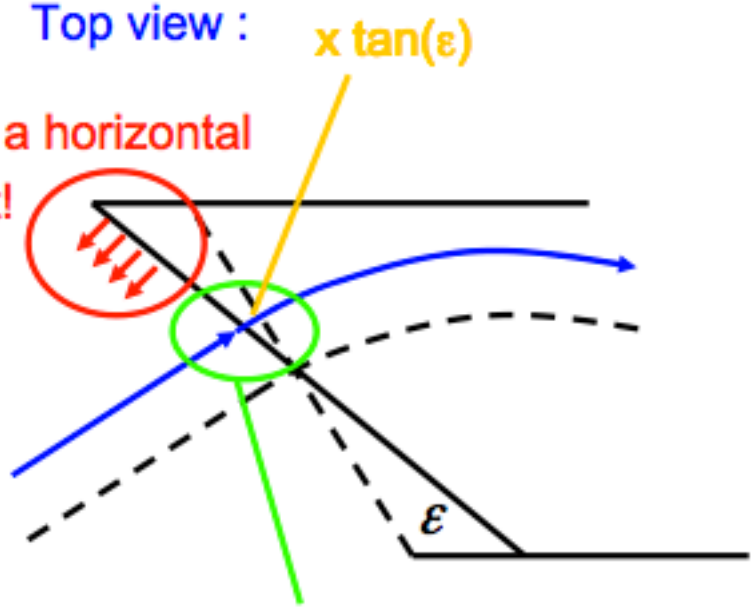
Top view :

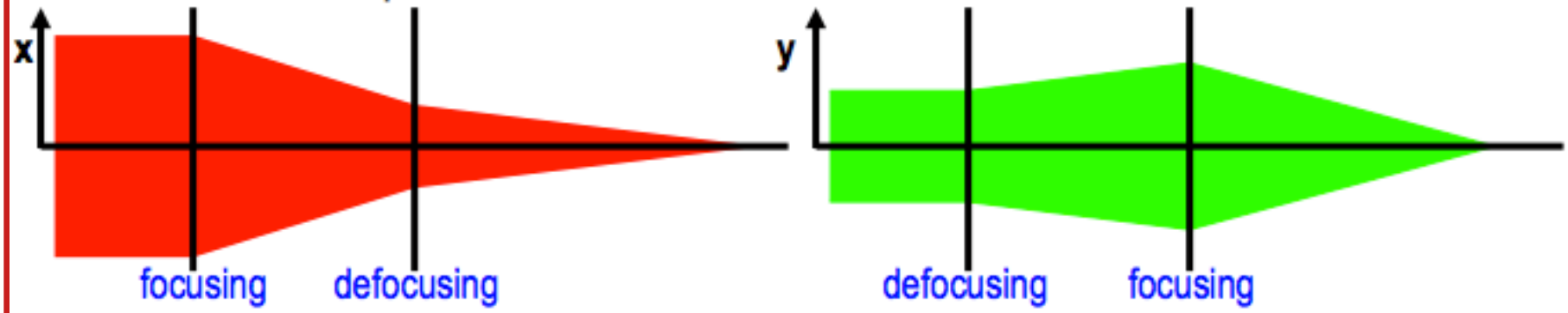


Horizontal focusing with $\Delta x' = -x\frac{\tan(\varepsilon)}{\rho}$

The longitudinal field above the enter plain defocuses, turns out to: $\Delta y' = y\frac{\tan(\varepsilon)}{\rho}$

