# The Thin Lens Dipole



$$\underline{M} = \begin{pmatrix} \begin{matrix} \cos(\kappa s) & \frac{1}{\kappa}\sin(\kappa s) \\ -\kappa\sin(\kappa s) & \cos(\kappa s) \end{matrix} & \underline{0} & \begin{matrix} 0 & \kappa^{-1}[1-\cos(\kappa s)] \\ 0 & \sin(\kappa s) \end{matrix} \\ \underline{0} & \begin{matrix} 1 & s \\ 0 & 1 \end{matrix} & \underline{0} \\ \begin{matrix} -\sin(\kappa s) & \kappa^{-1}[\cos(\kappa s)-1] \\ 0 & 0 \end{matrix} & \underline{0} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{pmatrix}$$

Weak magnet limit: $\kappa s << 1$

$$\underline{M}^{\text{thin}}_{\text{bend},x\tau}(s) = \underline{D}(-\tfrac{s}{2})\underline{M}_{\text{bend},x\tau}\underline{D}(-\tfrac{s}{2}) \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\kappa^2 s & 1 & 0 & \kappa s \\ -\kappa s & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Thin Combined Function Bend



$$\underline{M}_6 = \begin{pmatrix} \underline{M}_x & \underline{0} & \vec{0}\,\vec{D} \\ \underline{0} & \underline{M}_y & \underline{0} \\ \underline{T} & \underline{0} & \underline{1} \end{pmatrix}$$

Weak magnet limit: $\kappa s << 1$

$$\underline{M}_x = \begin{pmatrix} \cos(\sqrt{K}\,s) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}\,s) \\ -\sqrt{K}\sin(\sqrt{K}\,s) & \cos(\sqrt{K}\,s) \end{pmatrix}$$

$$\underline{M}_x^{\text{thin}} = \begin{pmatrix} 1 & 0 \\ -K\,s & 1 \end{pmatrix}$$

$$\underline{M}_y = \begin{pmatrix} \cosh(\sqrt{k}\,s) & \frac{1}{\sqrt{k}}\sinh(\sqrt{k}\,s) \\ \sqrt{k}\sinh(\sqrt{k}\,s) & \cosh(\sqrt{k}\,s) \end{pmatrix} \longrightarrow \underline{M}_y^{\text{thin}} = \begin{pmatrix} 1 & 0 \\ k\,s & 1 \end{pmatrix}$$

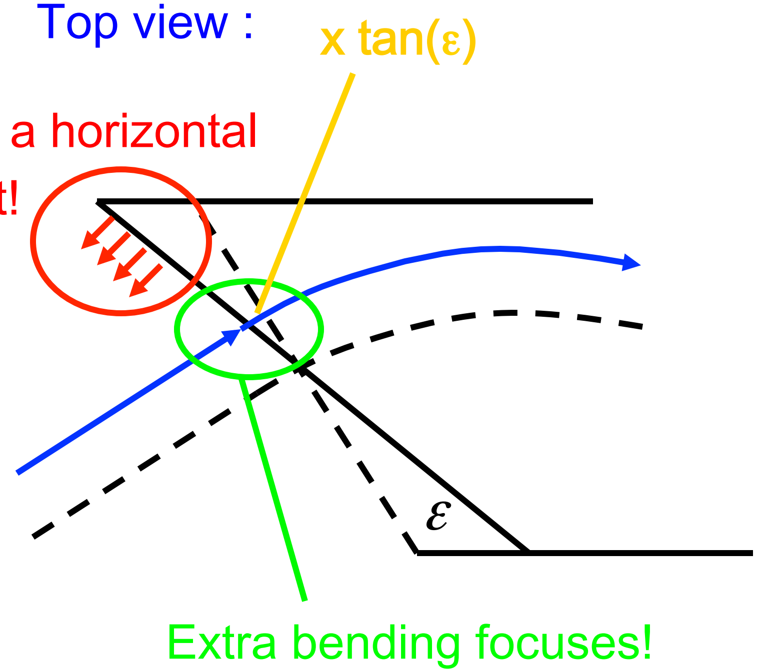
$$\vec{D} = \begin{pmatrix} \frac{\kappa}{K}[1-\cos(\sqrt{K}s)] \\ \frac{\kappa}{\sqrt{K}}\sin(\sqrt{K}s) \end{pmatrix}$$

$$\vec{D} = \begin{pmatrix} 0 \\ \kappa\,s \end{pmatrix}$$

# Edge Focusing



Top view :




Horizontal focusing with $\Delta x' = -x\frac{\tan(\varepsilon)}{\rho}$

$$B_x = \partial_y B_s\big|_{y=0}\, y\tan(\varepsilon) = \partial_s B_y\big|_{y=0}\, y\tan(\varepsilon)$$

$$y'' = \tfrac{q}{p}\,\partial_s B_y\big|_{y=0}\, y\tan(\varepsilon)$$

$$\Delta y' = \int y''\,ds = \tfrac{q}{p} B_y\, y\tan(\varepsilon) = y\frac{\tan(\varepsilon)}{\rho}$$

