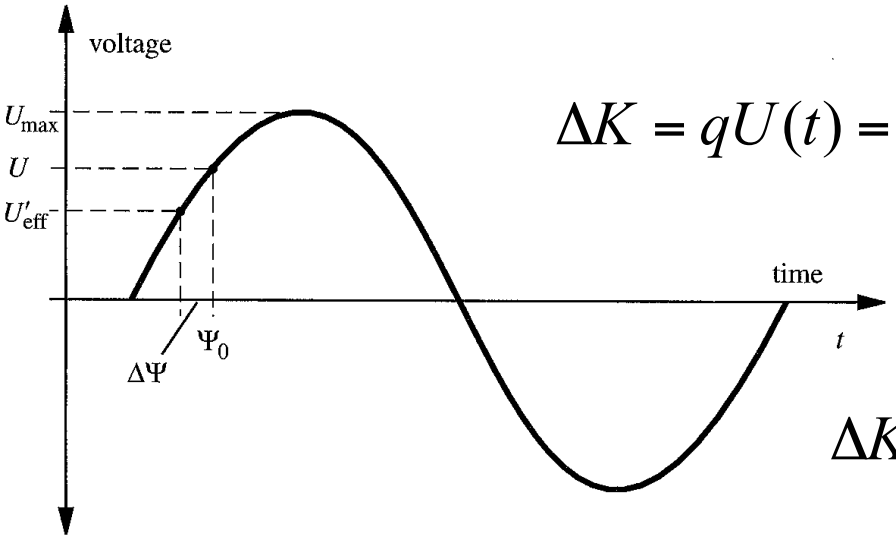
Phase focusing

- 1945: Veksler (UDSSR) and McMillan (USA) realize the importance of phase focusing



$$\Delta K = qU(t) = qU_{\max}\sin(\omega(t-t_0)+\psi_0)$$

Longitudinal position in the bunch:

$$\sigma = s - s_0 = -v_0(t-t_0)$$

$$\Delta K(\sigma) = qU_{\max}\sin(-\tfrac{\omega}{v_0}(s-s_0)+\psi_0)$$

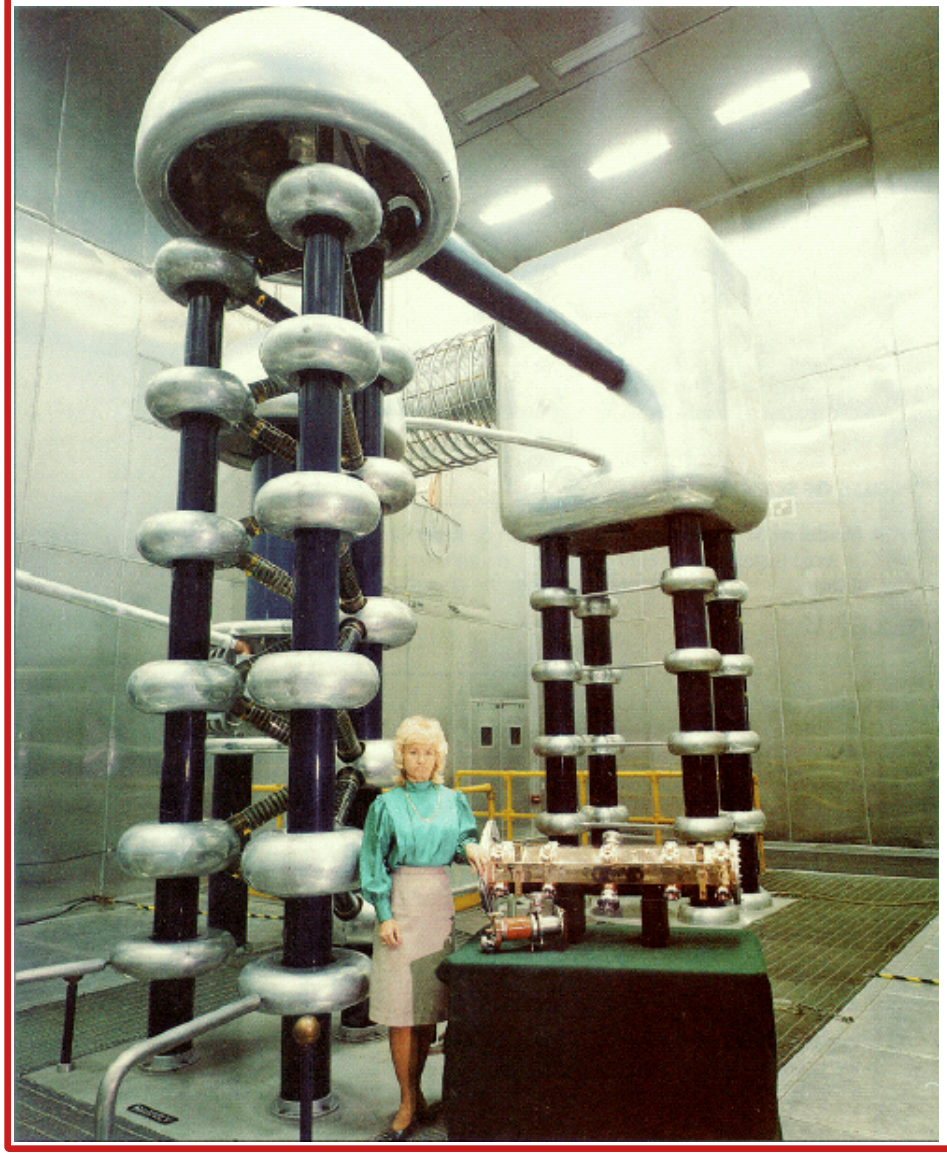
$$\Delta K(0) > 0 \quad \text{(Acceleration)}$$

