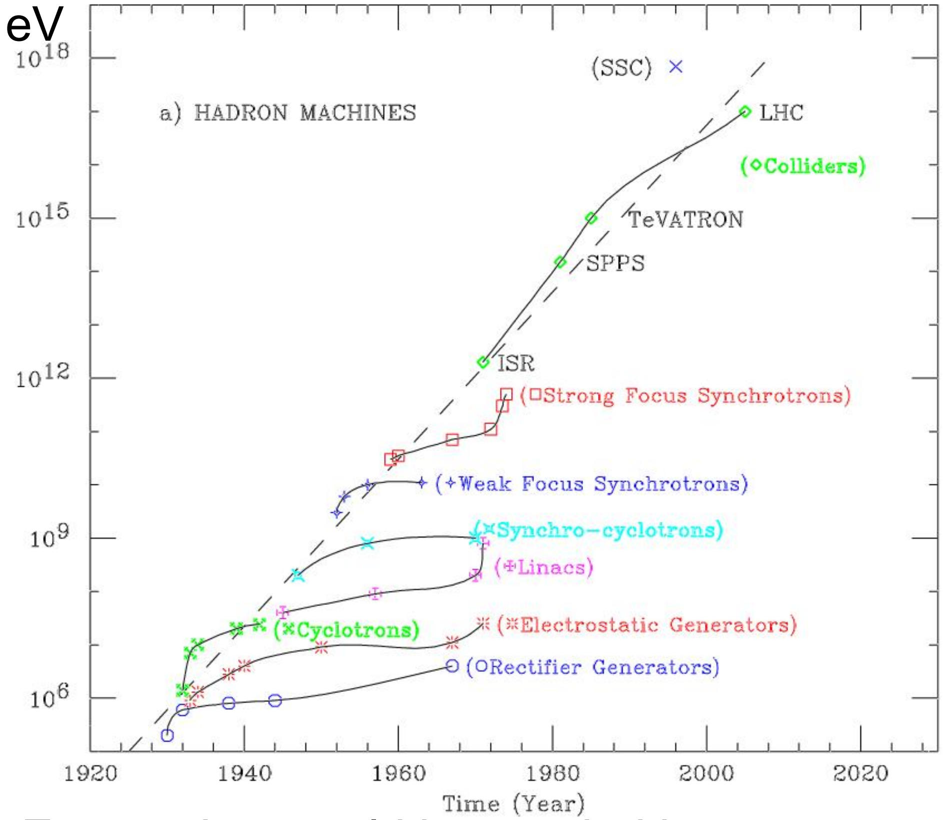
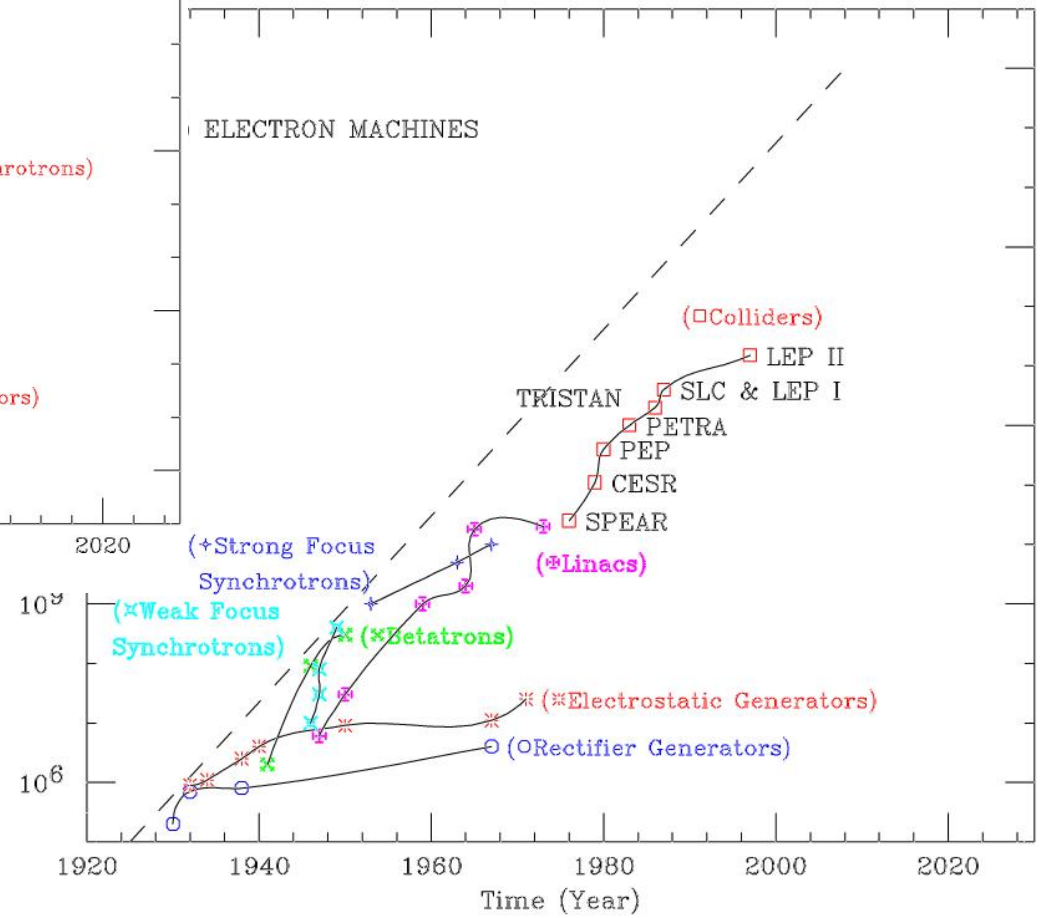
The Livingston Chart



