Rings for Synchrotron Radiation

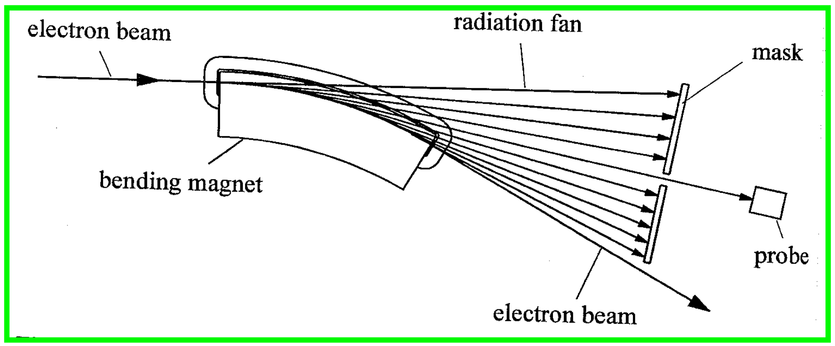
CHESS & LEPP

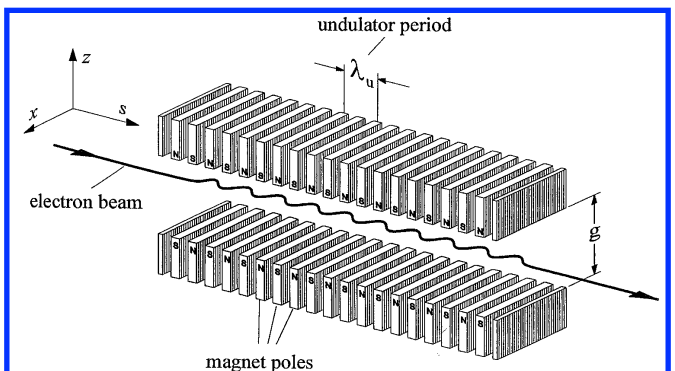
- 1947: First detection of synchrotron light at General Electrics.
- 1952: First accurate measurement of synchrotron radiation power by Dale Corson with the Cornell 300MeV synchrotron.
- 1968: TANTALOS, first dedicated storage ring for synchrotron radiation

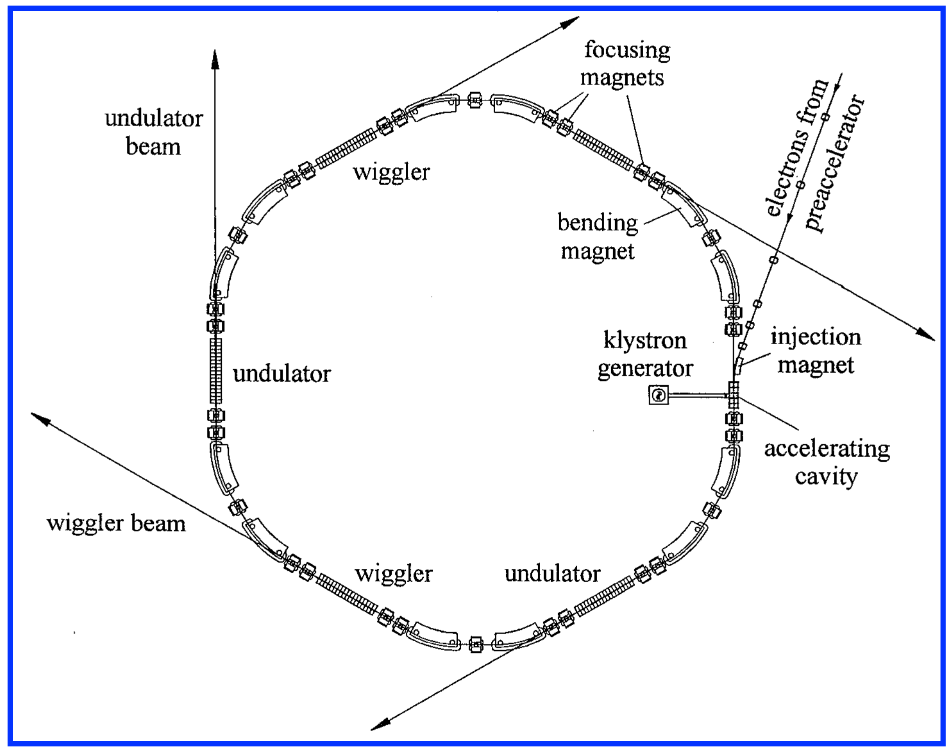
Dale Corson
Cornell's 8th president
USA 1914 –

