of either 0° or 180° only. Show that, with a random distribution of these phases among the sources, the average total intensity will be two times that for any one of them alone. (NOTE: There are four phase combinations.)

21. Three sources A, B, and C emit waves of equal amplitude with phase angles of either 0° or 180° only. Show that with a random distribution of these phases among the sources the average total intensity will be three times that for any one of them alone. (NOTE: There are eight phase combinations.)

22. In Wiener's experiment a photographic film 5 cm long is placed in contact with a mirror and one edge then raised by inserting a strip of paper, 0.002 cm thick, between the two. Find the band separation to be found on the film if light of wavelength 5000 A is used.

23. Write expressions for two simple periodic motions at right angles which when combined will produce resultant motions of the following types: (a) A linear motion of amplitude 3 and frequency 20 per sec, along a line making an angle of 60° with the positive x axis. (b) A circular motion of radius 3 and frequency 20 per sec, the center at the origin, and the initial position of the reference point at +3 on the y axis. (c) An elliptical motion with semiminor axis of 2 in the x direction, semimajor axis of 4 in the y direction, frequency of 20 per sec, and initial position of +2 on the x axis.

24. Write equations for two simple periodic motions at right angles [*i.e.*, $y = f_1(t)$, $x = f_2(t)$] that give (a) a linear motion of amplitude 5 and frequency 10 per sec along a line making an angle of 50° with the +x axis, (b) the same as (a) but with an angle of 150°, and (c) a circular motion of radius 5, frequency 10 per sec, and center at the origin.

25. For the type of waves described in Prob. 18, find the exact value of the wavelength for which the wave and group velocities are equal, and determine this velocity.

CHAPTER 13

INTERFERENCE OF TWO BEAMS OF LIGHT

It was stated at the beginning of the last chapter that two beams of light may be made to cross each other without either one producing any modification of the other after it passes beyond the region of crossing. In this sense the two beams do not interfere with each other. However, in the region of crossing, where both beams are acting at once, we are led to expect from the considerations of the preceding chapter that the resultant amplitude and intensity may be very different from the sum of those contributed by the two beams acting separately. This modification of intensity obtained by the superposition of two or more beams of light we call *interference.* If the resultant intensity is zero or in general less than we expect from the separate intensities, we have *destructive* interference, while if it is greater, we have *constructive* interference. The phenomenon in its simpler aspects is rather difficult to observe, because of the very short wavelength of light, and therefore was not recognized as such in the time prior to 1800 when the corpuscular theory of light was predominant. The first man successfully to demonstrate the interference of light, and thus establish its wave character, was Thomas Young. In order to understand his crucial experiment performed in 1801, we must first consider the application to light of an important principle which holds for any type of wave motion.

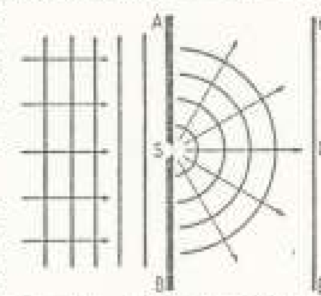


FIG. 13A. Diffraction of waves at a small aperture.

13.1. Huygens' Principle. When waves pass through an aperture, or past the edge of an obstacle, they always spread to some extent into the region which is not directly exposed to the oncoming waves. This phenomenon is called *diffraction.* In order to explain this bending of light, Huygens nearly three centuries ago proposed the rule that *each point on a wave front may be regarded as a new source of waves.** This principle

* The "waves" envisioned by Huygens were not continuous trains but rather a series of random pulses. Furthermore, he supposed the secondary waves to be effective only at the point of tangency to their common envelope, thus denying the possibility of diffraction. The correct application of the principle was first made by Fresnel, more than a century later.

has very far-reaching applications and will be used later in discussing the diffraction of light, but we shall consider here only a very simple proof of its correctness. In Fig. 13*A* let a set of plane waves approach the barrier *AB* from the left, and let the barrier contain an opening *S* of width somewhat smaller than the wavelength. At all points except *S* the waves will be either reflected or absorbed, but *S* will be free to produce a disturbance behind the screen. It is found experimentally, in agreement with the above principle, that the waves spread out from *S* in the form of semicircles.

Huygens' principle as shown in Fig. 13*A* can be illustrated very successfully with water waves. An arc lamp on the floor, with a glass-bottomed tray or tank above it, will cast shadows of waves on a white ceiling. A vibrating strip of metal or a wire fastened to one prong of a tuning fork of low frequency will serve as a source of waves at one end of the tray. If an electrically driven tuning fork is used, the waves may be made apparently to stand still by placing a slotted disk on the shaft of a motor in front of the arc lamp. The disk is set rotating with the same frequency as the tuning fork to give the stroboscopic effect. The latter experiment can be performed for a fairly large audience and is well worth doing. Descriptions of diffraction experiments in light will be given in Chap. 15.

If the experiment in Fig. 13*A* be performed with light, one would naturally expect, from the fact that light generally travels in straight lines, that merely a narrow patch of light would appear at *D*. However, if the slit is made very narrow, an appreciable broadening of this patch is observed, its breadth increasing as the slit is further narrowed. This remarkable evidence that light does not always travel in straight lines was mentioned at the very beginning of this book (Sec. 1.1 and Fig. 1A). When the screen *CE* is replaced by a photographic plate, a picture like the one shown in Fig. 13*B* is obtained. The light is most intense in the forward direction, but its intensity decreases slowly as the angle increases. If the slit is small compared with the wavelength of light, the intensity does not come to zero even when the angle of observation becomes 90° (Sec. 15.3). While this brief introduction to Huygens' principle will be sufficient for an understanding of the interference phenomena we are to discuss, we shall return in Chaps. 15 and 18 to a more detailed consideration of diffraction at a single opening.

13.2. Young's Experiment. The original experiment performed by Young is shown schematically in Fig. 13*C*. Sunlight was first allowed

to pass through a pinhole *S* and then, at a considerable distance away, through two pinholes S_1 and S_2. The two sets of spherical waves emerging from the two holes interfered with each other in such a way as to form a symmetrical pattern of varying intensity on the screen *AC*. Since this early experiment was performed, it has been found convenient to replace the pinholes by narrow slits and to use a source giving monochromatic light, *i.e.*, light of a single wavelength. In place of spherical wave fronts we now have cylindrical wave fronts, represented equally well in

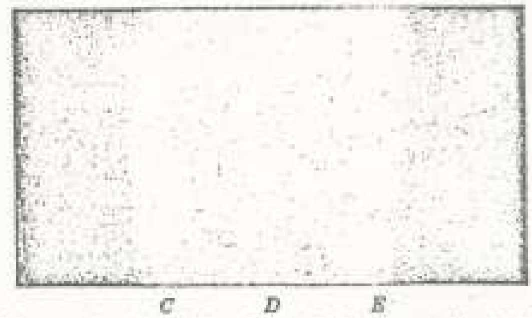


FIG. 13*B*. Photograph of the diffraction of light from a slit of width 0.001 mm.

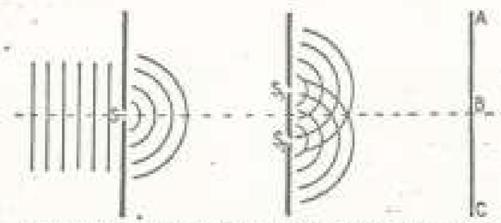


FIG. 13*C*. Experimental arrangement for Young's double-slit experiment.

two dimensions by the same Fig. 13*C*. If the circular lines represent crests of waves, the intersections of any two lines represent the arrival at those points of two waves with the same phase or with phases differing by a multiple of 2π. Such points are therefore those of maximum disturbance or brightness. A close examination of the light on the screen will reveal evenly spaced light and dark bands or fringes, similar to those shown in Fig. 13*D*. Such photographs are obtained by replacing the screen *AC* of Fig. 13*C* by a photographic plate.

A very simple demonstration of Young's experiment can be accomplished in the laboratory or lecture room by setting up a single-filament lamp *L* (Fig. 13*E*) at the front of the room. The straight vertical filament *S* acts as the source and first slit. Double slits for each observer can be easily made from small photographic plates about 1 to 2 in. square. The slits are made in the photographic emulsion by drawing the point of a penknife across the plate, guided by a straight edge. The plates need not be developed or blackened but can be used as they are. The lamp is now viewed by holding the double slit D close to the eye E and looking at the lamp filament. If the slits are close together, *e.g.*, 0.2 mm apart, they give widely spaced fringes, whereas slits farther apart, *e.g.*, 1 mm, give very narrow fringes. A piece of red glass F, placed adjacent to and above another of green glass in front of the lamp, will show that the red waves produce wider fringes than the green, which we shall see is due to their greater wavelength.