Comparison:
highest energy cosmic rays
have a few 10^{20}eV



Energy that would be needed in a fixed target experiment versus the year of achievement

$$E_1 = \frac{E_{\mathrm{cm}}^2}{2m_{02}c^2}$$

Rings for Synchrotron Radiation

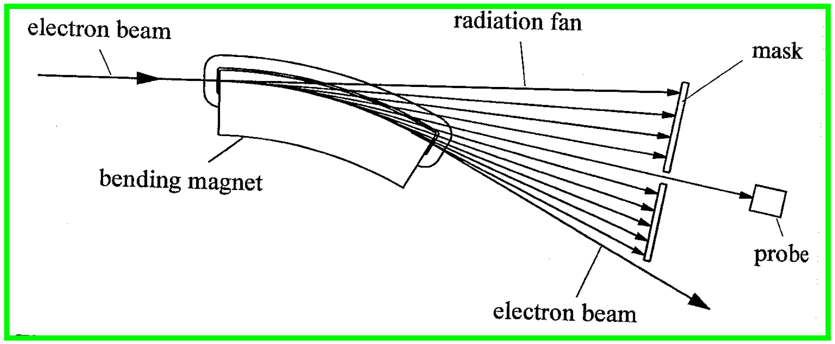
- 1947: First detection of synchrotron light at General Electrics.
- 1952: First accurate measurement of synchrotron radiation power by Dale Corson with the Cornell 300MeV synchrotron.
- 1968: TANTALOS, first dedicated storage ring for synchrotron radiation

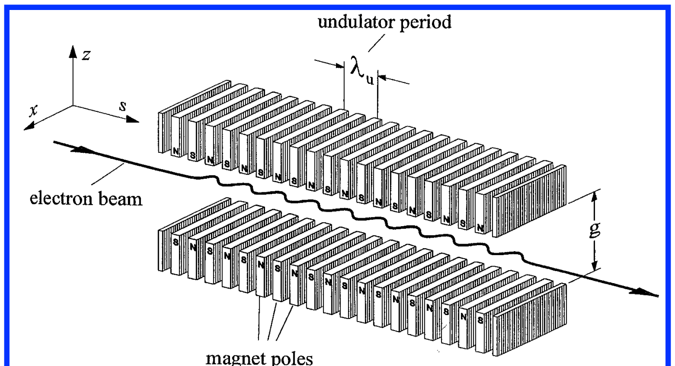
Dale Corson
Cornell's 8^{th} president
USA 1914 –