3 Generations of Light Sources

- 1st Genergation (1970s): Many HEP rings are parasitically used for X-ray production
- 2nd Generation (1980s): Many dedicated X-ray sources (light sources)
- 3rd Generation (1990s): Several rings with dedicated radiation devices (wigglers and undulators)
- Today (4th Generation): Construction of Free Electron Lasers (FELs) driven by LINACs







Macroscopic Fields in Accelerators

$$\frac{d}{dt}\vec{p} = q(\vec{E} + \vec{v} \times \vec{B})$$

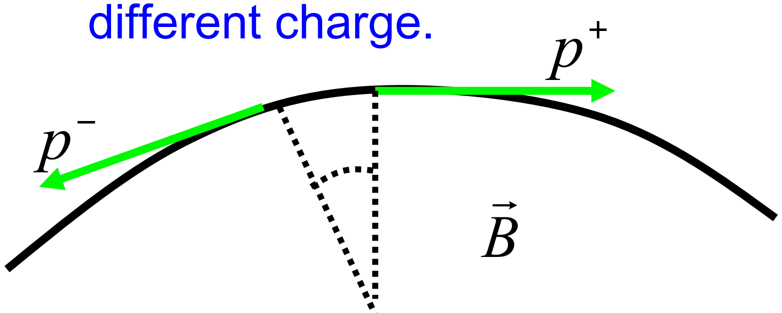
E has a similar effect as v B.

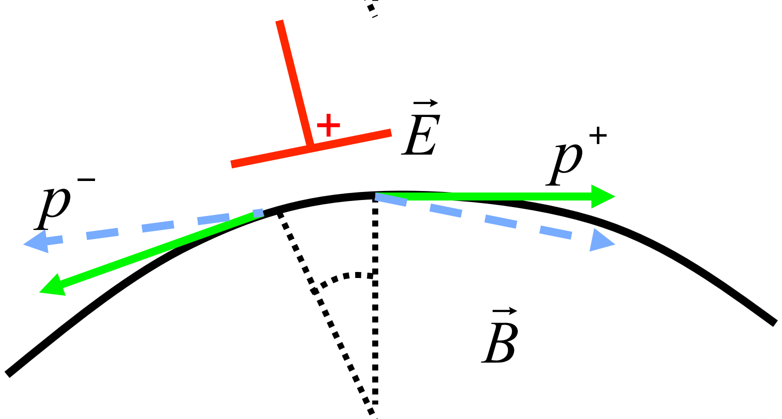
For relativistic particles B = 1T has a similar effect as

E = cB = $3\ 10^8$ V/m , such an

Electric field is beyond technical limits.

- Electric fields are only used for very low energies or
- For separating two counter rotating beams with different charge.





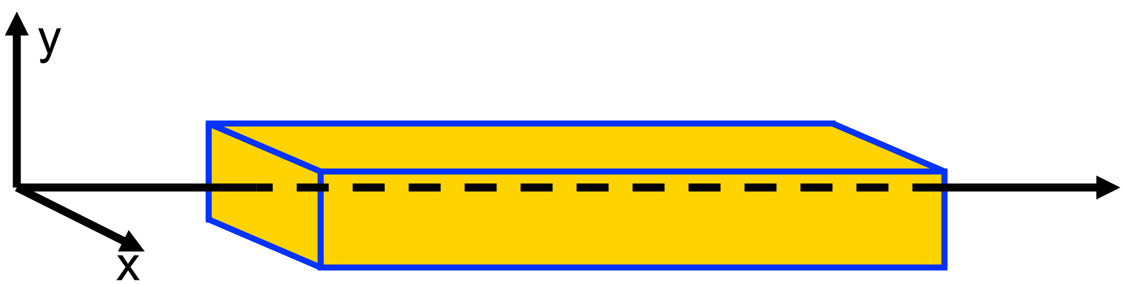
Electrostatic separators at CESR

Magnetic Fields in Accelerators

Static magnetic fileds: $\partial_t \vec{B} = 0\,; \quad \vec{E} = 0$ Charge free space: $\vec{j} = 0$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{j} + \varepsilon_0 \partial_t \vec{E}) = 0 \quad \Rightarrow \quad \vec{B} = -\vec{\nabla} \psi(\vec{r})$$

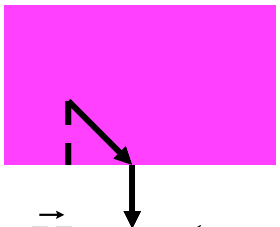
$$\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{\nabla}^2 \psi(\vec{r}) = 0$$