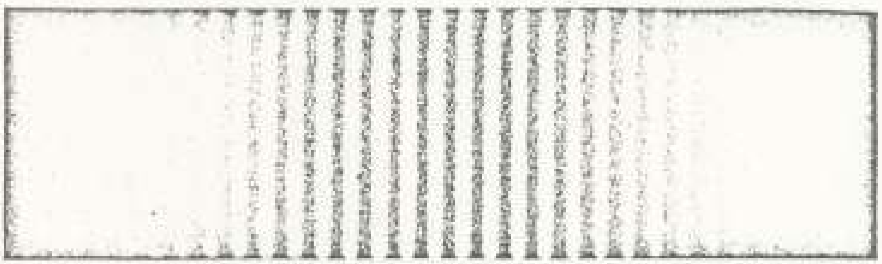


FIG. 13D. Interference fringes produced by a double slit using the arrangement shown in Fig. 13C.

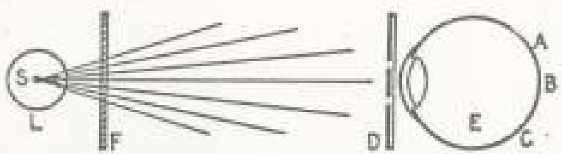


FIG. 13E. Simple method for observing interference fringes.

Frequently one wishes to perform accurate experiments by using more nearly monochromatic light than that obtained by white light and a red or green glass filter. Perhaps the most convenient method is to use the sodium arc now available on the market, or a d-c carbon arc arranged as follows: A small $\frac{1}{8}$-in. hole, 1 in. deep, is drilled in the end of the positive carbon and filled with common salt. When the arc has run several minutes, the hole is refilled. After several refillings a very bright source of sodium light, almost entirely of wavelength 5893 A, is obtained. Monochromatic green light can be obtained from any mercury arc by sending the light through a special glass filter now on the market. Such a filter transmits only the green line, λ5461.

13.3. Interference Fringes from a Double Source. We shall now give an equation for the intensity at any point P on the screen (Fig. 13F) and investigate the spacing of the interference fringes. Assuming that the source slit S is equidistant from S_1 and S_2, the light vibrations at the two slits will be in the same phase at any instant, and we may represent either of them by the equation (Eq. 11*a*) of simple periodic motion,

$$y = r \sin 2\pi \frac{t}{T}$$

where r is the amplitude, T the period, and t the time.

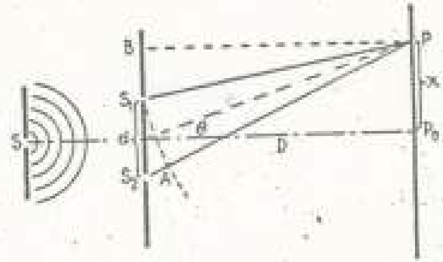


FIG. 13F. Schematic diagram of the optical paths of interfering beams in Young's experiment.

In the wave traveling from S_1 to P, the phase difference will have the value $2\pi S_1P/\lambda$ and hence at the point P will have the motion

$$y_1 = r_1 \sin 2\pi \left(\frac{t}{T} - \frac{S_1P}{\lambda}\right)$$

for the light from S_1, and the motion

$$y_2 = r_2 \sin 2\pi \left(\frac{t}{T} - \frac{S_2P}{\lambda}\right)$$

for the light from S_2. These two equations represent simple periodic motions of the same frequency, and we have exactly the problem treated in Sec. 12.1 by the principle of superposition. Our present equations have the form of Eqs. 12*a*, namely,

$$y_1 = r_1 \sin (\omega t + \alpha_1)$$
$$y_2 = r_2 \sin (\omega t + \alpha_2)$$

where now $\omega = 2\pi/T$, $\alpha_1 = -2\pi S_1P/\lambda$, and $\alpha_2 = -2\pi S_2P/\lambda$. Therefore, from Eq. 12*f* we may write for the resultant motion

$$y = y_1 + y_2 = R \sin (\omega t + \theta)$$

in which R and θ are to be found from Eqs. 12d and 12e, respectively. The new phase constant θ is of no interest to us here, as we are primarily interested in the resultant intensity, which is proportioned to R^2.

If, as is usually the case, the two slits S_1 and S_2 are of equal width and very close together, the amplitudes r_1 and r_2 will be so nearly equal that we may assume them to be so and put $r_1 = r_2 = r$. Then we have from Eq. 12g, for the resultant intensity,

$$I = R^2 = 4r^2 \cos^2 \frac{\delta}{2} \qquad (13a)$$

where δ is the phase difference $\alpha_1 - \alpha_2$ between the two superimposed vibrations, given by the relation

$$\delta = \frac{2\pi}{\lambda} \cdot (\text{path difference}) = \frac{2\pi}{\lambda}(S_2P - S_1P) \qquad (13b)$$

It now remains to find an expression for the path difference in terms of the distance x on the screen from the central point P_0, the separation of the slits d, and the distance D from the slits to the screen. From the relations between the squares on the sides of a right triangle, we first write for the triangle BPS_2

$$(S_2P)^2 = D^2 + \left(x + \frac{d}{2}\right)^2$$

and for the triangle BPS_1

$$(S_1P)^2 = D^2 + \left(x - \frac{d}{2}\right)^2$$

Taking the difference between these two equations, D drops out, and

$$(S_2P)^2 - (S_1P)^2 = \left(x + \frac{d}{2}\right)^2 - \left(x - \frac{d}{2}\right)^2$$

which gives

$$(S_2P)^2 - (S_1P)^2 = 2xd$$

Factoring the left side of this equation,

$$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

or

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

In general, when Young's experiment is performed, D is some thousand times larger than d or x, so that $S_2P + S_1P$ may be replaced by $2D$ without altering the equality by more than a very small fraction of 1 per cent. We find

$$S_2P - S_1P = \frac{2xd}{2D} = \frac{xd}{D} \qquad (13c)$$

This is the value of the path difference to be substituted in Eq. 13b to obtain the phase difference δ. Now Eq. 13a for the intensity has maximum values of $4r^2$ whenever δ is an integral multiple of 2π, and according to Eq. 13b this will occur when the path difference is an integral multiple of λ. Hence we have

$$\frac{xd}{D} = 0, \lambda, 2\lambda, 3\lambda, \cdots = m\lambda$$

or

$$x = m\lambda \frac{D}{d} \qquad \text{BRIGHT FRINGES} \qquad (13d)$$

The minimum value of the intensity is zero, and as is seen from Eq. 13a this occurs when $\delta = \pi, 3\pi, 5\pi, \cdots$. For these points

$$\frac{xd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \cdots = \left(m + \frac{1}{2}\right)\lambda$$

or

$$x = \left(m + \frac{1}{2}\right)\lambda \frac{D}{d} \qquad \text{DARK FRINGES} \qquad (13e)$$

The whole number m, which characterizes a particular bright fringe, is called the *order* of interference. Thus the fringes with $m = 0, 1, 2, \cdots$ are called the zero, first, second, etc., orders.

The distance on the screen between two fringes of orders m and $m + 1$ may be obtained from Eq. 13d by taking the difference

$$x_{m+1} - x_m = (m + 1)\lambda \frac{D}{d} - m\lambda \frac{D}{d} = \lambda \frac{D}{d} \qquad (13f)$$

It is the same as the separation between dark fringes,

$$x_{m+\frac{1}{2}} - x_{m-\frac{1}{2}} = \left(m + \frac{1}{2}\right)\lambda \frac{D}{d} - \left(m - \frac{1}{2}\right)\lambda \frac{D}{d} = \lambda \frac{D}{d} \qquad (13g)$$

According to these equations the spacing of the fringes is constant (independent of m), in agreement with the observed pattern of Fig. 13D. It is directly proportional to the slit-screen distance D, inversely proportional to the separation of slits d, and directly proportional to wavelength λ. A measurement of the spacing of the fringes thus gives us a direct determination of λ in terms of known quantities.

13.4. Intensity Distribution in the Fringe System. Let us now consider the physical reason for the formation of these dark and bright fringes. We have found that when the path difference $S_2P - S_1P$ is a whole number of wavelengths, the point P is the center of a bright fringe. For such a point the additional distance that one wave travels will be S_2A (Fig. 13F), provided the broken curve S_1A is the arc of a circle with P as a center. Thus it is clear that if S_2A contains a whole number of wavelengths, the two waves will reach P in the same phase, and the resultant amplitude will be twice that due to either wave alone. On the other hand, if S_2A is $(m + \frac{1}{2})\lambda$, *i.e.*, an integral number of wavelengths plus an additional half wavelength, the waves reach P exactly in opposite phase, and the resultant amplitude will be zero.