$$\Delta K(\sigma) < \Delta K(0) \text{ for } \sigma > 0 \Rightarrow \tfrac{d}{d\sigma}\Delta K(\sigma) < 0 \quad \text{(Phase focusing)}$$

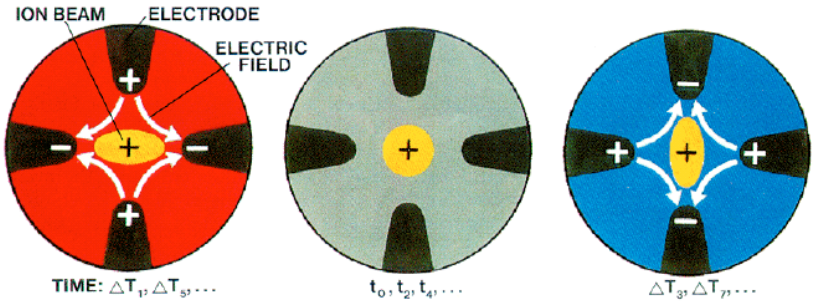
$$\left.\begin{aligned} qU(t) &> 0 \\ q\tfrac{d}{dt}U(t) &> 0 \end{aligned}\right\} \quad \psi_0 \in (0, \frac{\pi}{2})$$

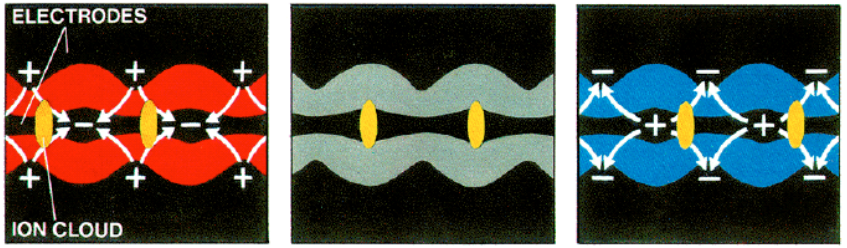
Phase focusing is required in any RF accelerator.