(x=0,y=0) is the beam's design curve

For finite fields on the design curve,
Ψ can be power expanded in x and y: $\psi(x,y,z) = \sum_{n,m=0}^{\infty} b_{nm}(z) x^n y^m$

Surfaces of Equal Potential



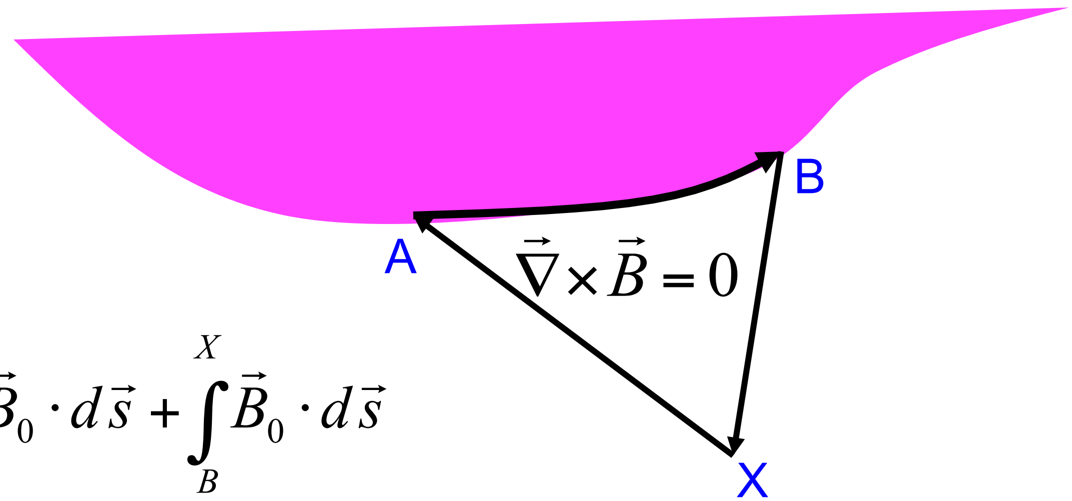
$$\vec{B}_\perp(\text{out}) = \vec{B}_\perp(\text{in})$$

$$\vec{H}_{\text{parallel}}(\text{out}) = \vec{H}_{\text{parallel}}(\text{in})$$

$$\vec{B}_{\text{parallel}}(\text{out}) = \frac{1}{\mu_r}\vec{B}_{\text{parallel}}(in)$$

$$\vec{B}(\vec{\mathrm{r}}) = -\vec{\nabla}\Psi(\vec{r})$$

For large permeability, H(out) is perpendicular to the surface.



$$0 = \oint \vec{B}\cdot d\vec{s} = \int_X^A \vec{B}_0\cdot d\vec{s} + \int_A^B \vec{B}_0\cdot d\vec{s} + \int_B^X \vec{B}_0\cdot d\vec{s}$$

$$= \int_X^A \vec{B}_0\cdot d\vec{s} + \frac{1}{\mu_r}\int_A^B \vec{B}_0\cdot d\vec{s} + \int_B^X \vec{B}_0\cdot d\vec{s}$$

$$\approx \int_X^A \vec{B}_0\cdot d\vec{s} + \int_B^X \vec{B}_0\cdot d\vec{s} = \Psi(A) - \Psi(B)$$

For highly permeable materials (like iron) surfaces have a constant potential.

Green's Theorem

$$\vec{\nabla}^2\psi = 0$$

Green function:

$$\vec{\nabla}_0^2 G(\vec{r},\vec{r}_0) = \delta(\vec{r}-\vec{r}_0)$$

$$\begin{aligned}
\psi(\vec{r}) &= \int_V \psi(\vec{r}_0)\delta(\vec{r}-\vec{r}_0)\,d^3\vec{r}_0 \\
&= \int_V \left[\psi(\vec{r}_0)\vec{\nabla}_0^2 G - G\vec{\nabla}_0^2\psi(\vec{r}_0)\right] d^3\vec{r}_0 \\
&= \int_V \vec{\nabla}_0\left[\psi(\vec{r}_0)\vec{\nabla}_0 G - G\vec{\nabla}_0\psi(\vec{r}_0)\right] d^3\vec{r}_0 \\
&= \int_{\partial V} \left[\psi(\vec{r}_0)\vec{\nabla}_0 G - G\vec{\nabla}_0\psi(\vec{r}_0)\right]\cdot d^2\vec{r}_0 \\
&= \int_{\partial V} \left[\psi(\vec{r}_0)\vec{\nabla}_0 G + \vec{B}(\vec{r}_0)G\right]\cdot d^2\vec{r}_0
\end{aligned}$$

Knowledge of the field and the scalar magnetic potential on a closed surface inside a magnet determines the magnetic field for the complete volume which is enclosed.