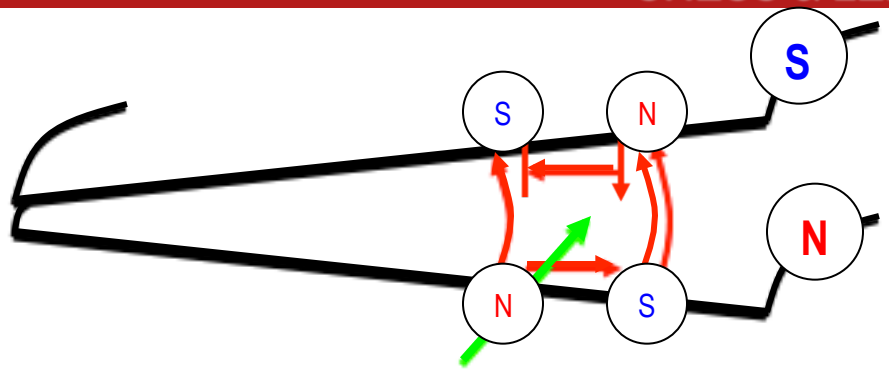
Quadrupole effect: focusing in x and defocusing in y or defocusing in x and focusing in y.

Transverse fields defocus in one plane if they focus in the other plane.
But two successive elements, one focusing the other defocusing,
can focus in both planes:



- The isocyclotron with constant

$$\omega_z = \frac{q}{m_0 \gamma(E)} B_z(r(E))$$



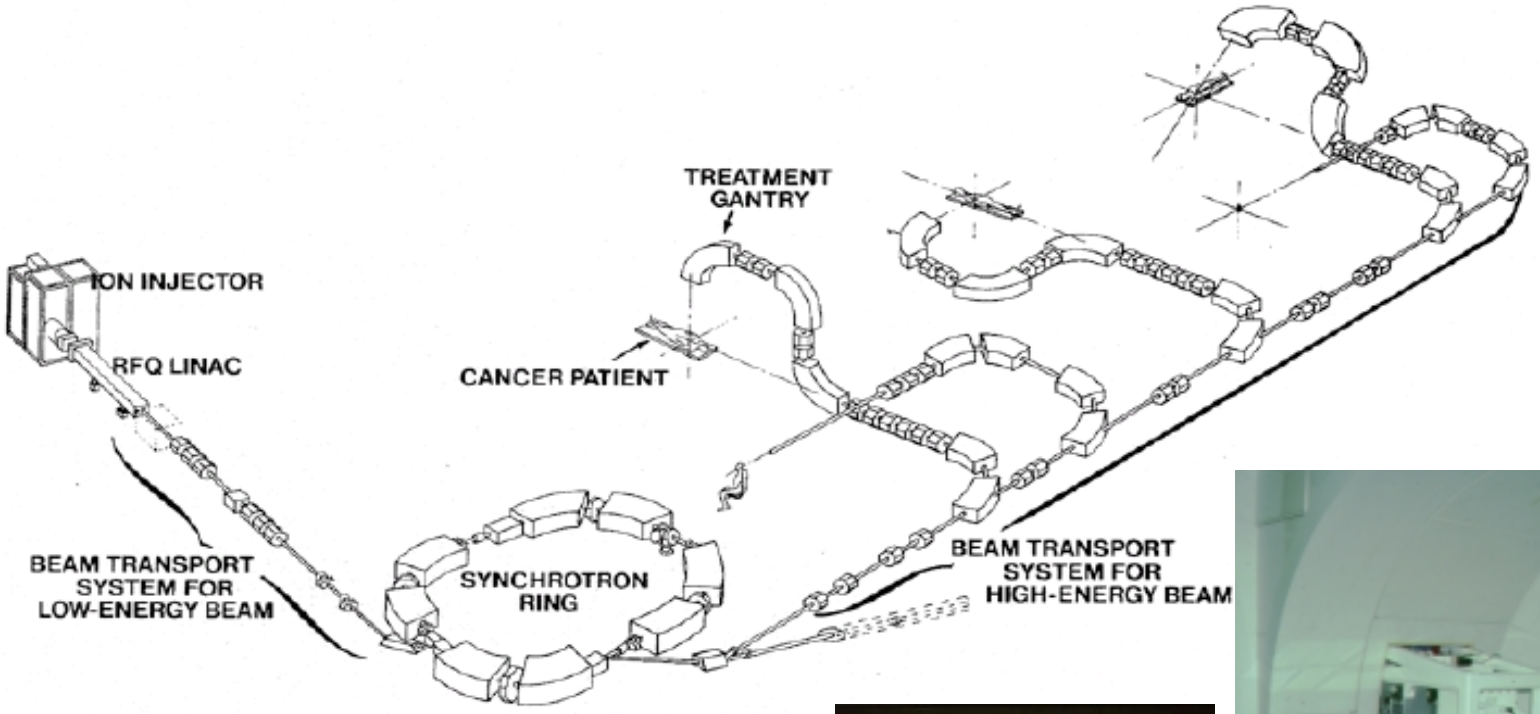
Up to 600MeV but
this vertically defocuses the beam.
Edge focusing is therefore used.