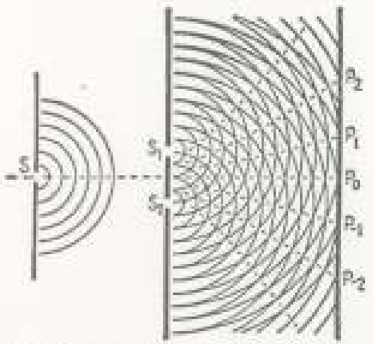


FIG. 13G. Wavelets from a double slit showing reinforcement along directions P_0, P_1, P_2, etc.

Figure 13G shows two complete sets of waves diverging from S_1 and S_2, the semicircles representing the crests of the waves, a distance λ apart. The intensity will be a maximum wherever a crest falls on a crest. This will obviously occur at P_0, which is equidistant from S_1 and S_2, and also at P_1, P_{-1}, P_2, P_{-2}, etc., because each of these points is some whole number of wavelengths farther from one slit than from the other. If the screen is moved toward or away from the slits the spacing of these maxima decreases or increases nearly in direct proportion to the distance; *i.e.*, the disturbance is a maximum at all points in space along the broken lines shown in the figure. These are not straight lines, as would be required by our simple equation 13d. They are actually hyperbolas, since the hyperbola is a curve for which the difference in the distance from two fixed points is a constant. However, when the wavelength is small and the distance to the screen large, the deviation from straight lines is small enough to be negligible for practical purposes.

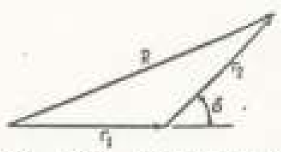


FIG. 13H. Illustrating the composition of two waves of the same frequency and amplitude but different phase.

To find the intensity on the screen at points between the maxima, we apply the vector method of compounding amplitudes described in Sec. 12.2 and illustrated for the present case in Fig. 13H. For the maxima, the angle δ is zero and the component amplitudes r_1 and r_2 are parallel, with the resultant $R = 2r$. For the minima, r_1 and r_2 are in opposite directions and $R = 0$. In general, for any value of δ, R is the

closing side of the triangle. The value of R^2, which measures the intensity, is then given by Eq. 13a and varies according to $\cos^2(\delta/2)$. In Fig. 13I the solid curve represents a plot of the intensity against the phase difference.

In concluding our discussion of these fringes, one question of fundamental importance should be considered. If the two beams of light arrive at a point on the screen exactly out of phase, they interfere destructively and the resultant intensity is zero. One may well ask what becomes of the *energy* of the two beams, since the law of conservation of energy tells us that energy cannot be destroyed. The answer to this question is that the energy which apparently disappears at the minima

FIG. 13I. Intensity distribution for the interference fringes from two beams.

actually is still present at the maxima, where the intensity is greater than would be produced by the two beams acting separately. In other words, the energy is not destroyed but merely redistributed in the interference pattern. The *average* intensity on the screen is exactly that which would exist in the absence of interference. Thus, as shown in Fig. 13I, the intensity in the interference pattern varies between $4r^2$ and zero. Now each beam acting separately would contribute r^2, and so without interference we would have a uniform intensity of $2r^2$, as indicated by the broken line. To obtain the average intensity on the screen for n fringes, we note that the average value of the square of the cosine is $\frac{1}{2}$. This gives, by Eq. 13a, $I = 2r^2$, justifying the statement made above, and it shows that no violation of the law of conservation of energy is involved in the interference phenomenon.

13.5. Fresnel's Biprism.* Soon after the double-slit experiment was performed by Young, the objection was raised that the bright fringes he observed were probably due to some complicated modification of the light by the edges of the slits and not to true interference. Thus the wave theory of light was still questioned. Not many years passed,

* Augustin Fresnel (1788–1827). Most notable French contributor to the theory of light. Trained as an engineer, he became interested in light, and in 1814–1815 he rediscovered Young's principle of interference and extended it to complicated cases of diffraction. His mathematical investigations gave the wave theory a sound foundation.

however, before Fresnel brought forward several new experiments in which the interference of two beams of light was proved in a manner not open to the above objection. The first of these is called the Fresnel biprism experiment.

A schematic diagram of the biprism experiment is shown in Fig. 13J. The thin double prism P refracts the light from the slit source S into

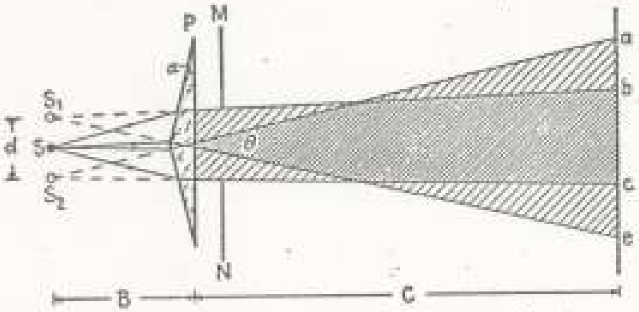


FIG. 13J. Diagram of Fresnel's biprism experiment.

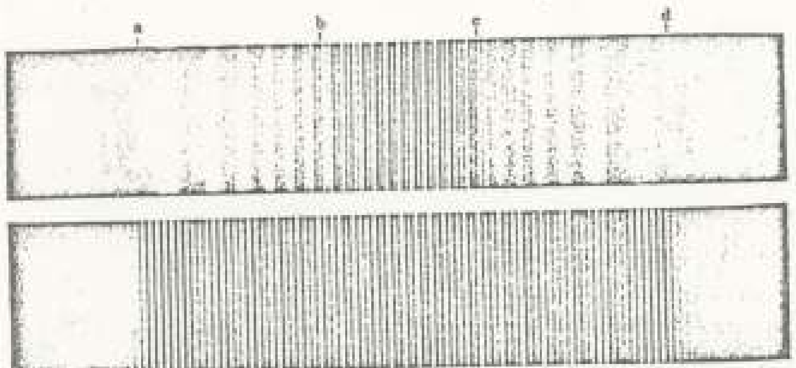


FIG. 13K. Interference and diffraction fringes produced in the Fresnel biprism experiment.

two overlapping beams ae and be. If screens M and N are placed as shown in the figure, interference fringes are observed only in the region bc. When the screen ae is replaced by a photographic plate, a picture like the upper one in Fig. 13K is obtained. The closely spaced fringes in the center of the photograph are due to interference, while the wide fringes at the edge of the pattern are due to diffraction. These wider bands are produced by the vertices of the two prisms, each of which acts as a straight edge, giving a pattern which will be discussed in detail in Chap. 18. When the screens M and N are removed from the light path, the two beams will overlap over the whole region ad. The lower photograph in Fig. 13K shows for this case the equally spaced interference

fringes superimposed on the diffraction pattern of a wide aperture. (For the diffraction pattern above, without the interference fringes, see Fig. 18T.) With such an experiment Fresnel was able to produce interference without relying upon diffraction to bring the interfering beams together.

Just as in Young's double-slit experiment, the wavelength of light can be determined from measurements of the interference fringes produced by the biprism. Calling B and C the distances of the source and screen, respectively, from the prism P, d the distance between the virtual images S_1 and S_2, and Δx the distance between the successive fringes on the screen, the wavelength of the light is given from Eq. 13f as

$$\lambda = \frac{\Delta x d}{B + C} \tag{13h}$$

Thus the virtual images S_1 and S_2 act as did the two slit sources in Young's experiment.

To find d, the separation of the virtual sources, we make use of the fact that for a prism of very small refracting angle the deviation of a ray is given by $(n - 1)\alpha$, n being the index of refraction of the prism and α its refracting angle. Hence, from Fig. 13J,

$$\frac{\theta}{2} = (n - 1)\alpha \tag{13i}$$

Now both α and n may be measured by placing the biprism on a spectrometer, so that θ can be found. Then $d = B\theta$, and we obtain for the wavelength

$$\lambda = \frac{B\theta}{B + C}\Delta x = \frac{2B(n - 1)\alpha}{B + C}\Delta x \tag{13j}$$

This method is rather laborious, and in practice it is much more convenient to measure the angle θ directly on the spectrometer. Parallel light from the collimator, when incident on both halves of the biprism, divides into two beams making an angle θ with each other, as does the central ray from S in Fig. 13J. The angular separation of the two slit images in the telescope is then equal to θ. A still simpler determination of θ may be made by holding the prism close to one eye and viewing a round frosted light bulb. At a certain distance from the light the two images may be brought to the point where their inner edges just touch. The diameter of the bulb divided by the distance from the bulb to the prism then gives θ directly.