Potential Expansion

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If field data in a plane (for example the midplane of a cyclotron or of a beam line magnet) is known, the complete filed is determined:

$$\psi(x,y,z)=\sum_{n=0}^{\infty} b_n(x,z)\,y^n \quad\Rightarrow\quad \vec{B}(x,0,z)=-\begin{pmatrix}\partial_x b_0(x,z)\\ b_1(x,z)\\ \partial_z b_0(x,z)\end{pmatrix}$$

$$0=\vec{\nabla}^2\psi=\sum_{n=0}^{\infty}(\partial_x^2+\partial_z^2)\,b_n\,y^n+\sum_{n=2}^{\infty} n(n-1)\,b_n\,y^{n-2}$$

$$=\sum_{n=0}^{\infty}\Big[(\partial_x^2+\partial_z^2)\,b_n+(n+2)(n+1)\,b_{n+2}\Big]y^n$$

$$b_{n+2}(x,z)=-\frac{1}{(n+2)(n+1)}(\partial_x^2+\partial_y^2)\,b_n(x,z)$$

Data of the magnetic field in the plane y=0 is used to determine $b_0(x,z)$ and $b_1(x,z)$.

Complex Potentials

$$w = x + iy \quad , \quad \overline{w} = x - iy$$

$$\partial_x = \partial_w + \partial_{\overline{w}} \, , \quad \partial_y = i\partial_w - i\partial_{\overline{w}} = i(\partial_w - \partial_{\overline{w}})$$

$$\vec{\nabla}^2 = \partial_x^2 + \partial_y^2 + \partial_z^2 = (\partial_w + \partial_{\overline{w}})^2 - (\partial_w - \partial_{\overline{w}})^2 + \partial_z^2 = 4\partial_w \partial_{\overline{w}} + \partial_z^2$$

$$\psi = \mathrm{Im}\{\sum_{\nu,\lambda=0}^{\infty} a_{\nu\lambda}(z) \cdot (w\overline{w})^\lambda \, \overline{w}^\nu\}$$

$$\vec{\nabla}^2\psi = \mathrm{Im}\{\sum_{\nu=0,\lambda=1}^{\infty} 4a_{\nu\lambda}(\lambda+\nu)\lambda(w\overline{w})^{\lambda-1}\overline{w}^\nu + \sum_{\nu=0,\lambda=0}^{\infty} a''_{\nu\lambda}(w\overline{w})^\lambda \overline{w}^\nu\}$$

$$= \mathrm{Im}\{\sum_{\nu,\lambda=0}^{\infty}[4(\lambda+1+\nu)(\lambda+1)a_{\nu\lambda+1} + a''_{\nu\lambda}](w\overline{w})^\lambda \overline{w}^\nu\} = 0$$

Iteration equation: $a_{\nu\lambda+1} = \dfrac{-1}{4(\lambda+1+\nu)(\lambda+1)} a''_{\nu\lambda} \quad , \quad a_{\nu 0} = \Psi_\nu(z)$

The functions $\Psi_\nu(z)$ along a line determine the complete field inside a magnet.

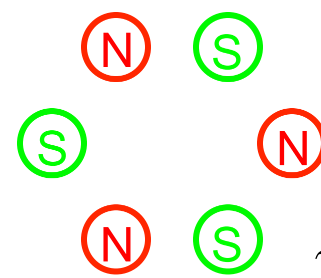
Multipole Coefficients

$\Psi_\nu(z)$ are called the z-dependent multipole coefficients

$$\psi(x,y,z) = \mathrm{Im}\{\sum_{\nu,\lambda=0}^{\infty} \frac{(-1)^\lambda \nu!}{(\lambda+\nu)!\lambda!}\left(\frac{w\bar{w}}{4}\right)^\lambda \bar{w}^\nu \Psi_\nu^{[2\lambda]}(z)\}$$

$$\psi(r,\varphi,z) = \sum_{\nu,\lambda=0}^{\infty} \frac{(-1)^\lambda \nu!}{(\lambda+\nu)!\lambda!}\left(\frac{r}{2}\right)^{2\lambda} r^\nu \,\mathrm{Im}\{\Psi_\nu^{[2\lambda]}(z)e^{-i\nu\varphi}\}$$

The index ν describes C_ν Symmetry around the z-axis $\vec{e}_z$ due to a sign change after $\Delta\varphi = \frac{\pi}{\nu}$



$\nu = 3$