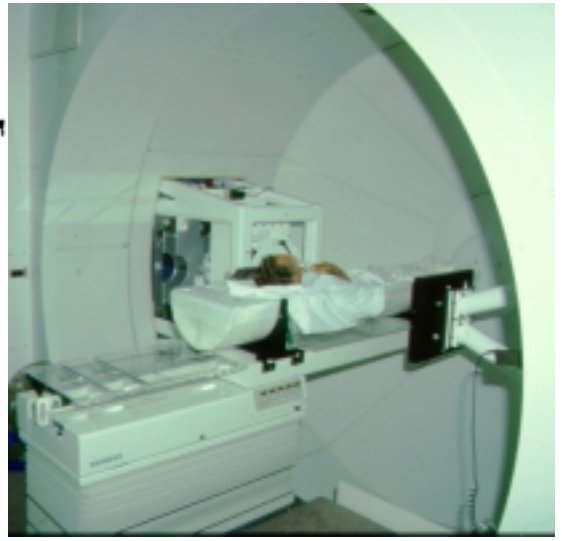
First Medical Applications

- 1939: Lawrence uses 60’ cyclotron for 9MeV protons, 19MeV deuterons, and 35MeV 4He. First tests of tumor therapy with neutrons via $d + t \mapsto n + \alpha$ With 200-800keV d to get 10MeV neutrons.