Fresnel biprisms are easily made from a small piece of glass, such as half a microscope slide, by beveling about $\frac{1}{8}$ to $\frac{1}{4}$ in. on one side. This requires very little grinding with ordinary abrasive materials and polishing with rouge, since the angle required is only about 1°.

13.6. Fresnel's Mirrors. Another experiment illustrating the interference between two beams of light is known as the Fresnel mirror experiment. Light from a narrow slit S is split into two beams by reflection from two mirrors placed close together so that their planes make a small angle θ with each other, as illustrated in Fig. 13L. On that part of the screen where the two beams overlap, interference fringes are obtained of the type shown in Fig. 13M. The explanation of these fringes is similar to that for the double slit and biprism. After reflection from the mirrors, the light arriving at the screen appears to come from virtual sources

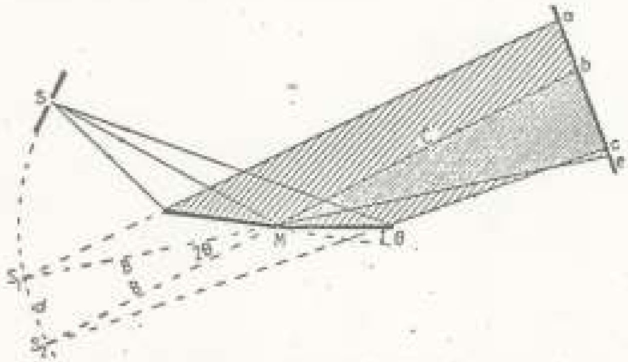


Fig. 13L. Diagram of the Fresnel double-mirror experiment.

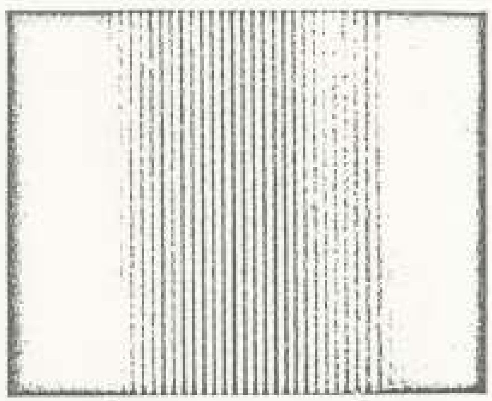


Fig. 13M. Interference fringes produced in the Fresnel double-mirror experiment.

S_1 and S_2. The relations between fringe separation and the geometry of the figure are such that, if we call d the distance between the virtual sources, C the distance Mb, and λ the wavelength, the distance Δx between fringes on the screen will be related to these quantities by Eq. 13h. The interval d can be measured directly by the second or third method described above for the biprism, or it can be calculated from the fact that $d = 2B\theta$ (see Fig. 13L), θ now being the angle between the mirrors. In terms of this angle, the wavelength is given by

$$\lambda = \frac{2B\theta}{B + C}\Delta x \tag{13k}$$

The Fresnel double-mirror experiment is usually performed on an optical bench with the light reflected from the mirrors at nearly grazing angles. Two pieces of ordinary plate glass about 2 in. square make a very good double mirror. One plate should have an adjusting screw for changing the angle θ, and the other a screw for making the two mirror edges parallel.

13.7. Coherent Sources. It will be noticed that the three successful methods of demonstrating interference we have discussed so far have one important feature in common: The two interfering beams are always derived from the same source of light. We find by experiment that it is impossible to obtain interference fringes from two separate sources, such as two lamp filaments set side by side. This failure is caused by the fact that the light from any one source is not an infinite train of waves. On the contrary, there are sudden changes in phase occurring in very short intervals of time (of the order of 10^{-8} sec). This point has already been mentioned in Sec. 11.3. Thus, although interference fringes may exist on the screen for such a short interval, they will shift their position each time there is a phase change, with the result that no fringes at all will be seen. In Young's experiment and in Fresnel's mirrors and biprism, the two sources S_1 and S_2 always have a point-to-point correspondence of phase, since they are both derived from the same source. If the phase of the light from a point in S_1 suddenly shifts, that of the light from the corresponding point in S_2 will shift simultaneously. The result is that the *difference* in phase between any pair of points in the two sources always remain constant, and so the interference fringes are stationary. It is a characteristic of any interference experiment with light that the sources must have this point-to-point phase relation, and sources that have this relation are called *coherent sources*.

If in Young's experiment the source slit S (Fig. 13C) is made too wide, or the angle between the rays which leave it too large, the double slit no longer represents two coherent sources, and the interference fringes disappear. This subject will be discussed in more detail at the end of Chap. 16, The Double Slit.

13.8. Lloyd's Mirror. The experiment known as Lloyd's mirror is important in any treatment of the nature of light, for it shows, in addition to the interference of two coherent beams of light, the phase change of light as it is reflected at grazing incidence from the surface of glass.

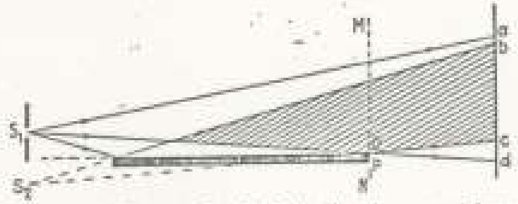
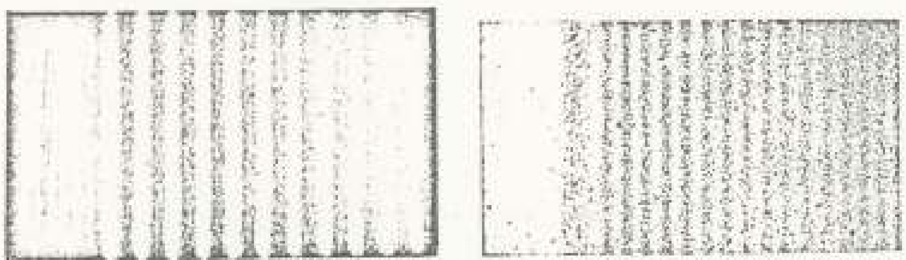


FIG. 13N. Diagram for the Lloyd's-mirror experiment.



(a) Taken with visible light λ4358 A. (*After White.*)

(b) Taken with X rays λ8.33 A. (*After Kellstrom.*)

FIG. 13O. Interference fringes produced with Lloyd's mirror.

Light from a narrow slit S_1, in Fig. 13N, is incident at a grazing angle on the surface of a fairly long and flat strip of glass. The light is reflected from the glass in such a manner that its arrival at the screen is essentially the same as though it started from the virtual source S_2. In addition to the reflected light arriving at the screen there is also the light coming directly from the source S_1 without reflection. In the region of overlapping of these two beams, interference occurs, and it can be observed as a system of fringes on the screen in the region bc.

An important feature of the Lloyd's-mirror experiment lies in the fact that when the screen is placed in contact with the end of the mirror (in the position MN, Fig. 13N), the edge O of the reflecting surface comes at the center of a *dark* fringe, instead of a bright one as might be expected. This means that one of the two beams has undergone a phase change of π.

Since the direct beam could not change phase, this experimental obse... tion is interpreted to mean that the reflected light has changed phase at reflection. Two photographs of fringes formed by the Lloyd's-mirror experiment are reproduced in Fig. 13O, one taken with visible light and the other with X rays.

If the light from source S_1 in Fig. 13N is allowed to enter the end of the glass plate by moving the latter up, and to be internally reflected from the upper glass surface, fringes will again be observed in the interval OP, with a dark fringe at O. This again shows that there is a phase change of π at reflection. As will be shown in Chap. 28, this is not in contradiction with the discussion of phase change given in Sec. 11.8. In this instance the light is incident at an angle greater than the critical angle for total reflection.

The Lloyd's-mirror experiment is readily set up for demonstration purposes as follows: A carbon arc, followed by a colored glass filter and a narrow slit, serves as a source. A strip of ordinary plate glass 1 to 2 in. wide and a foot or more long makes an excellent mirror. A magnifying glass focused on the far end of the mirror enables one to observe the fringes shown in Fig. 13O. Internal fringes can be observed by polishing the ends of the mirror to allow the light to enter and leave the glass, and by roughening one of the glass faces with coarse emery.

