The FODO Cell

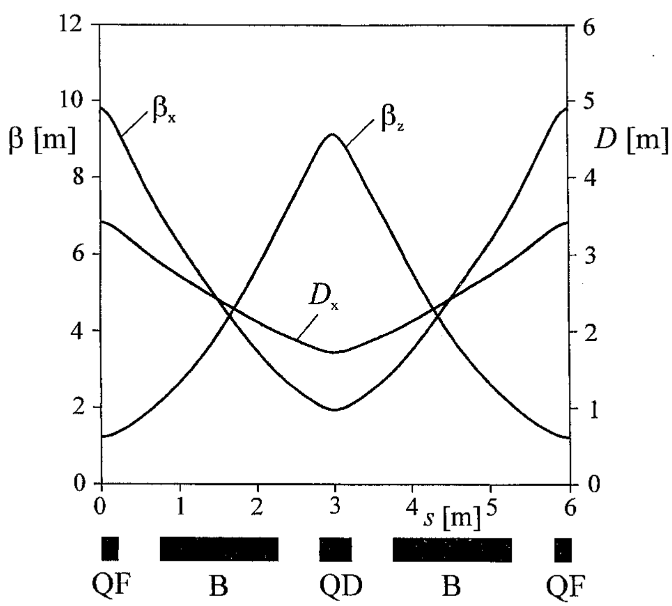
CHESS & LEPP

Alternating gradients allow focusing in both transverse plains. Therefore focusing and defocusing quadrupoles are usually alternated and interleaved with bending magnets.

$$L_{FoDo} \approx 6\text{m}\,,\quad \varphi \approx 22.5^\circ\,,\quad \mu_{FoDo} \approx \tfrac{\pi}{2}$$

$$\bar{\beta} \approx 3.8\text{m}$$

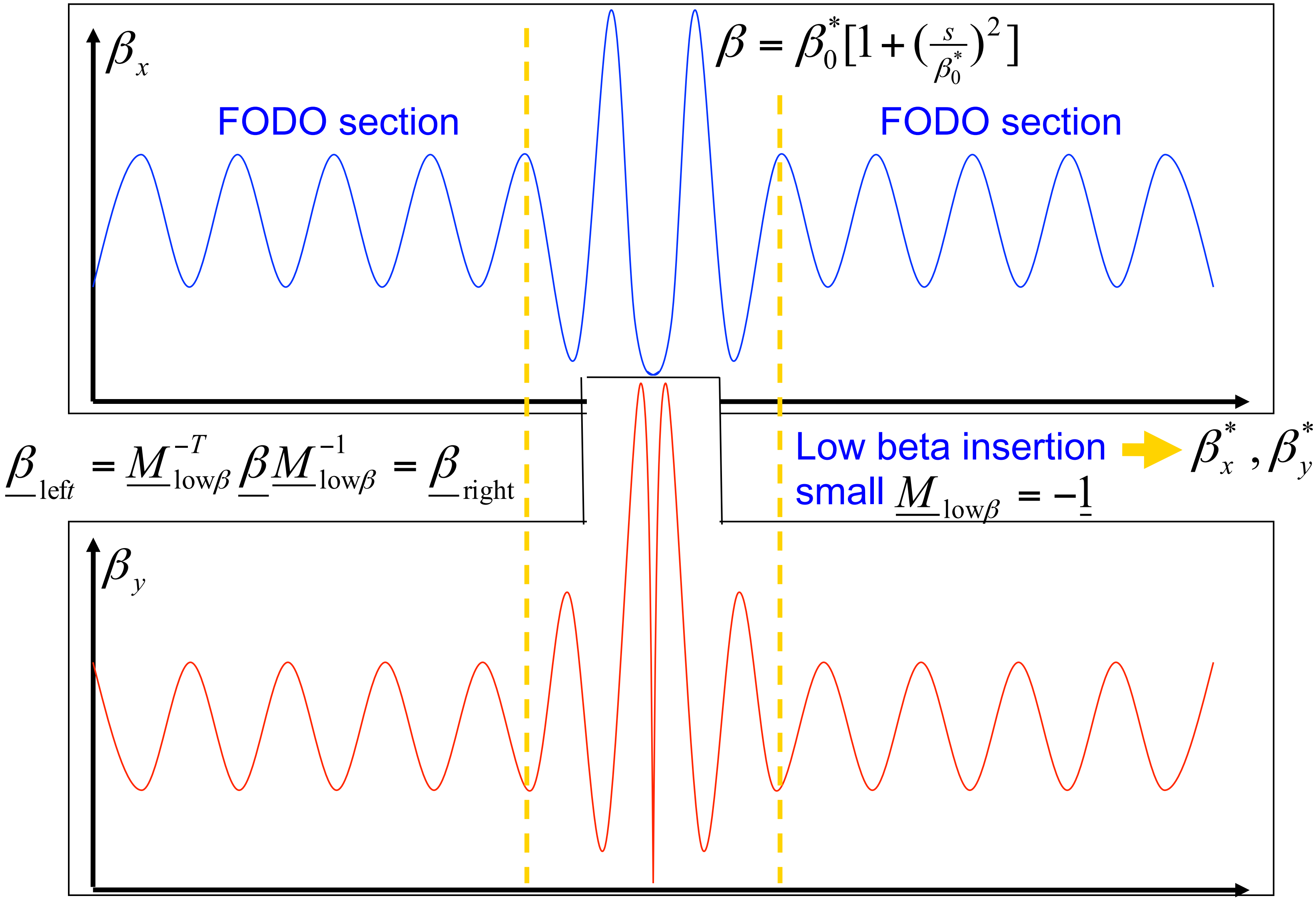
$$\beta_{\max} \approx 10.2\text{m}\,,\quad \beta_{\min} \approx 1.8\text{m}$$