Modern Nuclear Therapy

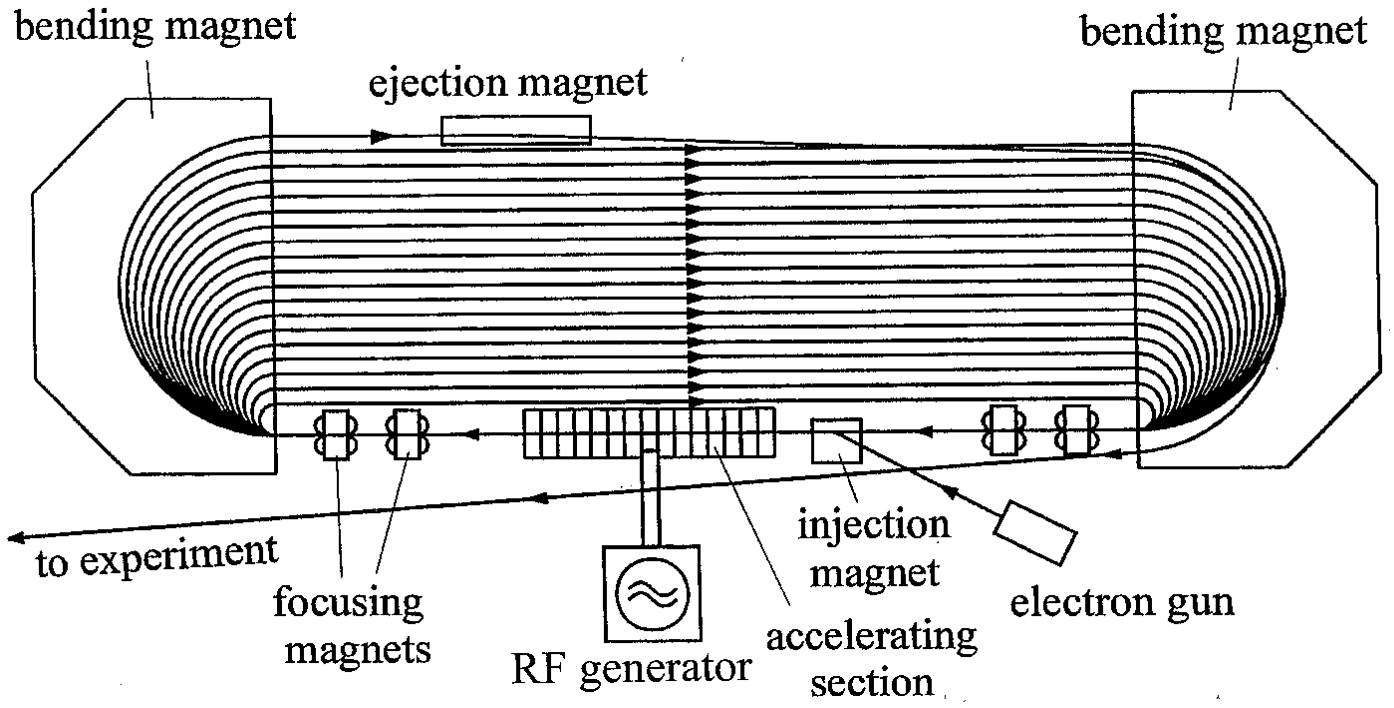
The Loma Linda proton therapy facility





The microtron

- Electrons are quickly relativistic and cannot be accelerated in a cyclotron.
- In a microtron the revolution frequency changes, but each electron misses an integer number of RF waves.

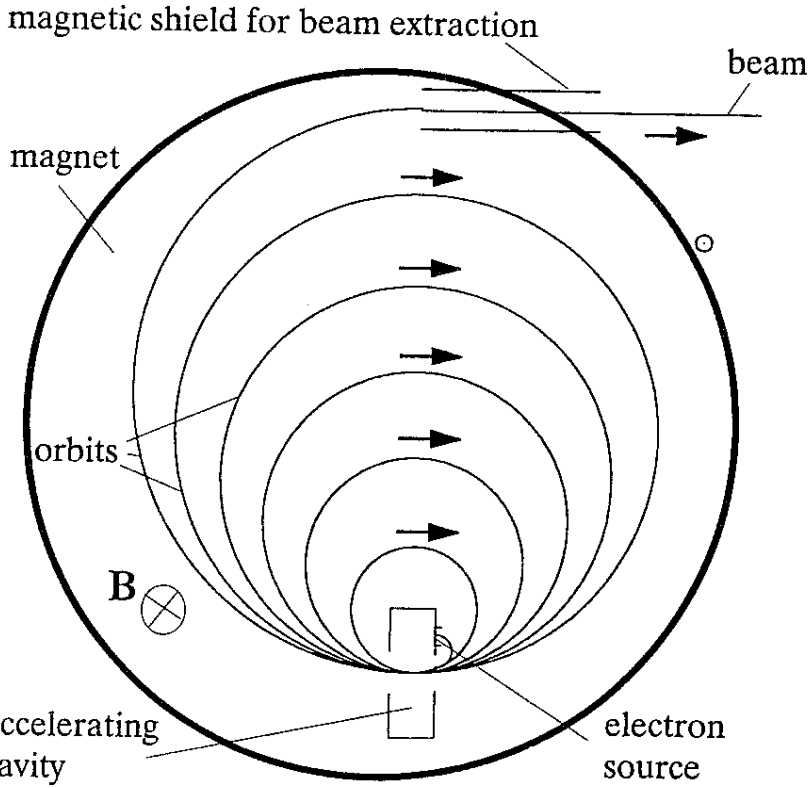


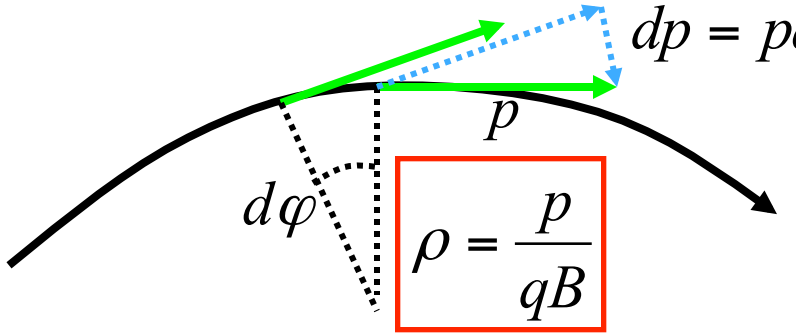
- Today: Used for medical applications with one magnet and 20MeV.
- Nuclear physics: MAMI designed for 820MeV as race track microtron.