13.9. Billet's Split Lens. Another device for producing interference fringes is known as Billet's split lens. In this experiment (Fig. 13P) half lenses are placed close together to form two real images S_1 and S_2 of the slit S. S_1 and S_2 now act in the same way as the double slit in Young's experiment. Fringes are observed in the overlapping region bc. An ordinary lens and a biplate, consisting of two identical plane-parallel plates inclined slightly to each other, will give the same result as a split lens.

13.10. Michelson* Interferometer. This is an instrument designed by Michelson in which light from an extended source is divided into two

* A. A. Michelson (1852–1931). American physicist of great genius. He early became interested in the velocity of light, and began experiments while an instructor in physics and chemistry at the Naval Academy, from which he graduated in 1873. It is related that the superintendent of the Academy asked young Michelson why he wasted his time on such useless experiments. Years later Michelson was awarded the Nobel prize (1907) for his work on light. Much of his work on the velocity of light (Sec. 19.5) was done during 10 years spent at the Case Institute of Technology. During the latter part of his life he was professor of physics at the University of Chicago, where many of his famous experiments on the interference of light were done.

parts by partial reflection. These beams are sent in quite different directions against plane mirrors, whence they are brought together again to form interference fringes. The arrangement is shown schematically in Fig. 13Q. The main optical parts consist of two highly polished plane mirrors M_1 and M_2 and two plane-parallel plates of glass G_1 and G_2. Sometimes the rear side of the plate G_1 is lightly silvered (shown by the

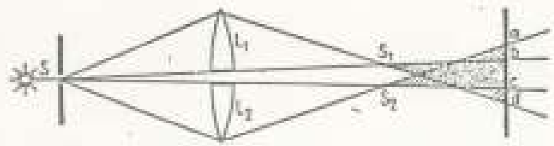


FIG. 13P. Diagram of Billet's split lens for producing interference fringes.

heavy line in the figure) so that the light coming from the source S is divided into (1) a reflected and (2) a transmitted beam of equal intensity. The light reflected normally from mirror M_1 passes through G_1 a third time and reaches the eye as shown. The light reflected from the mirror M_2 passes back through G_2 for the second time, is reflected from the surface of G_1 and into the eye. The purpose of the plate G_2, called the compensating plate, is to render the path *in glass* of the two rays equal. This is not essential for producing fringes in monochromatic light, but it is indispensable when white light is used (Sec. 13.13). The mirror M_1 is mounted on a carriage C and can be moved along the well-machined ways or tracks T. This slow and accurately controlled motion is accomplished by means of the screw V which is calibrated to show the exact distance the mirror has been moved. To obtain fringes, the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws shown on mirror M_2.

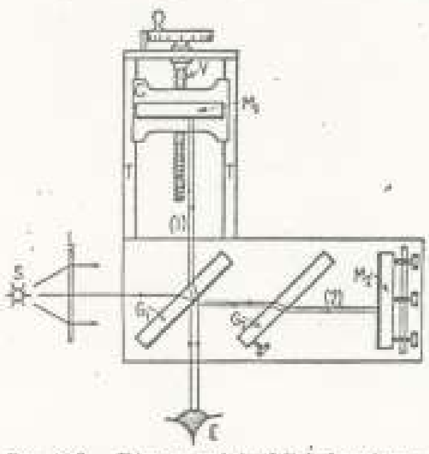


FIG. 13Q. Diagram of the Michelson interferometer.

Even when the above adjustments have been made, fringes will not be seen unless two important requirements are fulfilled. First, the light must originate from an *extended* source. A point source or a slit source, as used in the methods previously described, will not produce the desired system of fringes in this case. The reason for this will appear when we consider the origin of the fringes. Second, the light must in general be

monochromatic, or nearly so. Especially is this true if the distances of M_1 and M_2 from G_1 are appreciably different.

An extended source suitable for use with a Michelson interferometer may be obtained in any one of several ways. A sodium flame or a mercury arc, if large enough, may be used without the screen L shown in Fig. 13Q. If the source is small, a ground glass screen or a lens at L will extend the field of view. Looking at the mirror M_1 through the plate G_1, one then sees the whole mirror filled with light. In order to obtain the fringes, the next step is to measure the distances of M_1 and M_2 to the back surface of G_1 roughly with a millimeter scale, and to move M_1 until they are the same to within a few millimeters. The mirror M_2 is now adjusted to be perpendicular to M_1 by observing the images of a common pin, or any sharp point, placed between the source and G_1.

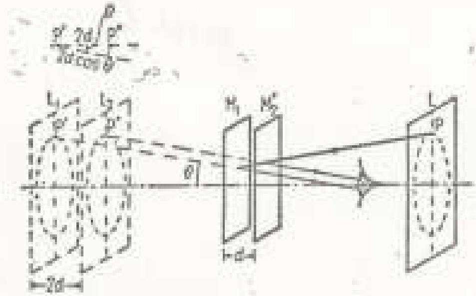


FIG. 13R. Diagram illustrating the formation of circular fringes in the Michelson interferometer.

Two pairs of images will be seen, one coming from reflection at the front surface of G_1 and the other from reflection at its back surface. When the tilting screws on M_2 are now turned until one pair of images falls exactly on the other, the interference fringes should appear. When they first appear, the fringes will not be clear unless the eye is focused on or near the back mirror M_1, so the observer should look constantly at this mirror while searching for the fringes. When the fringes have been found, the adjusting screws are turned in such a way as to continually increase the width of the fringes, and finally a set of concentric circular fringes will be obtained. M_2 is then exactly perpendicular to M_1, if the latter is at an angle of 45° with G_1.

13.11. Circular Fringes. These are produced with monochromatic light when the mirrors are in exact adjustment, and are undoubtedly the most important type of fringes obtained with the Michelson interferometer. Their origin may be understood by reference to the diagram of

Fig. 13R. Here the real mirror M_2 has been replaced by its virtual image M_2' formed by reflection in G_1. M_2' is then parallel to M_1. Owing to the several reflections in the real interferometer, we may now think of the extended source as being behind the observer at L and forming two virtual images L_1 and L_2 in M_1 and M_2'. These virtual sources are coherent in that the phases of corresponding points in the two are exactly the same at all instants. If d is the separation M_1M_2', the virtual sources will be separated by $2d$. When d is exactly an integral number of half wavelengths, *i.e.*, the path difference $2d$ equal to an integral number of whole wavelengths, all rays of light reflected normal to the mirrors will be in phase. Rays of light reflected at an angle, however, will in general not be in phase. The path difference between the two rays coming to the eye from corresponding points P' and P'' is $2d \cos \theta$, as shown in the figure. The angle θ is necessarily the same for the two rays when M_1 is parallel to M_2' so that the rays are parallel. Hence when the eye is focused to receive parallel rays (a small telescope is more satisfactory here, especially for large values of d) the rays will reinforce each other to produce maxima for those angles θ satisfying the relation*

$$2d \cos \theta = m\lambda \qquad (13l)$$

Since for a given m, λ, and d the angle θ is constant, the maxima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors. By expanding the cosine, it can be shown from Eq. 13l that the radii of the rings are proportional to the square roots of integers, as in the case of Newton's rings (Sec. 14.4). The intensity distribution across the rings follows Eq. 13a, in which the phase difference δ is given by

$$\delta = \frac{2\pi}{\lambda} 2d \cos \theta$$

With monochromatic light the circular fringes are visible for very large path differences, the limit being set only by the fact that no actual source gives perfectly monochromatic light. If there is even a small range of wavelengths present in the light from the source, the fringes formed by the different components will be differently spaced and will mask all interference at sufficiently large values of d. Using the very nearly monochromatic light of the red cadmium line, the fringes remain visible up to path differences of about 50 cm, or $d = 25$ cm. A study of the change of clearness or "visibility" of the fringes† with increasing path

* Under these conditions *minima* may be observed (see discussion at the end of Sec. 13.13).

† The visibility of fringes is quantitatively defined as $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$, where I_{max} and I_{min} are the intensities at the maxima and minima of the fringe pattern.

difference gives information about the sharpness of a spectral line used for the source of light. The larger the path difference over which the fringes remain clear, the more monochromatic the light, or the sharper the line. This was one of the first uses to which Michelson put his interferometer. Maxima and minima in the visibility curve indicate that the line has a fine structure of two or more components. Thus it is found that with light of the sodium D lines the fringes will become alternately sharp and diffuse, as the fringes formed by the two lines get in and out of step. The number of fringes between two positions of maximum sharpness is about 1000, indicating that the wavelengths of the D lines differ by about 1 part in 1000. Since Michelson's method of inferring the structure of lines has been superseded by a more direct method (Sec. 14.6), it will not be described in detail here.