$$\underline{M}_0 = \underline{M}_{FoDo}^N$$

The periodic beta function and dispersion for each FODO is also periodic for an accelerator section that consists of many FODO cells. Often large sections of an accelerator consist of FODOs.

The Low Beta Insertion



Quadrupole Errors

$$\vec{z}' = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z}, s)$$

$$\vec{z}(s) = \vec{z}_H(s) + \int_0^s \underline{M}(s,\hat{s})\Delta\vec{f}(\vec{z},\hat{s})\, d\hat{s} \approx \vec{z}_H(s) + \int_0^s \underline{M}(s,\hat{s})\Delta\vec{f}(\vec{z}_H,\hat{s})\, d\hat{s}$$

$$x'' = -(\kappa^2 + k)x - \Delta k(s)\, x \quad \Rightarrow \quad \begin{pmatrix} x' \\ a' \end{pmatrix} = \begin{pmatrix} a \\ -(\kappa^2 + k)x \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ \Delta k(s) & 0 \end{pmatrix}\begin{pmatrix} x \\ a \end{pmatrix}$$

$$\vec{z}(s) = \left\{ \underline{M}(s) - \int_0^s \underline{M}(s,\hat{s}) \begin{pmatrix} 0 & 0 \\ \Delta k(\hat{s}) & 0 \end{pmatrix} \underline{M}(\hat{s},0)\, d\hat{s} \right\} \vec{z}_0$$

One quadrupole error:

$$\underline{M}(s,\hat{s}) + \Delta\underline{M}(s,\hat{s}) = \underline{M}(s,\hat{s}) - \underline{M}(s,\hat{s}) \begin{pmatrix} 0 & 0 \\ \Delta kl(\hat{s}) & 0 \end{pmatrix}$$

Quadrupole Error and Phase advance

$$\Delta\underline{M}(s,\hat{s}) = -\underline{M}(s,\hat{s})\begin{pmatrix} 0 & 0 \\ \Delta kl(\hat{s}) & 0 \end{pmatrix}$$

$$\underline{M}(s) = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}[\cos\tilde{\psi} + \alpha_0 \sin\tilde{\psi}] & \sqrt{\beta_0\beta}\sin\tilde{\psi} \\ \sqrt{\frac{1}{\beta_0\beta}}[(\alpha_0 - \alpha)\cos\tilde{\psi} - (1+\alpha_0\alpha)\sin\tilde{\psi}] & \sqrt{\frac{\beta_0}{\beta}}[\cos\tilde{\psi} - \alpha\sin\tilde{\psi}] \end{pmatrix}$$