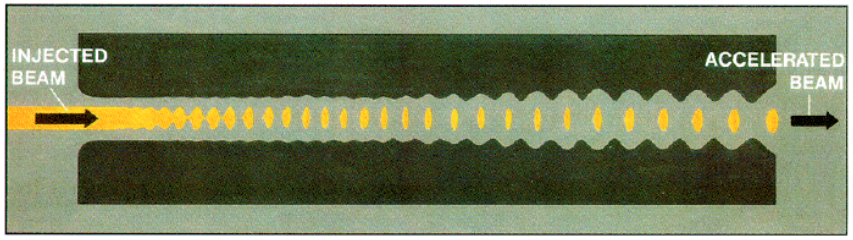
The RF quadrupole (RFQ)



- 1970: Kapchinskii and Teplyakov invent the RFQ



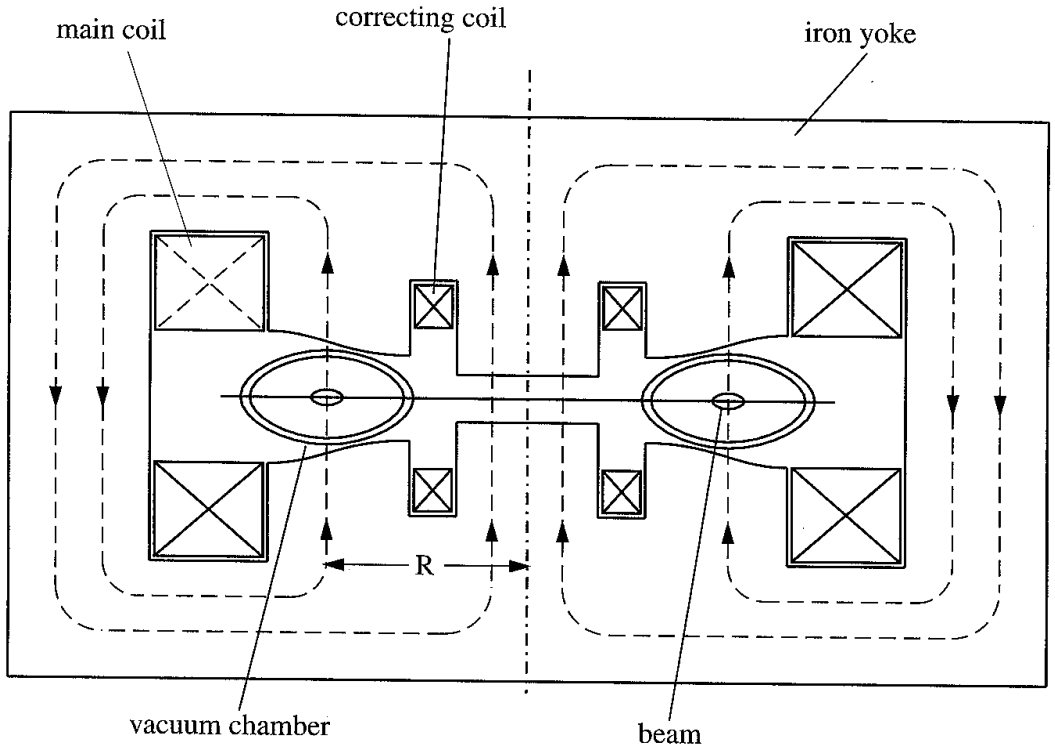




Three historic lines of accelerators

Direct Voltage Accelerators Resonant Accelerators Transformer Accelerator

- 1924: Wideroe invents the betatron
- 1940: Kerst and Serber build a betatron for 2.3MeV electrons and understand betatron (transverse) focusing (in 1942: 20MeV)



Betatron:

R=const, B=B(t)

Whereas for a cyclotron:

R(t), B=const

No acceleration section is needed since

$$\oint_{\partial A} \vec{E}\cdot d\vec{s} = -\iint_{A} \frac{d}{dt}\vec{B}\cdot d\vec{a}$$