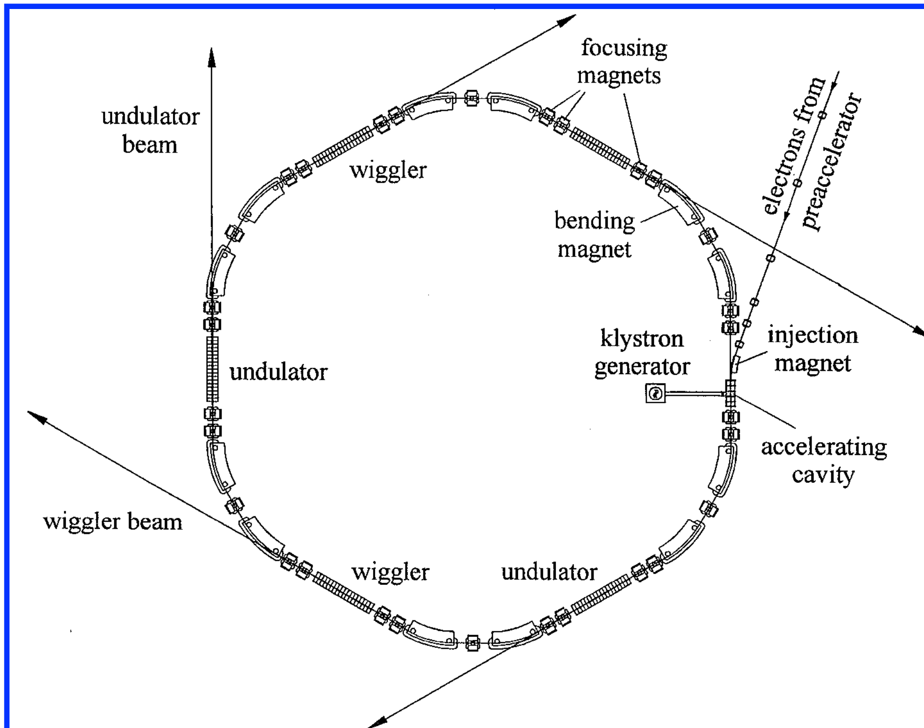
3 Generations of Light Sources

CHESS & LEPP

- 1st Genergation (1970s): Many HEP rings are parasitically used for X-ray production
- 2nd Generation (1980s): Many dedicated X-ray sources (light sources)
- 3rd Generation (1990s): Several rings with dedicated radiation devices (wigglers and undulators)
- Today (4th Generation): Construction of Free Electron Lasers (FELs) driven by LINACs