$$\Delta\underline{M}(s,\hat{s}) = -\Delta kl(\hat{s})\begin{pmatrix} \sqrt{\hat{\beta}\beta}\sin\psi & 0 \\ \sqrt{\frac{\hat{\beta}}{\beta}}[\cos\psi - \alpha\sin\psi] & 0 \end{pmatrix} \quad , \qquad \tilde{\psi} = \psi - \hat{\psi}$$

$$= \begin{pmatrix} \frac{\frac{1}{2}\Delta\beta[\cos\psi + \hat{\alpha}\sin\psi] + \Delta\psi\beta[\hat{\alpha}\cos\psi - \sin\psi]}{\sqrt{\hat{\beta}\beta}} & \sqrt{\hat{\beta}}\left(\frac{\frac{\Delta\beta}{2}\sin\psi + \Delta\psi\beta\cos\psi}{\sqrt{\beta}}\right) \\ \cdots & \cdots \end{pmatrix}$$

$$\Delta\psi = -\frac{\Delta\beta}{2\beta}\tan\tilde{\psi}$$

$$\frac{1}{2}\Delta\beta\cos\tilde{\psi} + \frac{1}{2}\Delta\beta\frac{\sin^2\tilde{\psi}}{\cos\tilde{\psi}} = \frac{1}{2}\Delta\beta\frac{1}{\cos\tilde{\psi}} = -\Delta kl(\hat{s})\beta\hat{\beta}\sin\tilde{\psi}$$

Quadrupole Error correction

$$\Delta\psi = \Delta kl(\hat{s})\hat{\beta}\sin^2(\psi - \hat{\psi})$$

→ More focusing always increases the tune

$$\frac{\Delta\beta}{\beta} = -\Delta kl(\hat{s})\hat{\beta}\sin(2[\psi - \hat{\psi}])$$

→ Beta beat oscillates twice as fast as orbit.

$$\Delta\psi = \sum_j \Delta kl_j \beta_j \tfrac{1}{2}[1 - \cos(2[\psi - \psi_j])]$$

$$\frac{\Delta\beta}{\beta} = -\sum_j \Delta kl_j(\hat{s})\beta_j \sin(2[\psi - \psi_j])$$

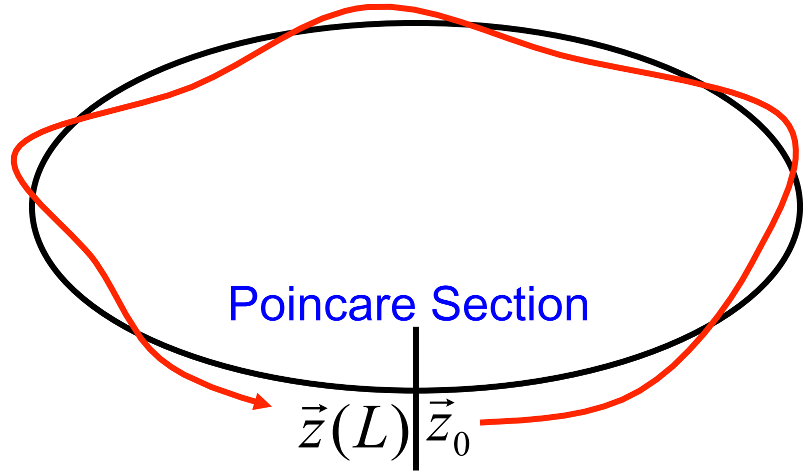
When beta functions and betatron phases have been measured at many places, quadrupoles can be changed with these formulas to correct the Twiss errors.

Tune shift in a ring due to quadrupole error

Average phase advance change per turn:

$$\overline{\Delta\psi} = \tfrac{1}{2}\Delta kl(\hat{s})\overline{\hat{\beta}} = \tfrac{1}{2}\Delta kl(\hat{s})\beta_0$$

Tune change: $\Delta\nu = \frac{1}{4\pi}\Delta kl(\hat{s})\beta_0$



$$\cos(\mu+\Delta\mu) \approx \cos\mu - \Delta\mu\sin\mu =$$

$$\tfrac{1}{2}\mathrm{Tr}[\begin{pmatrix} 1 & 0 \\ -\Delta kl(\hat{s}) & 1 \end{pmatrix}\begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}] = \cos\mu - \tfrac{1}{2}\Delta kl(\hat{s})\beta\sin\mu$$

Oscillation frequencies can be measured relatively easily and accurately.

Measurement of beta function: Change k and measure tune.