The Betatron Condition

Condition: $R = \dfrac{-p_\varphi(t)}{qB_z(R,t)} = \text{const.}$ given $\oint_{\partial A} \vec{E}\cdot d\vec{s} = -\iint_A \frac{d}{dt}\vec{B}\cdot d\vec{a}$

$$E_\varphi(R,t) = -\frac{1}{2\pi R}\int \frac{d}{dt} B_z(r,t)\, r dr d\varphi = -\frac{R}{2}\left\langle \frac{d}{dt} B_z \right\rangle$$

$$\frac{d}{dt} p_\varphi(t) = qE_\varphi(R,t) = -q\frac{R}{2}\left\langle \frac{d}{dt} B_z \right\rangle$$

$$p_\varphi(t) = p_\varphi(0) - q\frac{R}{2}[\langle B_z\rangle(t) - \langle B_z\rangle(0)] = -RqB_z(R,t)$$

$$B_z(R,t) - B_z(R,0) = \frac{1}{2}[\langle B_z\rangle(t) - \langle B_z\rangle(0)]$$

Small deviations from this condition lead to transverse beam oscillations called betatron oscillations in all accelerators.

- Today: Betatrons with typically about 20MeV for medical applications

The Synchrotron

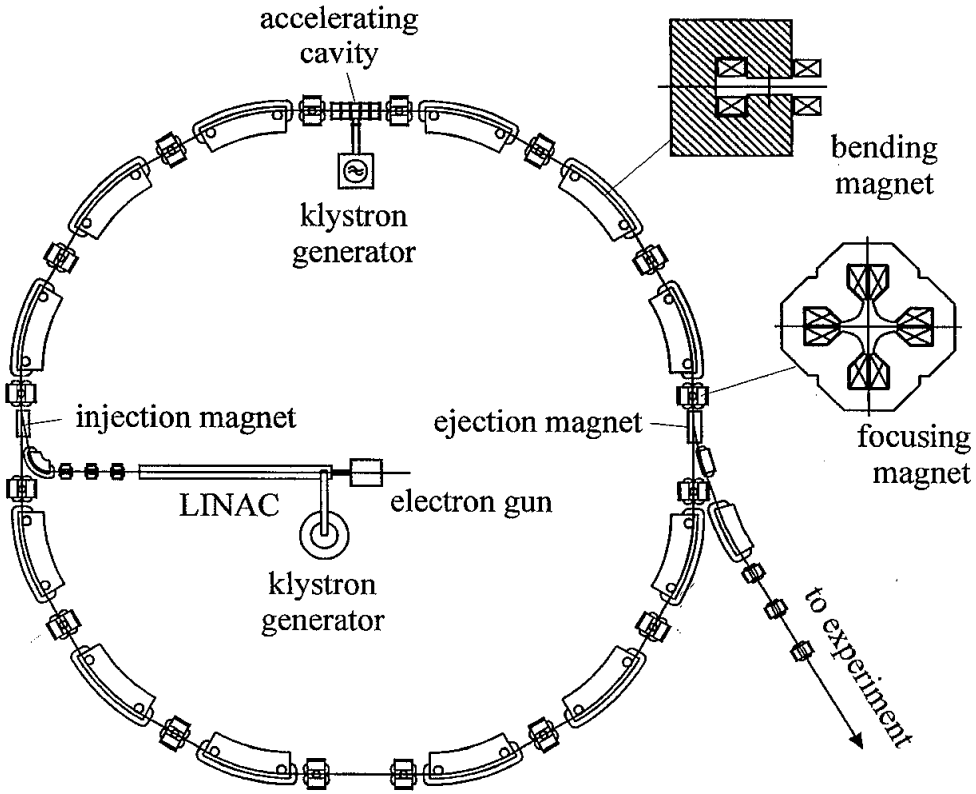
- 1945: Veksler (UDSSR) and McMillan (USA) invent the synchrotron
- 1946: Goward and Barnes build the first synchrotron (using a betatron magnet)
- 1949: Wilson et al. at Cornell are first to store beam in a synchrotron (later 300MeV, magnet of 80 Tons)
- 1949: McMillan builds a 320MeV electron synchrotron

