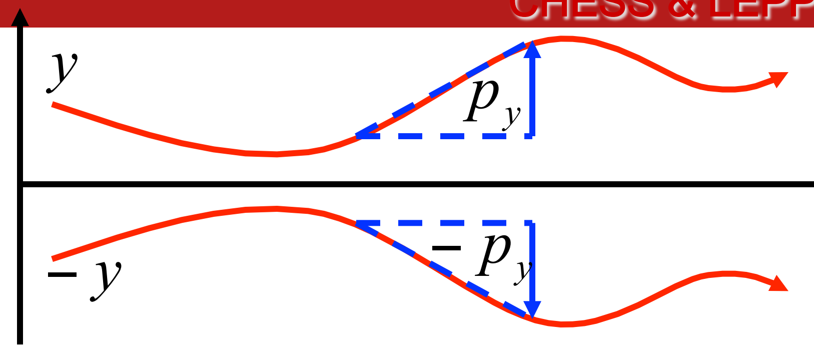
Midplane Symmetric Motion

$$\vec{r}^{\oplus} = (x, -y, z)$$

$$\vec{p}^{\oplus} = (p_x, -p_y, p_z)$$

$$\frac{d}{dt}\vec{p} = \vec{F}(\vec{r}, \vec{p}) \quad \Rightarrow \quad \frac{d}{dt}\vec{p}^{\oplus} = \vec{F}(\vec{r}^{\oplus}, \vec{p}^{\oplus})$$



$$\begin{aligned} v_y B_z - v_z B_y &= -v_y B_z(x,-y,z) - v_z B_y(x,-y,z) & & B_x(x,-y,z) = -B_x(x,y,z) \\ v_z B_x - v_x B_z &= -v_z B_x(x,-y,z) + v_x B_z(x,-y,z) & \Rightarrow \quad & B_y(x,-y,z) = B_y(x,y,z) \\ v_x B_y - v_y B_x &= v_x B_y(x,-y,z) + v_y B_x(x,-y,z) & & B_z(x,-y,z) = -B_z(x,y,z) \end{aligned}$$

$$\boxed{\psi(x,-y,z) = -\psi(x,y,z)}$$

$$\Psi_n \operatorname{Im}\left\{ e^{in\vartheta_n}(x+iy)^n \right\} = -\Psi_n \operatorname{Im}\left\{ e^{in\vartheta_n}(x-iy)^n \right\}$$

$$\Rightarrow \quad \Psi_n \operatorname{Im}\left[(e^{in\vartheta_n} - e^{-in\vartheta_n})(x+iy)^n \right] = 0 \quad \Rightarrow \quad \boxed{\vartheta_n = 0}$$

The discussed multipoles

produce midplane symmetric motion. When the field is rotated by $\pi/2$,
i.e $\vartheta_n = \pi/2n$, one speaks of a skew multipole.

Superconducting Magnets

Above 2T the field from the bare coils dominate over the magnetization of the iron.