The microtron condition

- The extra time that each turn takes must be a multiple of the RF period.

$$\frac{dp}{dt} = qvB \Rightarrow \rho = \frac{dl}{d\varphi} = \frac{vdt}{dp/p} = \frac{p}{qB}$$





$$\rho = \frac{p}{qB}$$

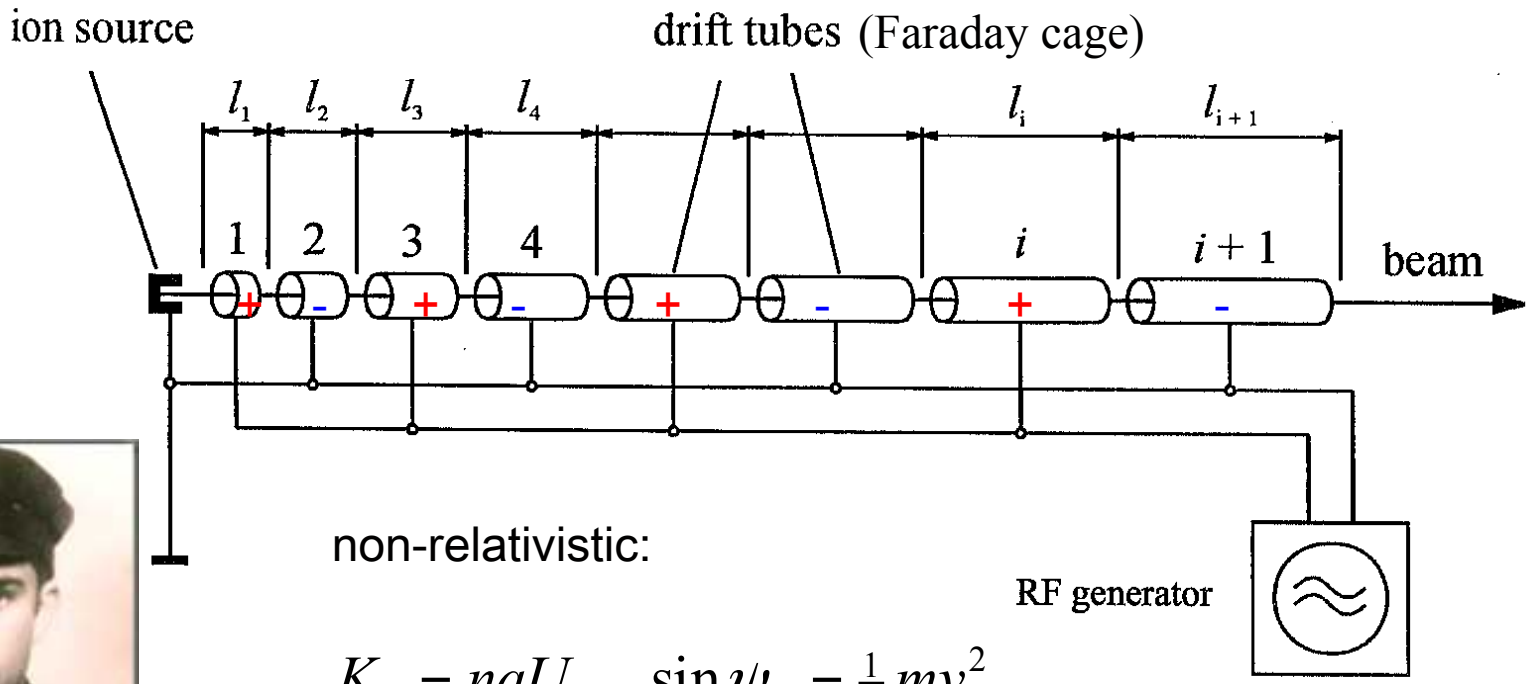
$$\Delta t = 2\pi\left(\frac{\rho_{n+1}}{v_{n+1}} - \frac{\rho_n}{v_n}\right)$$