- Many smaller magnets instead of one large magnet
- Only one acceleration section is needed, with

$$R = \frac{p(t)}{qB(R,t)} = \text{const.}$$

$$\omega = 2\pi \frac{v_{\text{particle}}}{L} n$$

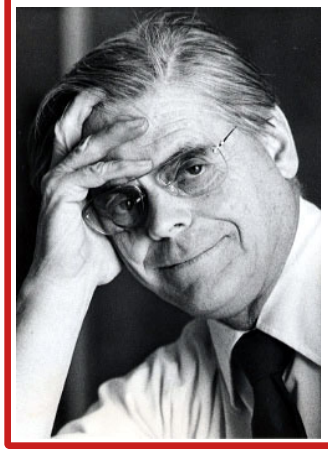
for an integer n called the harmonic number



Rober R Wilson, Architecture

Science Ed Center, FNAL (1990)

Wilson Hall, FNAL



Robert R Wilson
USA 1914-2000

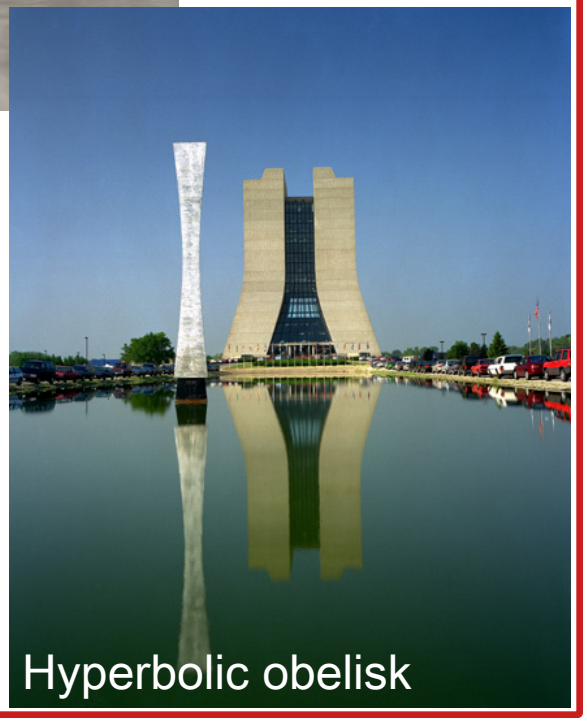
Rober R Wilson, Cornell & FNAL


Broken symmetry


Mobius strip


Tractricious

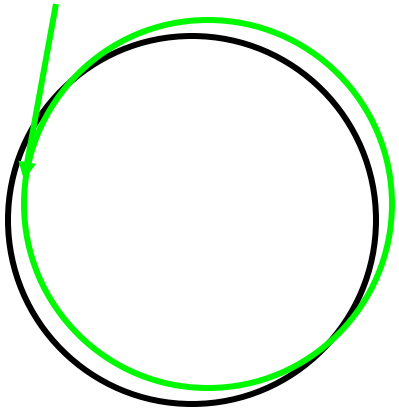

π lines

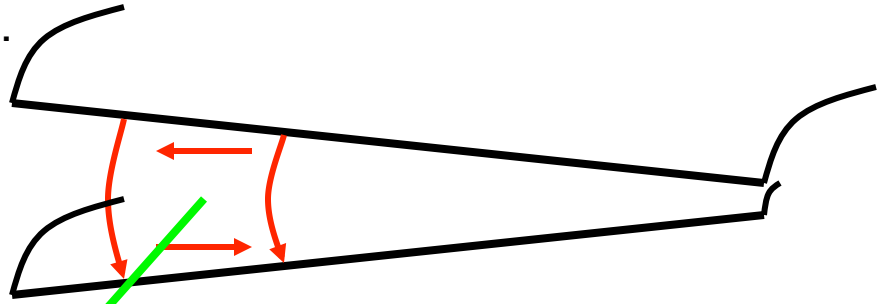

Hyperbolic obelisk

Weak focusing Synchrotrons

- 1952: Operation of the Cosmotron, 3.3 GeV proton synchrotron at Brookhaven: Beam pipe height: 15cm.