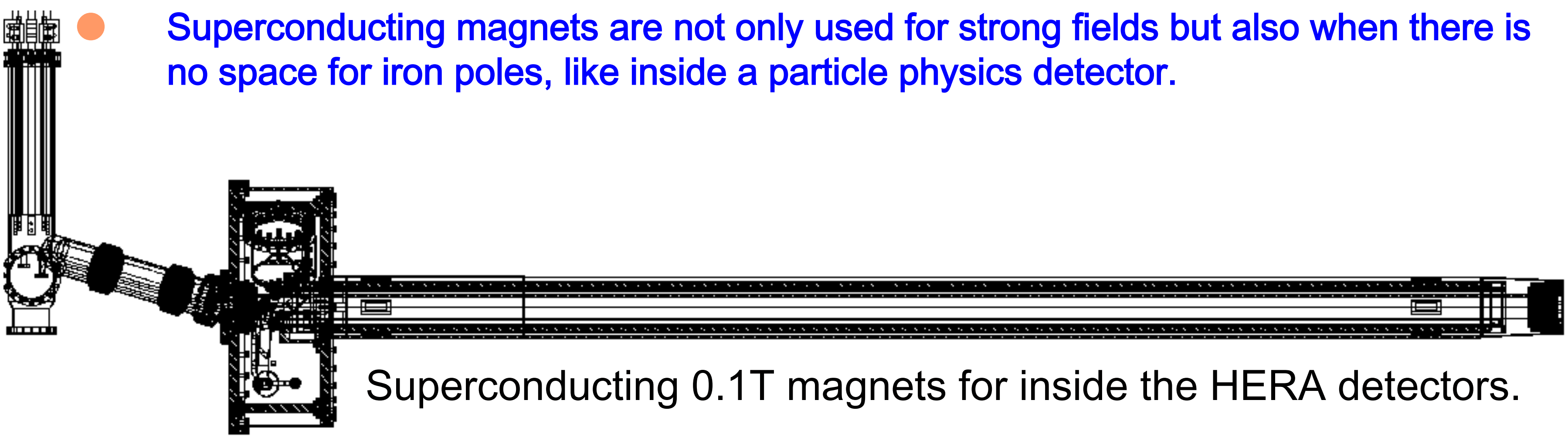
But Cu wires cannot create much filed without iron poles:

5T at 5cm distance from a 3cm wire would require a current density of

$$j = \frac{I}{d^2} = \frac{1}{d^2}\frac{2\pi\, r\, B}{\mu_0} = 1389\,\frac{\text{A}}{\text{mm}^2}$$

Cu can only support about 100A/mm^2.

- Superconducting cables routinely allow current densities of 1500A/mm^2 at 4.6 K and 6T. Materials used are usually Nb aloys, e.g. NbTi, Nb_3Ti or Nb_3Sn.
- Superconducting magnets are not only used for strong fields but also when there is no space for iron poles, like inside a particle physics detector.

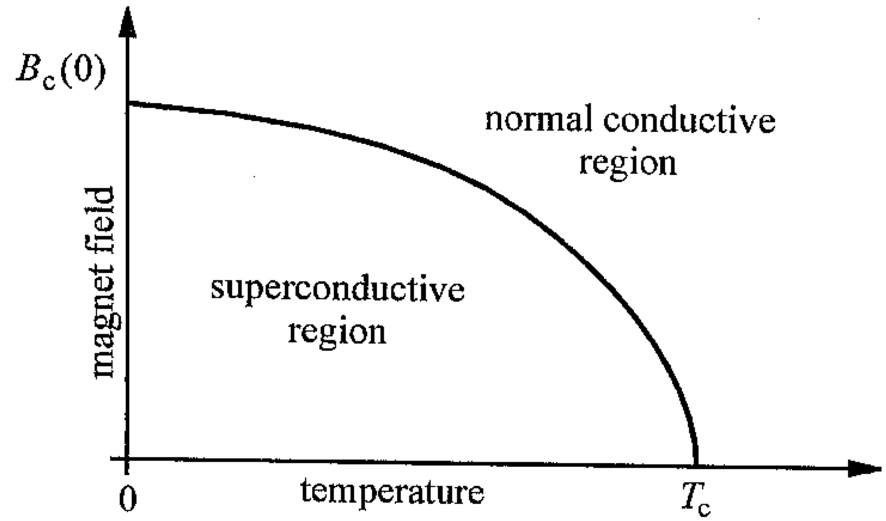


Superconducting 0.1T magnets for inside the HERA detectors.

