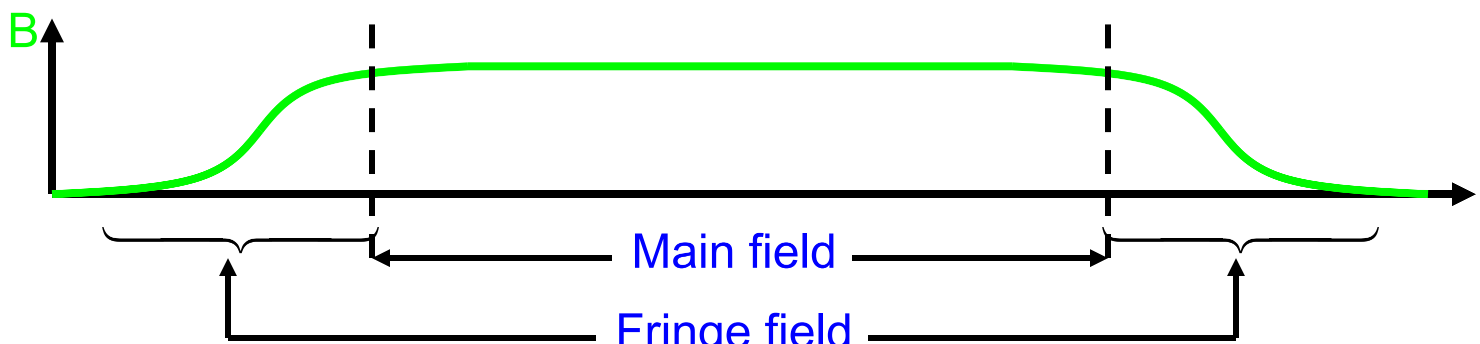
Fringe Fields and Main Fields

CHESS & LEPP



Only the fringe field region has terms with $\lambda \neq 0$ and $\partial_z^2 \psi \neq 0$

Main fields in accelerator physics: $\lambda = 0\,,\quad \partial_z^2 \psi = 0$

$$\Psi_\nu = \begin{cases} e^{i\nu\vartheta_\nu} |\Psi_\nu| & \text{for} \quad \nu \neq 0 \\ i \quad |\Psi_0| & \text{for} \quad \nu = 0 \end{cases}$$

$$\psi(r,\varphi) = \sum_{\nu=1}^{\infty} r^\nu |\Psi_\nu| \,\mathrm{Im}\{e^{-i\nu(\varphi-\vartheta_\nu)}\} + |\Psi_0|$$

Main Field Potential

Main field potential: $$\psi = |\Psi_0| - \sum_{\nu=1}^{\infty} r^{\nu} |\Psi_{\nu}| \sin[\nu(\varphi - \vartheta_{\nu})]$$

The isolated multipole: $$\psi = -r^{\nu} |\Psi_{\nu}| \sin(\nu\varphi)$$

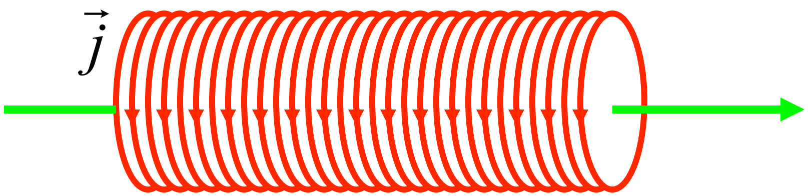
Where the rotation ϑ_{ν} of the coordinate system is set to 0

The potentials produced by different multipole components Ψ_{ν} have

a) Different rotation symmetry C_{ν}

b) Different radial dependence r^{ν}

Multipoles in Accelerators ν=0: Solenoids



$$m\gamma \begin{pmatrix}\ddot x\\ \ddot y\\ \ddot z\end{pmatrix} = q\begin{pmatrix}\dot x\\ \dot y\\ \dot z\end{pmatrix}\times\begin{pmatrix}-\frac{x}{2}B_z'\\ -\frac{y}{2}B_z'\\ B_z\end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix}\ddot x\\ \ddot y\end{pmatrix} = \frac{qB_z}{m\gamma}\begin{pmatrix}\dot y\\ -\dot x\end{pmatrix} + \frac{qB_z'\dot z}{2m\gamma}\begin{pmatrix}y\\ -x\end{pmatrix}$$

$$\Downarrow$$

$$\ddot w = -i\frac{qB_z}{m\gamma}\dot w - i\frac{q\dot B_z}{2m\gamma} w$$

$$\psi = \Psi_0(z) - \frac{w\bar w}{4}\Psi_0''(z) \pm \dots$$

$$\vec B = \begin{pmatrix}\frac{x}{2}\Psi_0''\\ \frac{y}{2}\Psi_0''\\ -\Psi_0'\end{pmatrix} \quad\Rightarrow\quad \vec\nabla\cdot\vec B = 0$$

$$g = \frac{qB_z}{2m\gamma}\ ,\quad w_0 = w\, e^{i\int_0^t g\,dt}$$

$$\ddot w_0 = (\ddot w + i2g\dot w + i\dot g w - g^2 w)\, e^{i\int_0^t g\,dt}$$
$$= -g^2 w_0$$

$$\ddot x_0 = -g^2 x_0$$
$$\ddot y_0 = -g^2 y_0$$

Focusing in a rotating coordinate system.

For const. g: $x_0(t) = Cos[g(t-t_0))x_0(0)$