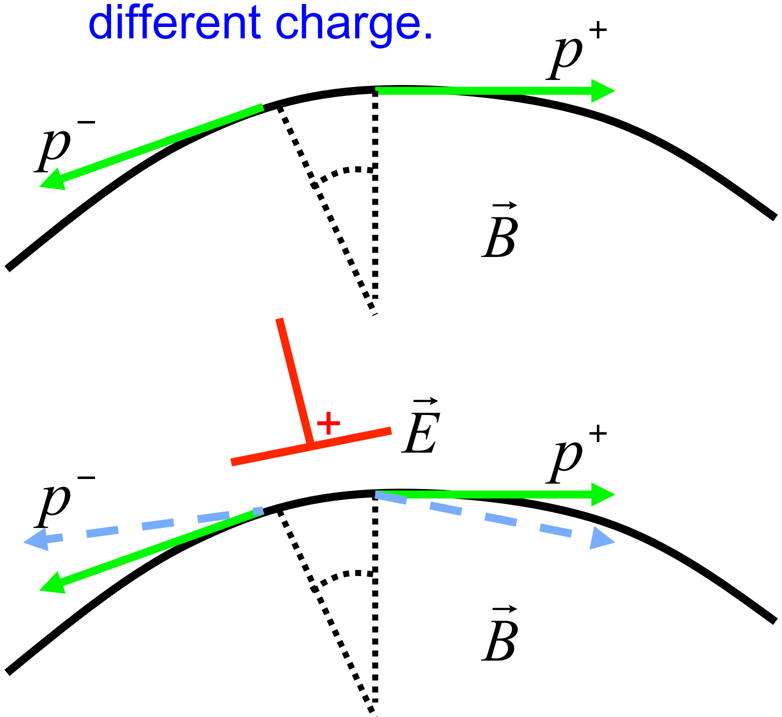


Macroscopic Fields in Accelerators

$$\frac{d}{dt}\vec{p} = q(\vec{E} + \vec{v} \times \vec{B})$$

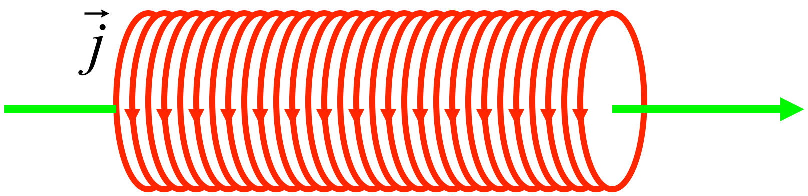
E has a similar effect as v B.
For relativistic particles B = 1T has a similar effect as
E = cB = 3 10^8 V/m , such an
Electric field is beyond technical limits.

- Electric fields are only used for very low energies or
- For separating two counter rotating beams with different charge.



Electrostatic separators at CESR

Multipoles in Accelerators   ν=0: Solenoids



$$m\gamma\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix} = q\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix}\times\begin{pmatrix}-\frac{x}{2}B_z'\\ -\frac{y}{2}B_z'\\ B_z\end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = \frac{qB_z}{m\gamma}\begin{pmatrix}\dot{y}\\ -\dot{x}\end{pmatrix} + \frac{qB_z'\dot{z}}{2m\gamma}\begin{pmatrix}y\\ -x\end{pmatrix}$$

$$\Downarrow$$

$$\ddot{w} = -i\frac{qB_z}{m\gamma}\dot{w} - i\frac{q\dot{B}_z}{2m\gamma}w$$

$$\psi = \Psi_0(z) - \frac{w\bar{w}}{4}\Psi_0''(z) \pm \ldots$$

$$\vec{B} = \begin{pmatrix}\frac{x}{2}\Psi_0''\\ \frac{y}{2}\Psi_0''\\ -\Psi_0'\end{pmatrix} \quad\Rightarrow\quad \vec{\nabla}\cdot\vec{B} = 0$$

$$g = \frac{qB_z}{2m\gamma}\ , \quad w_0 = w\,e^{i\int_0^t g\,dt}$$

$$\ddot{w}_0 = (\ddot{w} + i2g\dot{w} + i\dot{g}w - g^2 w)e^{i\int_0^t g\,dt}$$

$$= -g^2 w_0$$

$$\boxed{\begin{aligned}\ddot{x}_0 &= -g^2 x_0\\ \ddot{y}_0 &= -g^2 y_0\end{aligned}}$$

Focusing in a rotating coordinate system.

For const. g: $x_0(t) = Cos[g(t-t_0))x_0(0)$

Solenoid vs. Strong Focusing

If the solenoids field was perpendicular to the particle's motion,

its bending radius would be $\rho_z = \dfrac{p}{qB_z}$

$$\ddot{\vec{r}} = -\left(\frac{qB_z}{2m\gamma}\right)^2 \vec{r} = -\frac{qv_z}{m\gamma} B_z \frac{\vec{r}}{4\rho_z}$$

Solenoid focusing is weak compared to the deflections created by a transverse magnetic field.

Transverse fields: $\vec{B} = B_x \, \vec{e}_x + B_y \, \vec{e}_y$

$$m\gamma \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = q \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \frac{qv_z}{m\gamma} \begin{pmatrix} -B_y \\ B_x \end{pmatrix}$$ Strong focusing

Weak focusing < Strong focusing by about r/ρ

Solenoid Focusing

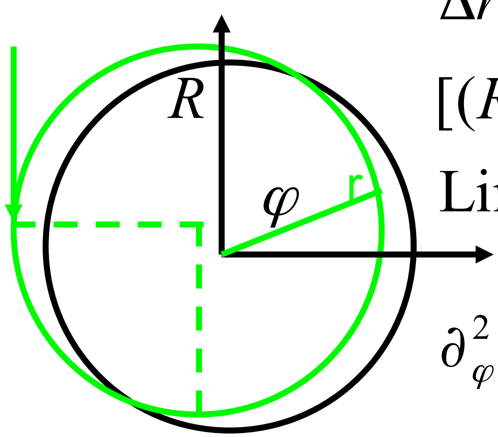
Solenoid magnets are used in detectors for particle identification via $\rho = \frac{p}{qB}$

The solenoid's rotation $\dot{\varphi} = -\frac{qB_z}{2m\gamma}$ of the beam is often compensated by a reversed solenoid called compensator.