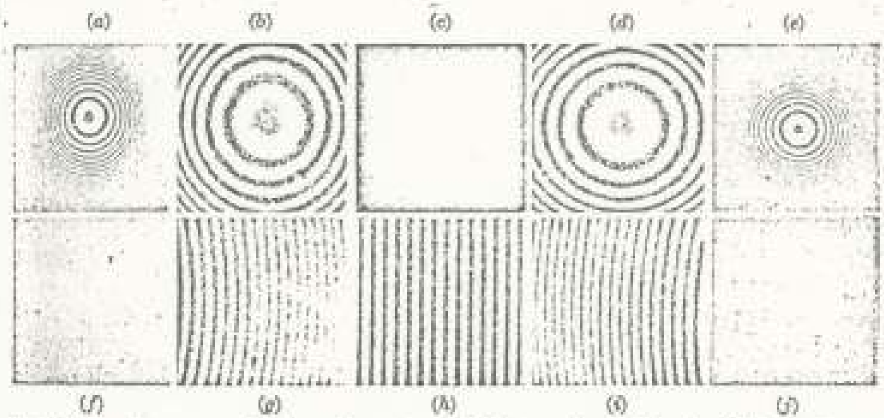


Fig. 13S. Appearance of the various types of fringes observed in the Michelson interferometer.

Starting with M_1 a few centimeters beyond M_2', the fringe system will have the general appearance shown in (a) of Fig. 13S, with the rings very closely spaced. If M_1 is now moved slowly toward M_2' so that d is decreased, Eq. 13l shows that a given ring, characterized by a given value of the order m, must decrease its radius because the product $2d \cos \theta$ must remain constant. The rings therefore shrink and vanish at the center, a ring disappearing each time $2d$ decreases by λ, or d by $\lambda/2$. This follows from the fact that at the center $\cos \theta = 1$, so that Eq. 13l becomes

$$2d = m\lambda \qquad (13m)$$

To change m by unity, d must change by $\lambda/2$. Now as M_1 approaches M_2' the rings become more widely spaced, as indicated in Fig. 13S(b), until finally we reach a critical position where the central fringe has spread out to cover the whole field of view, as shown in (c). This happens when M_1 and M'_2 are exactly coincident, for it is clear that under these conditions the path difference is zero for all angles of incidence. If the mirror is moved still farther, it effectively passes through M'_2, and new widely spaced fringes appear, growing out from the center. These will gradually become more closely spaced as the path difference increases, as indicated in (d) and (e) of the figure.

13.12. Localized Fringes. If the mirrors M'_2 and M_1 are not exactly parallel, fringes will still be seen with monochromatic light for path differences not exceeding a few millimeters. In this case the space between the mirrors is wedge-shaped, as indicated in Fig. 13T. The two rays* reaching the eye from a point P on the source are now no longer parallel, but appear to diverge from a point P' near the mirrors. Thus to see these fringes clearly, the eye must be focused on or near the rear mirror M_1. The localized fringes are practically straight, because the variation of the path difference across the field of view is now due primarily to the variation of the thickness of the "air film" between the mirrors. With a wedge-shaped film, the locus of points of equal thickness is a straight line parallel to the edge of the wedge. The fringes are not exactly straight, however, if d has an appreciable value, because there is also some variation of the path difference with angle. They are in general curved and are always convex toward the thin edge of the wedge. Thus, with a certain value of d, we might observe fringes shaped like those of Fig. 13S(g). Decreasing d, they move to the left across the field, a new fringe crossing the center of the field each time d changes by $\lambda/2$. As we approach zero path difference, the fringes become straighter, until the point is reached where M_1 actually intersects M'_2, when they are perfectly straight, as in (h). Beyond this point, they begin to curve in the opposite direction (i). The blank fields (f) and (j) indicate that this type of fringe cannot be observed for large path differences.

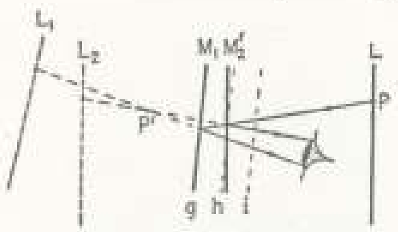


FIG. 13T. Diagram illustrating the formation of fringes with inclined mirrors in the Michelson interferometer.

13.13. White-light Fringes. If a source of white light is used, no fringes will be seen at all except for a path difference so small that it does not exceed a few wavelengths. In observing these fringes, the mirrors are tilted slightly as for localized fringes, and the position of M_1

* When the term "ray" is used, here and elsewhere in discussing interference phenomena, it merely indicates the direction of the *perpendicular to a wave front*, and is in no way to suggest an infinitesimally narrow pencil of light.

is found where it intersects M'_2. With white light there will then be observed a central dark fringe, bordered on either side by 8 or 10 colored fringes. This position is often rather troublesome to find using white light only. It is best located approximately beforehand by finding the place where the localized fringes in monochromatic light become straight. Then a very *slow* motion of M_1 through this region, using white light, will bring these fringes into view.

The fact that only a few fringes are observed with white light is easily accounted for when we remember that such light contains all wavelengths between 4000 and 7500 A. The fringes for a given color are more widely spaced the greater the wavelength. Thus the fringes in different colors will only coincide for $d = 0$, as indicated in Fig. 13U. The solid curve represents the intensity distribution in the fringes for green light, and the broken curve that for red light. Clearly, only the central fringe will be uncolored, and the fringes of different colors will begin to separate at once on either side, producing various impure colors which are not the

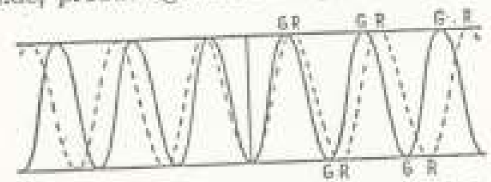


FIG. 13U. Illustrating the origin of white-light fringes with a dark fringe at the center.

saturated spectral colors. After 8 or 10 fringes, so many colors are present at a given point that the resultant color is essentially white. Interference is still occurring in this region, however, because a spectroscope will show a continuous spectrum with dark bands at those wavelengths for which the condition for destructive interference is fulfilled. White-light fringes are also observed in all the other methods of producing interference described above, if white light is substituted for monochromatic light. They are particularly important in the Michelson interferometer, where they may be used to locate the position of zero path difference, as we shall see in Sec. 13.14.

An excellent reproduction in color of these white-light fringes will be found in Michelson, "Light Waves and Their Uses," Plate II. The fringes in three different colors are also shown separately, and a study of these in connection with the white-light fringes is instructive as showing the origin of the various impure colors in the latter.

It was stated above that the central fringe in the white-light system, *i.e.*, that corresponding to zero path difference, is black when observed in the Michelson interferometer. One would ordinarily expect this fringe to be white, since the two beams should be in phase with each other for any wavelength at this point, and in fact this is the case in the fringes formed with the other arrangements, such as the biprism. In the present case, however, it will be seen by referring to Fig. 13Q that while ray (1) undergoes an internal reflection in the plate G_1, ray (2) undergoes an external reflection, with a consequent change of phase (Sec. 11.8). Hence the central fringe is black, if the black surface of G_1 is unsilvered. If it is silvered, the conditions are different and the central fringe may be white.

13.14. Applications of the Michelson Interferometer. The principal advantage of this form of interferometer over the earlier arrangements for producing interference lies in the fact that the two beams are here widely separated, and the path difference between them can be varied at will by moving the mirror M_1 or by introducing a refracting material in the path of one of the beams. Corresponding to these two ways of varying the path difference, there are two types of measurement which can be made with this interferometer. The first is the accurate measurement of distance in terms of the wavelength of light, which we shall discuss in this section. The second is the determination of indices of refraction, which will be briefly referred to at the beginning of Sec. 13.16.

When the mirror M_1 is moved slowly from one position to another, counting the number of fringes in monochromatic light which cross the center of the field of view will give a measure of the distance the mirror has moved in terms of λ, since by Eq. 13m we have, for the position d_1 corresponding to the bright fringe of order m_1,

$$2d_1 = m_1\lambda$$

and for d_2, giving a bright fringe of order m_2,

$$2d_2 = m_2\lambda$$

Subtracting these two equations, we find

$$d_1 - d_2 = (m_1 - m_2)\frac{\lambda}{2} \qquad (13n)$$

Hence the distance moved equals the number of fringes counted, multiplied by a half wavelength. Of course, the distance measured need not correspond to an integral number of half wavelengths. Fractional parts of a whole fringe displacement can easily be estimated to one-tenth of a fringe, and, with care, to one-fiftieth. The latter figure then gives the distance to an accuracy of $\frac{1}{100}\lambda$, or 5×10^{-7} cm for green light.