Natural ring focusing:



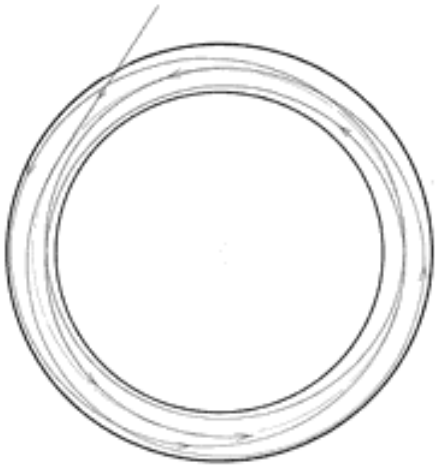


Vertical focusing

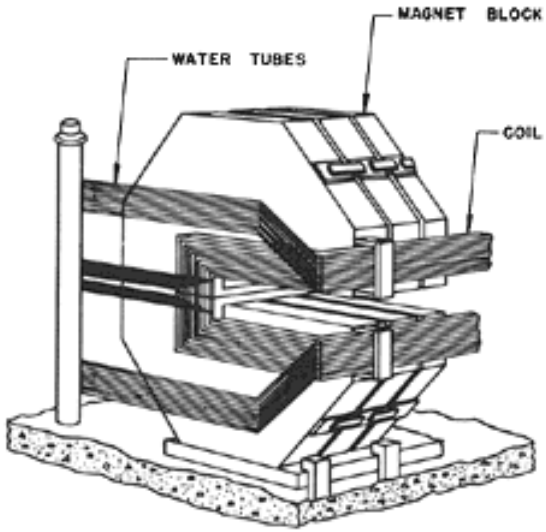
+ Horizontal defocusing + ring focusing

Focusing in both planes

The Cosmotron



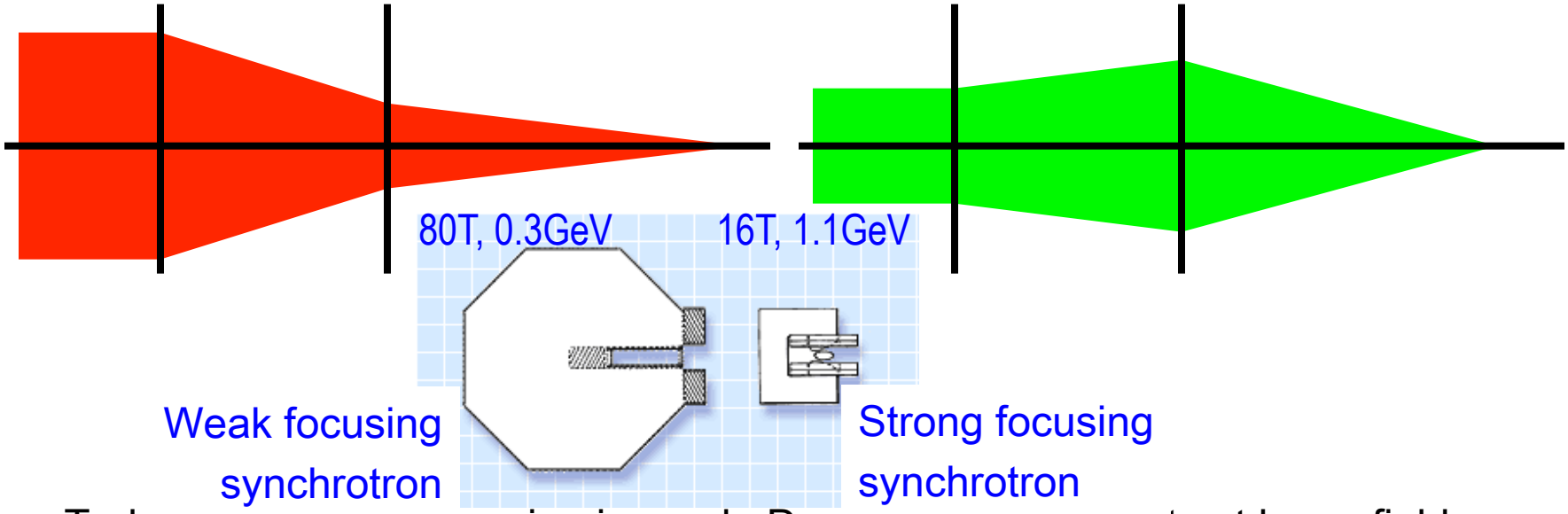
Weak focusing accelerator



Strong focusing Synchrotrons

- 1952: Courant, Livingston, Snyder publish about strong focusing
- 1954: Wilson et al. build first synchrotron with strong focusing for 1.1MeV electrons at Cornell, 4cm beam pipe height, only 16 Tons of magnets.
- 1959: CERN builds the PS for 28GeV after proposing a 5GeV weak focusing accelerator for the same cost (still in use)

Transverse fields defocus in one plane if they focus in the other plane.
But two successive elements, one focusing the other defocusing,