Solenoid or Weak Focusing:

Solenoids are also used to focus low γ beams: $\ddot{w} = -\boxed{\left(\frac{qB_z}{2m\gamma}\right)^2} w$

Weak focusing from natural ring focusing:



$$\Delta r = r - R$$

$$[(R+\Delta r)\cos\varphi - \Delta x_0]^2 + [(R+\Delta r)\sin\varphi - \Delta y_0]^2 = R^2$$

$$\text{Linearization in } \Delta: \quad \Delta r = (\cos\varphi\,\Delta x_0 + \sin\varphi\,\Delta y_0)$$

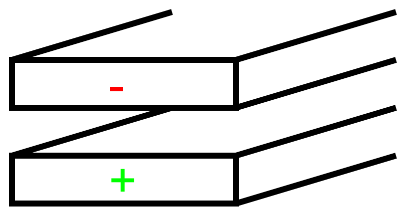
$$\partial_\varphi^2 \Delta r = -\Delta r \quad \Rightarrow \quad \Delta\ddot{r} = -\dot{\varphi}^2 \Delta r = -\left(\frac{v}{\rho}\right)^2 \Delta r = -\boxed{\left(\frac{qB}{m\gamma}\right)^2} \Delta r$$

Multipoles in Accelerators ν=1: Dipoles

$$\psi = \Psi_1 \,\mathrm{Im}\{x - iy\} = -\Psi_1 \cdot y \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_1 \,\vec{e}_y$$

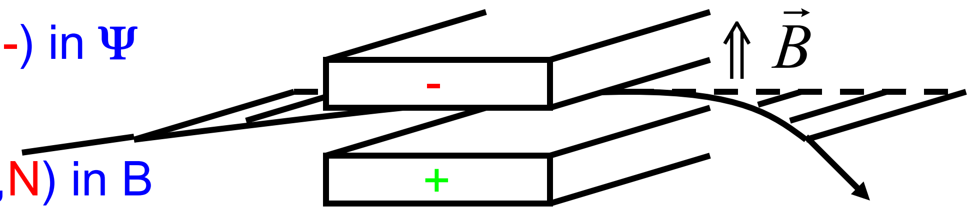
Equipotential
$y = \text{const.}$

C$_1$ Symmetry

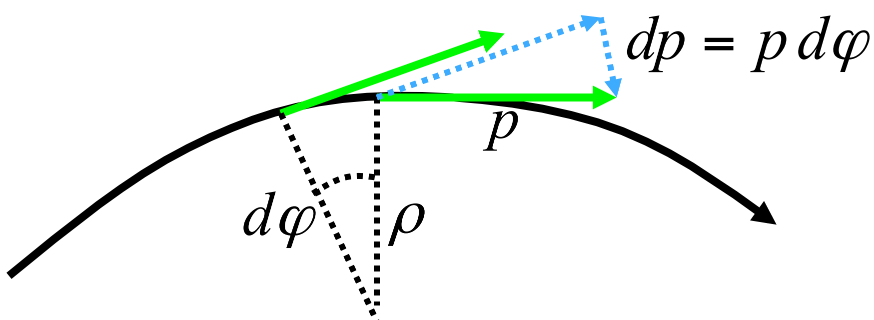


(+,-) in Ψ

(S,N) in B



Dipole magnets are used for steering the beams direction

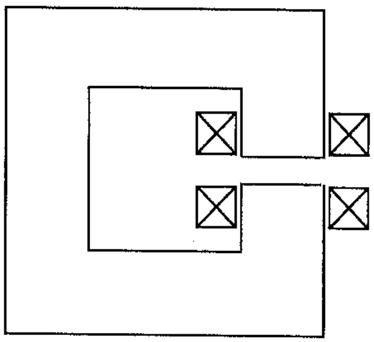


$$\frac{d\vec{p}}{dt} = q\,\vec{v} \times \vec{B} \quad \Rightarrow \quad \frac{dp}{dt} = qvB_\perp \quad \Rightarrow \quad \rho = \frac{dl}{d\varphi} = \frac{vdt}{dp/p} = \frac{p}{qB_\perp}$$