A small Michelson interferometer in which a microscope is attached to the moving carriage carrying M_1 is frequently used in the laboratory for measuring the wavelength of light. The microscope is focused on a fine glass scale, and the number of fringes, $m_1 - m_2$, crossing the mirror between two readings d_1 and d_2 on the scale gives λ, by Eq. 13n. The bending of a beam, or even of a brick wall, under pressure from the hand can be made visible and measured by attaching M_1 directly to the beam or wall.

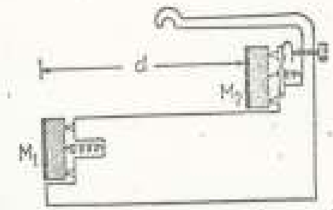


Fig. 13V. Diagram of one of the nine etalons used by Michelson.

The most important measurement made with the interferometer was the comparison of the standard meter in Paris with the wavelengths of intense red, green, and blue lines of cadmium by Michelson and Benoit. Clearly it would be impossible to count directly the number of fringes for a displacement of the movable mirror from one end of the standard meter to the other. Instead, nine intermediate standards (etalons) were used, of the form shown in Fig. 13V, each approximately twice the length of the other. The two shortest etalons were first mounted in an interferometer of special design (Fig. 13W), with a field of view covering the four mirrors, M_1, M_2, M_1', and M_2'. With the aid of the white light

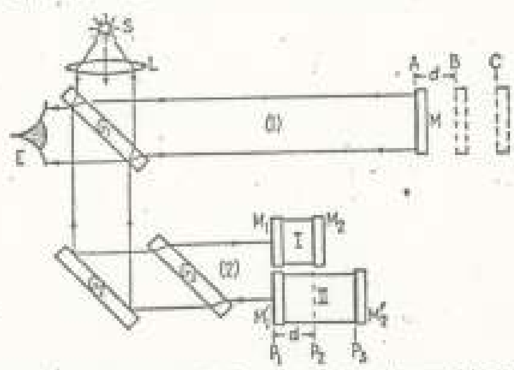


Fig. 13W. Diagram of the special Michelson interferometer used in accurately comparing the wavelength of light with the standard meter.

fringes the distances of M, M_1, and M_1' from the eye were made equal, as shown in the figure. Substituting the light of one of the cadmium lines for white light, M was then moved slowly from A to B, counting the number of fringes passing the cross hair. The count was continued until M reached the position B, which was exactly coplanar with M_2, as judged by the appearance of the white-light fringes in the upper mirror of the shorter etalon. The fraction of a cadmium fringe in excess of an integral number required to reach this position was determined, giving the distance M_1M_2 in terms of wavelengths. The shorter etalon was then moved through its own length, without counting fringes, until the white-light fringes reappeared in M_1. Finally M was moved to C, when the white-light fringes appeared in M_2' as well as in M_2. The additional displacement necessary to make M coplanar with M_2' was measured in terms of cadmium fringes, thus giving the exact number of wavelengths in the longer etalon. This was in turn compared with the length of a third etalon of approximately twice the length of the second, by the same process.

The longest intermediate standard was about 10 cm in length. This was compared with the prototype meter as shown in Fig. 13X. Starting

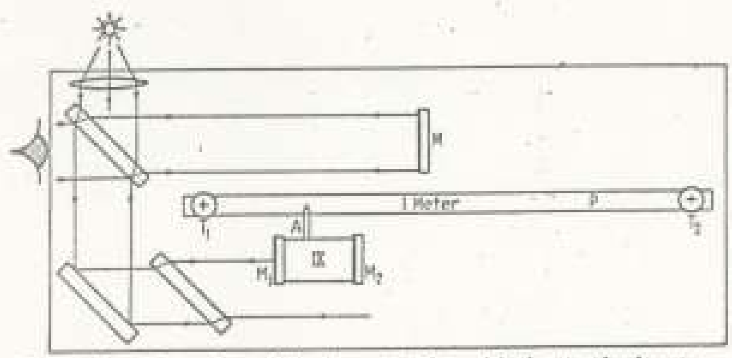


FIG. 13X. Comparison of the longest etalon with the standard meter.

with the pointer A in coincidence with the end mark under the microscope T_1 and M coplanar with M_2 as observed with white-light fringes, the etalon is moved through its own length until the fringes are centered in M_1. M is then moved until they reappear in M_2, and the etalon moved again, repeating the process until after nine displacements the pointer A appears in T_2. The number of cadmium fringes required to make A coincide with the second end mark is finally determined.

It is important to notice that the error in the intercomparison of etalons is not cumulative. Thus the fractional part of a fringe measured in comparing twice the length of the first etalon with that of the second is only used to make sure of the *whole number* of the fringe nearest the cross hair when M is coplanar with M_2'. The final stepping-off process with the longest etalon does involve an accumulated error, but this is at most much smaller than that made in setting on the end marks with the microscope.

The final results were, for the three cadmium lines:

Red line............	1 m = 1,553,163.5λ	or λ = 6438.4722 A
Green line..........	1 m = 1,966,249.7λ	or λ = 5085.8240 A
Blue line............	1 m = 2,083,372.1λ	or λ = 4799.9107 A

Not only has the standard meter been determined in terms of what we now believe to be an invariable unit, the wavelength of light, but we have also obtained absolute determinations of the wavelength of three spectrum lines, the red line of which is at present the primary standard in spectroscopy. More recent measurements on the red cadmium line have been made (see Sec. 14.7). It now is internationally agreed that in dry atmospheric air at 15°C and a pressure of 760 mm Hg the red cadmium line, produced under the conditions described by Michelson, has the wavelength

$$\lambda_r = 6438.4696 \text{ A}$$

13.15. Twyman and Green Interferometer. If a Michelson interferometer is illuminated with strictly parallel monochromatic light, produced by a point source at the principal focus of a well-corrected lens, it becomes a very powerful instrument for testing the perfection of optical parts such as prisms and lenses. The piece to be tested is placed in one of the light beams, and the mirror behind it is so chosen that the reflected waves, after traversing the test piece a second time, again become plane. These waves are then brought to interference with the plane waves from the other arm of the interferometer by another lens, at the focus of which the eye is placed. If the prism or lens is optically perfect, so that the returning waves are strictly plane, the field will appear uniformly illuminated. Any local variation of the optical path will, however, produce fringes in the corresponding part of the field, which are essentially the "contour lines" of the distorted wave front. Even though the surfaces of the test piece may be accurately made, the glass may contain regions that are slightly more or less dense. With the Twyman and Green interferometer these may be detected, and corrected for by local polishing of the surface.

13.16. Determination of Index of Refraction by Interference Methods. If a thickness t of a substance having an index of refraction n is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because of the fact that light travels more slowly in the substance and consequently has a shorter wavelength. The optical path (Eq. 11p) is now nt through the medium, whereas it was practically t through the corresponding thickness of air ($n = 1$). Thus the increase in optical path due to insertion of the substance is $(n-1)t$.* This will introduce $(n-1)t/\lambda$ extra waves in the path of one beam, so if we call Δm the number of fringes by which the fringe system is displaced when the substance is placed in the beam, we have

$$(n-1)t = (\Delta m)\lambda \tag{13o}$$

In principle a measurement of Δm, t, and λ thus gives a determination of n.

In practice, the insertion of a plate of glass in one of the beams produces a discontinuous shift of the fringes so that the number Δm cannot be counted. With monochromatic fringes it is impossible to tell which fringe in the displaced set corresponds to one in the original set. With white light, the displacement in the fringes of different colors is very different because of the variation of n with wavelength, and the fringes disappear entirely. This illustrates the necessity of the compensating plate G_2 in Michelson's interferometer if white-light fringes are to be observed. If the plate of glass is very thin, these fringes may still be visible, and this affords a method of measuring n for very thin films. For thicker pieces, a practicable method is to use two plates of identical thickness, one in each beam, and to turn one gradually about a vertical axis, counting the number of monochromatic fringes for a given angle of rotation. This angle then corresponds to a certain known increase in effective thickness.

For the measurement of the index of refraction of gases, which can be introduced gradually into the light path by allowing the gas to flow into an evacuated tube, the interference method is the most practicable one. Several forms of refractometers have been devised especially for this purpose, of which we shall describe two, the Jamin refractometer and the Rayleigh refractometer.