can focus in both planes:



- Today: only strong focusing is used. Due to bad field quality at lower field excitations the injection energy is 20-500MeV from a linac or a microtron.

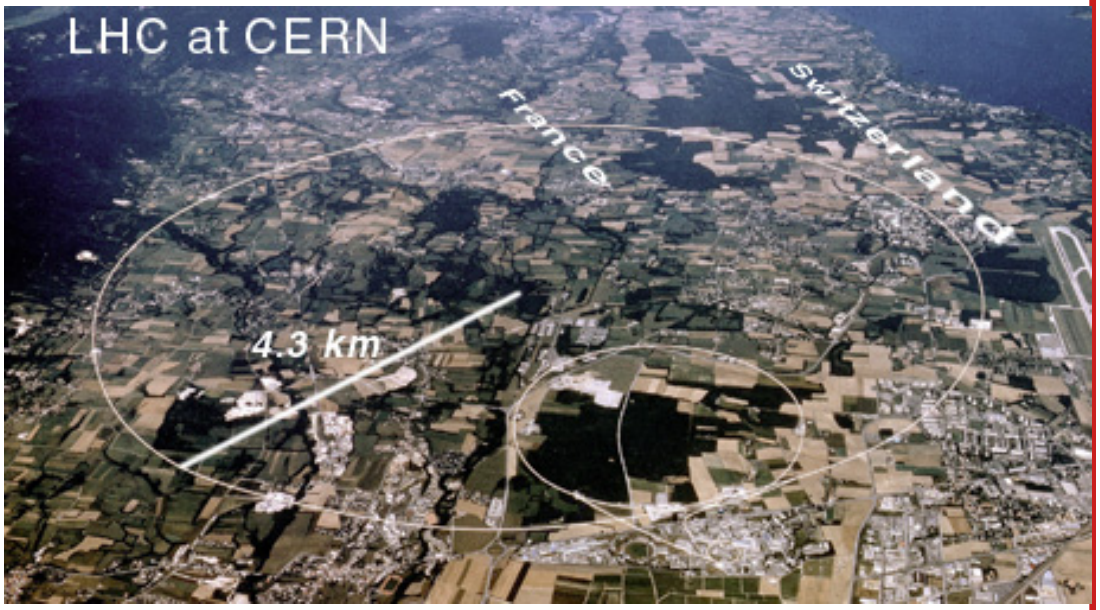
Limits of Synchrotrons

$$\rho = \frac{p}{qB} \quad \Rightarrow \quad \text{The rings become too long}$$

Protons with p = 20 TeV/c , B = 6.8 T would require a 87 km SSC tunnel
Protons with p = 7 TeV/c , B = 8.4 T require CERN's 27 km LHC tunnel

$$P_{\text{radiation}} = \frac{c}{6\pi\varepsilon_0} N \frac{q^2}{\rho^2} \gamma^4 \quad \Downarrow$$

Energy needed to compensate
Radiation becomes too large



Electron beam with p = 0.1 TeV/c in CERN's 27 km LEP tunnel radiated 20 MW
Each electron lost about 4GeV per turn, requiring many RF accelerating sections.