$$= \frac{2\pi}{qB}(m_0\gamma_{n+1} - m_0\gamma_n) = \frac{2\pi}{qBc^2}\Delta K$$

B=1T, n=1, and f_{RF}=3GHz leads to 4.78MeV
This requires a small linear accelerator.

$$\Delta K = n\frac{qBc^2}{\omega_{RF}} \quad \text{for an integer n}$$

Wideroe linear accelerator



Wideroe

non-relativistic:

$$K_n = nqU_{\max}\sin\psi_0 = \tfrac{1}{2}mv_n^2$$

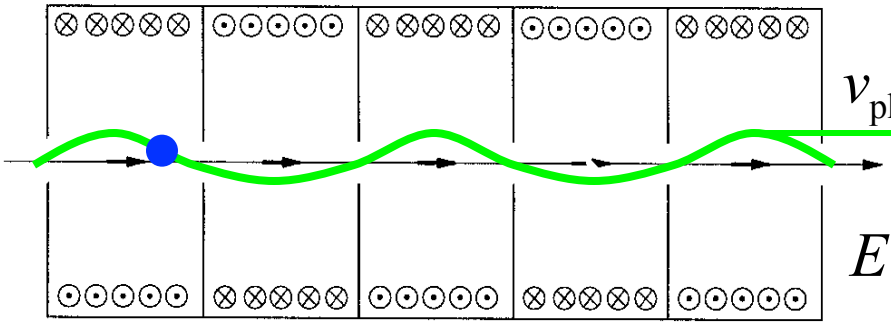
$$l_n = \tfrac{1}{2}v_n T_{RF} = \tfrac{1}{2}\beta_n\lambda_{RF} \propto \sqrt{n}$$

Called the π or the 1/2βλ mode

Accelerating cavities

- 1933: J.W. Beams uses resonant cavities for acceleration

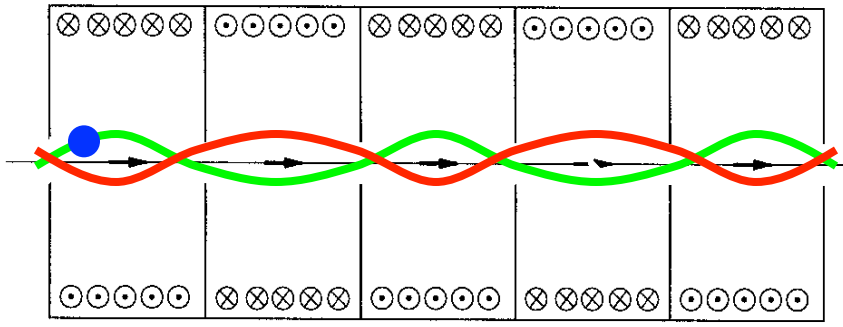
Traveling wave cavity:



$$v_{\text{phase}} = \frac{\omega}{k} = v_{\text{particle}}$$ Here v=c for electrons

$$E(t,s) \approx E_{\max} \sin(\omega t - ks)$$

Standing wave cavity:



$$\frac{\omega}{k} = v_{\text{particle}}$$

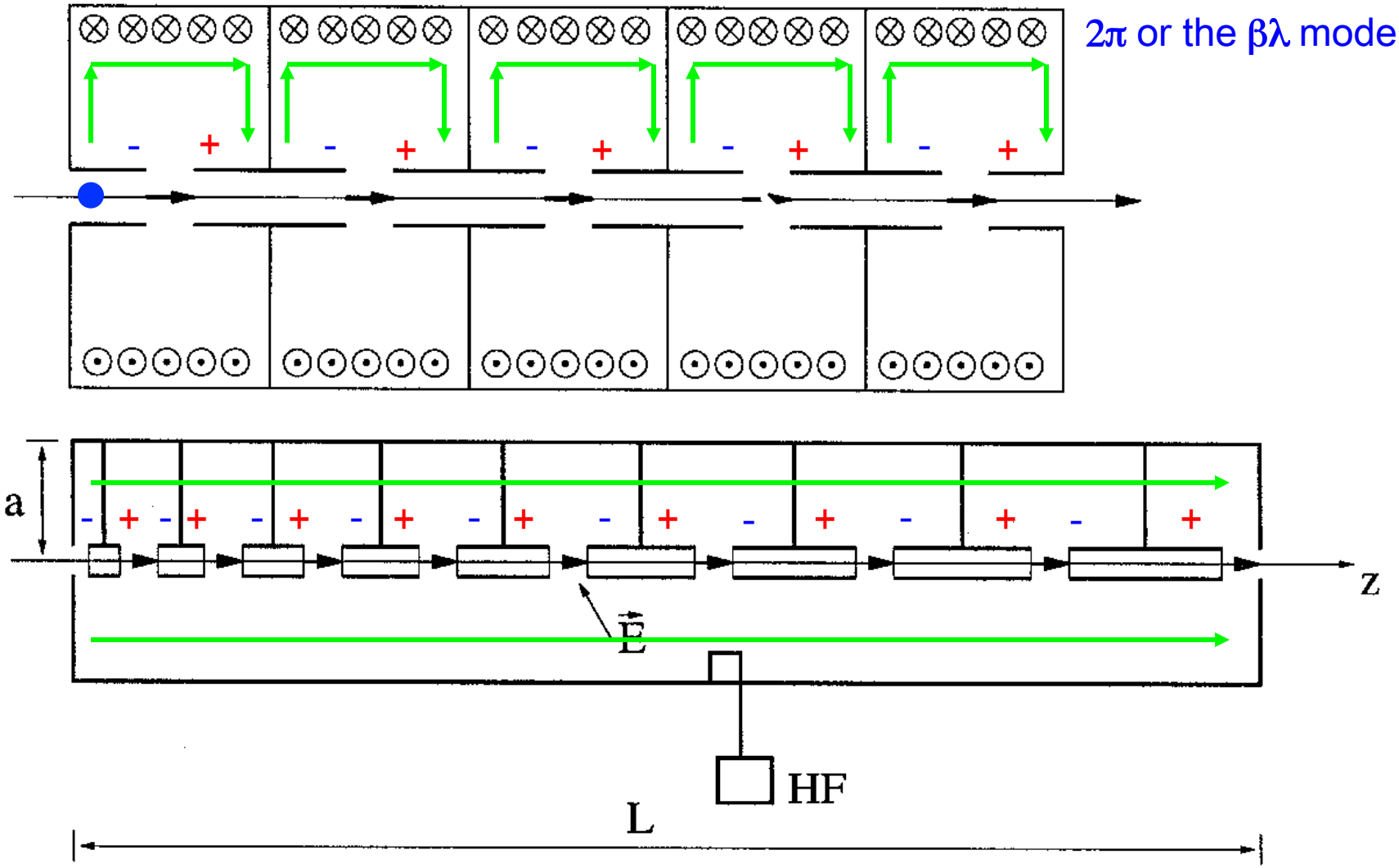
$$E(t,s) \approx E_{\max} \sin(\omega t)\sin(ks)$$

$$E(\tfrac{s}{v_{\text{particle}}},s) \approx E_{\max} \sin^2(ks)$$

π or the 1/2βλ mode

Transit factor (for this example): $\langle E \rangle = \frac{1}{\lambda_{RF}} \int_0^{\lambda_{RF}} E(\tfrac{s}{v_{\text{particle}}},s)\, ds \approx \tfrac{1}{2} E_{\max}$

The Alvarez Linear Accelerator



Needs only one power input coupler and walls do not dissipate energy.