Jamin's refractometer is shown schematically in Fig. 13Y. Monochromatic light from a broad source S is broken into two parallel beams (1) and (2) by reflection at the two parallel faces of a thick plate of glass G_1. These two rays pass through to another identical plate of glass G_2 to recombine after reflection, forming interference fringes known as Brewster's fringes (see Sec. 14.7). If now the plates are parallel, the light paths will be identical. Suppose as an experiment we wish to measure the index of refraction of a certain gas at different temperatures and pressures. Two similar evacuated tubes T_1 and T_2 of equal length are placed in the two parallel beams. Gas is slowly admitted to tube T_2. Counting the number of fringes Δm crossing the field while the gas

* In the Michelson interferometer, where the beam traverses the substance twice in its back-and-forth path, t is *twice* the actual thickness.

reaches the desired pressure and temperature, the value of n can be found by applying Eq. 13o. It is found experimentally that at a given temperature the value $(n-1)$ is directly proportional to the pressure. This is a special case of a theoretical law known as the *Lorentz-Lorenz* law* according to which

$$\frac{n^2-1}{n^2+2} = (n-1)\frac{(n+1)}{(n^2+2)} = \text{const.} \times \rho \tag{13p}$$

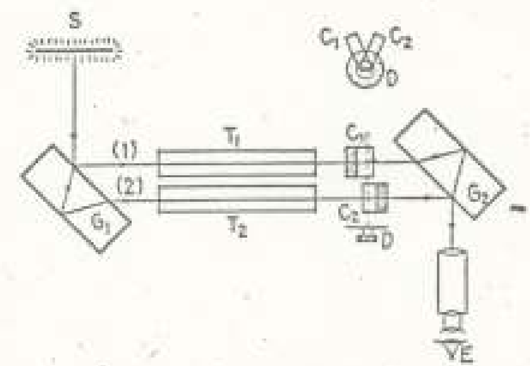


Fig. 13Y. Diagram of the Jamin refractometer.

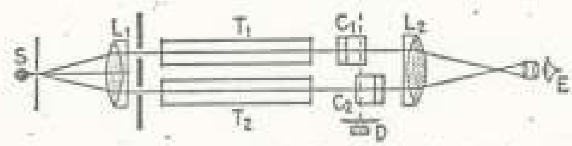


Fig. 13Z. Diagram of the Rayleigh refractometer.

Here ρ is the density of the gas. When n is very nearly unity, the factor $(n+1)/(n^2+2)$ is nearly constant, as required by the above experimental observation.

In Rayleigh's† refractometer (Fig. 13Z) monochromatic light from a linear source S is made parallel by a lens L_1, split into two beams by a

* H. A. Lorentz (1853–1928). For many years professor of mathematical physics at the University of Leyden, Holland. Awarded the Nobel prize (1902) for his work on the relations between light, magnetism, and matter, he also contributed notably to other fields of physics. Gifted with a charming personality and kindly disposition, he traveled a great deal, and was widely known and liked. By a strange coincidence L. Lorenz of Copenhagen derived the above law from the elastic-solid theory only a few months before Lorentz obtained it from the electromagnetic theory.

† Lord Rayleigh (third Baron) (1842–1919). Professor of physics at Cambridge University and the Royal Institution of Great Britain. Gifted with great mathematical ability and physical insight, he made important contributions to many fields of physics. His work on sound and on the scattering of light (Sec. 22.9) are the best known. He was a Nobel prize winner in 1904.

fairly wide double slit and sent through similar tubes and two compensating plates to be brought together again by lens L_2 to interfere in front of the observer.

The purpose of the compensating plates C_1 and C_2 in each of the above refractometers is to speed up the measurement and determination of the refractive index. As the two plates of equal thickness are rotated together by the single knob and dial D, one light path is shortened and the other lengthened. The device can therefore compensate for the path difference in the two tubes. The dial, if previously calibrated by counting fringes, can be made to read directly the index of refraction. The sensitivity of this device can be varied at will, a high sensitivity being obtained when the angle between the two plates is small and a low sensitivity when the angle is large. See Tolansky

Problems

1. Red light of wavelength 6800 A from a narrow slit falls on a double slit of separation (between centers) of $d = 0.026$ cm. If the interference pattern is formed on a screen 100 cm away, what will be the linear separation between fringes on the screen?

2. Under the conditions of Prob. 1, what is the sign and magnitude of the percentage error in the distance of the tenth fringe from the center one, resulting from the approximation mentioned in the text above Eq. 13c?

3. Green light of wavelength 5120 A from a narrow slit is incident on a double slit of separation $d = 0.35$ mm. Plot a curve giving the fringe separation as a function of the distance from the double slit.

4. Yellow light of wavelength 5800 A from a narrow slit is incident on a double slit. If the over-all separation of 10 fringes on a screen 100 cm away is 1.2 cm, find the double slit separation.

5. White light falling on a double slit of separation 1.5 mm forms colored fringes on a screen 100 cm away. If a pinhole is located in this screen at a distance of 2 mm from the central white fringe, what wavelengths within the visible spectrum will be absent from the transmitted light?

6. Solve Prob. 5 if the pinhole is located 4 mm from the central white fringe.

7. Interference fringes formed on a screen 80 cm from a double slit of separation 0.52 mm are measured to be 0.8 mm apart. Find the wavelength of the light and give its color.

8. A Fresnel biprism with refracting angles of 1° and index 1.53 is used to form interference fringes. Find the fringe separation for green light, λ5000, when the distance between the source and the prism is 30 cm and the distance between the prism and the screen 70 cm.

9. Solve Prob. 8 if the distances 30 cm and 70 cm are interchanged.

10. Interference fringes of yellow light, λ5800, are formed by Billet's split lens (see Fig. 13P). The distance from the source S to the lens L is 25 cm. The focal length of lens is 15 cm. The lens halves are separated 0.08 mm and the source-to-screen distance is 200 cm. Find the fringe separation.

11. Solve Prob. 10 if the distance S to L is 20 cm and other dimensions remain unchanged.

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12. In moving one mirror M_1 of Michelson's interferometer a distance of 0.3220 mm, 1204 fringes are counted. Calculate the wavelength of light.

13. Solve Prob. 12 if 368 fringes are counted in moving the mirror 0.1220 mm.

14. Calculate the number of fringes that must be counted for green cadmium light for the shortest etalon used with Michelson's special interferometer, which had a length of 0.390 mm.

15. Solve Prob. 14 for red cadmium light.

16. Solve Prob. 14 for blue cadmium light.

17. The two tubes of a Jamin refractometer are 25.0 cm long. One contains a gas at a pressure of 10 cm Hg and the other is evacuated. If on removing the gas a shift of 20 fringes of green light 5760 A is counted, what is the index of refraction of the gas at atmospheric pressure?

18. Solve Prob. 17 if 16 fringes are counted with blue light, $\lambda = 4500$ A.

19. From Eq. 13l prove that the radii of the circular fringes in the Michelson interferometer are proportional to the square roots of whole numbers.

20. If the path difference between the mirrors of a Michelson interferometer is 5 mm, what will be the angular radius of the fifth bright fringe in the circular pattern of fringes observed with green light $\lambda = 5000$ A? (NOTE: Orders of interference m decrease from center of ring pattern outward.)

21. A source of microwaves, $\lambda = 1$ cm, is located at one end of a table 2 m long and 2 cm above the flat metal table top. Interference fringes are located at the far end of the table with a crystal detector. Determine the positions of the first three principal maxima, measured along a line at and perpendicular to the far end.

22. A pair of Fresnel mirrors making an angle of 1° with each other are located 1 m from a slit source emitting light of wavelength 6000 A. Calculate the fringe separation on a screen 2 m beyond the intersection of the Fresnel mirrors.

23. A thin film of plastic of index $n = 1.45$ for light of wavelength 5800 A is inserted in one arm of a Michelson interferometer. If a shift of 6.5 fringes is observed, find the film thickness.

24. The two compensating plates of a Jamin refractometer are inclined at a fixed angle of 5° with each other. One plate is vertical when fringes are first observed. Through what angle should they be rotated to produce a shift of 20 fringes of green light, 5500 A, if the refractive index is $n = 1.500$? Assume plate thicknesses of 5 mm.

25. Two sources of sound having a pitch of 225 cycles per sec, and vibrating in phase, are separated by a distance of 20 ft. The velocity of sound is 1100 ft/sec. (a) Draw a sketch showing approximately the location of points of maximum intensity. How far apart, along the line joining the sources, are the points of maximum intensity? (b) The frequency of one of the sources is now increased to 226 cycles per sec. What now happens to the intensity pattern? At what rate does it sweep by a stationary observer situated on the line joining the sources?