Quadrupole effect with

$$kl = \frac{\tan(\varepsilon)}{\rho}$$

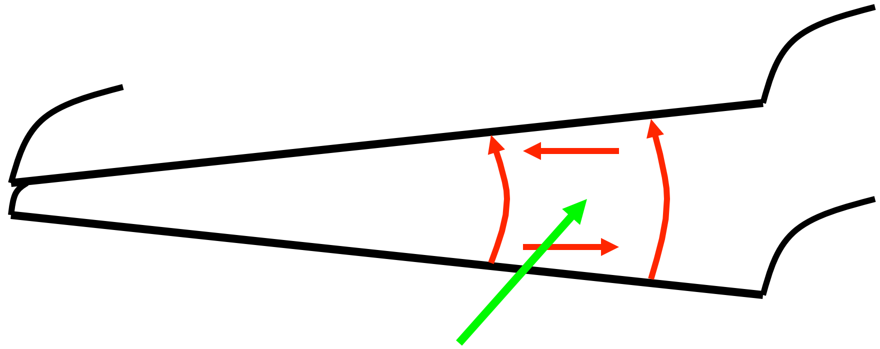
$$\vec{z} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{\tan(\varepsilon)}{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{\tan(\varepsilon)}{\rho} & 1 \end{pmatrix} \vec{z}_0$$

# Cyclotrons with edge focusing



- The isocyclotron with constant

$$\omega_z = \frac{q}{m_0\gamma(E)} B_z(r(E))$$



Up to 600MeV but

this vertically defocuses the beam.

Edge focusing is therefore used.

# Variation of Constants



$$\vec{z}\,' = \vec{f}(\vec{z},s)$$

$$\vec{z}\,' = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z},s)$$ Field errors, nonlinear fields, etc can lead to $\Delta\vec{f}(\vec{z},s)$

$$\vec{z}\,'_H = \underline{L}(s)\vec{z}_H \quad \Rightarrow \quad \vec{z}_H(s) = \underline{M}(s)\vec{z}_{H0} \quad \text{with} \quad \underline{M}'(s)\vec{a} = \underline{L}(s)\underline{M}(s)\vec{a}$$

$$\vec{z}(s) = \underline{M}(s)\vec{a}(s) \quad \Rightarrow \quad \vec{z}\,'(s) = \underline{M}'(s)\vec{a} + \underline{M}(s)\vec{a}\,'(s) = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z},s)$$

$$\vec{a}(s) = \vec{z}_0 + \int_0^s \underline{M}^{-1}(\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}),\hat{s})\,d\hat{s}$$

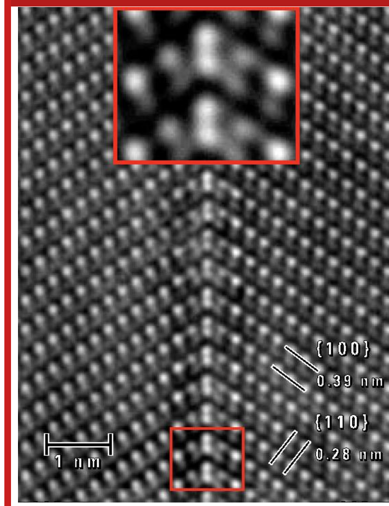
$$\vec{z}(s) = \underline{M}(s)\left\{\vec{z}_0 + \int_0^s \underline{M}^{-1}(\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}),\hat{s})\,d\hat{s}\right\}$$

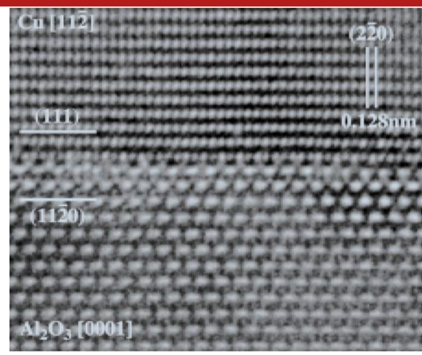
$$= \vec{z}_H(s) + \int_0^s \underline{M}(s-\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}),\hat{s})\,d\hat{s}$$

Perturbations are propagated from s to s’

# Aberration Correction

CHESS & LEPP




$$w_2(s) = w_H(s) + C(s) w_0^2 \overline{w}_0 + \dots$$

$$w_2(s) = w_H(s) + A(s) \overline{w}_0^2 + B(s) w_0^2 \overline{w}_0 + \dots$$

$$w_2(s) = w_H(s) + A(s) \overline{w}_0^2 + 2B(s) w_0^2 \overline{w}_0 + \dots$$

2B cancels C !

Quadratic in sextupole strength

Linear in solenoid strength