Superconducting Magnets

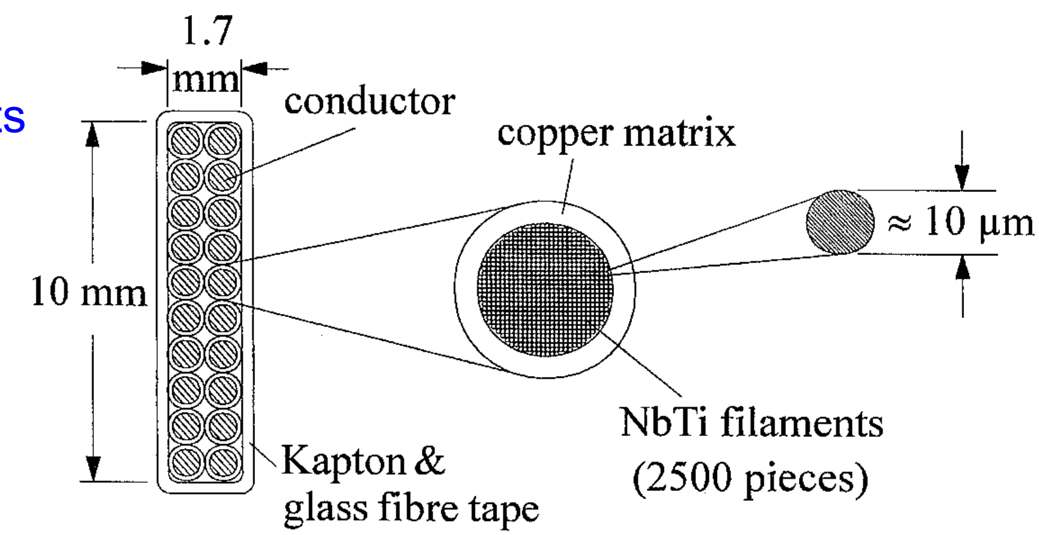
Problems:

- Superconductivity brakes down for too large fields
- Due to the Meissner-Ochsenfeld effect superconductivity current only flows on a thin surface layer.



Remedy:

- Superconducting cable consists of many very thin filaments (about 10μm).



Complex Potential of a Wire

Straight wire at the origin: $\vec{\nabla}\times\vec{B} = \mu_0\,\vec{j} \quad\Rightarrow\quad \vec{B}(r) = \frac{\mu_0\,I}{2\pi\,r}\vec{e}_\varphi = \frac{\mu_0\,I}{2\pi\,r}\begin{pmatrix}-y\\ x\end{pmatrix}$

Wire at $\vec{a}$:

$$\vec{B}(x,y) = \frac{\mu_0\,I}{2\pi\,(\vec{r}-\vec{a})^2}\begin{pmatrix}-[y-a_y]\\ x-a_x\end{pmatrix}$$

This can be represented by complex multipole coefficients Ψ_ν

$$\vec{B}(x,y) = -\vec{\nabla}\Psi \quad\Rightarrow\quad B_x + iB_y = -\left(\partial_x + i\partial_y\right)\psi = -2\partial_{\overline{w}}\psi$$

$$B_x + iB_y = \tfrac{\mu_0 I}{2\pi}\frac{-i(w_a - w)}{(w_a - w)(\overline{w}_a - \overline{w})} = i\,\tfrac{\mu_0 I}{2\pi}\frac{-\frac{w_a}{a^2}}{1-\frac{\overline{w}w_a}{a^2}}$$

$$= i\,\tfrac{\mu_0 I}{2\pi}\,\partial_{\overline{w}}\ln(1-\tfrac{\overline{w}w_a}{a^2}) = -2\partial_{\overline{w}}\,\mathrm{Im}\left\{\tfrac{\mu_0 I}{2\pi}\ln(1-\tfrac{\overline{w}w_a}{a^2})\right\}$$

$$\psi = \mathrm{Im}\left\{\tfrac{\mu_0 I}{2\pi}\ln(1-\tfrac{\overline{w}w_a}{a^2})\right\} = -\,\mathrm{Im}\left\{\tfrac{\mu_0 I}{2\pi}\sum_{\nu=1}^{\infty}\tfrac{1}{\nu}\left(\tfrac{w_a}{a^2}\right)^\nu\overline{w}^\nu\right\} \quad\Rightarrow\quad \Psi_\nu = \tfrac{\mu_0 I}{2\pi}\,\tfrac{1}{\nu}\,\tfrac{1}{a^\nu}\,e^{i\nu\varphi_a}$$