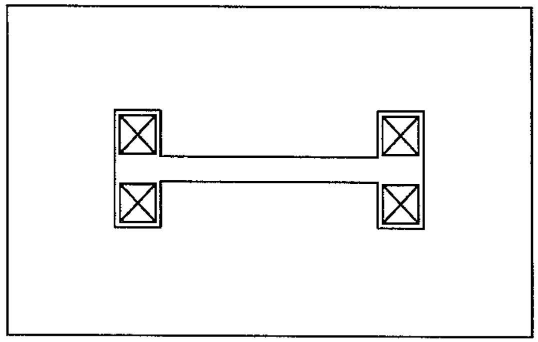
Bending radius: $\rho = \dfrac{p}{qB}$

Different Dipoles

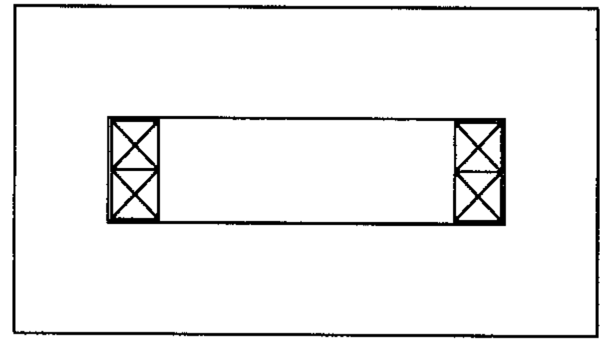
C-shape magnet:

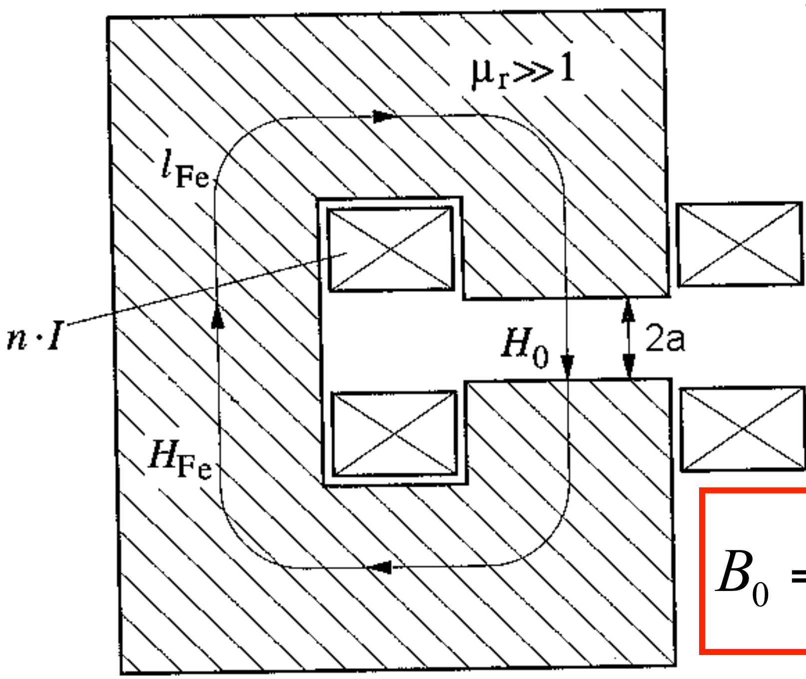


H-shape magnet:



Window frame magnet:





$$\vec{B}_\perp(\text{out}) = \vec{B}_\perp(\text{in})$$

$$\vec{H}_\perp(\text{out}) = \mu_r \vec{H}_\perp(\text{in})$$

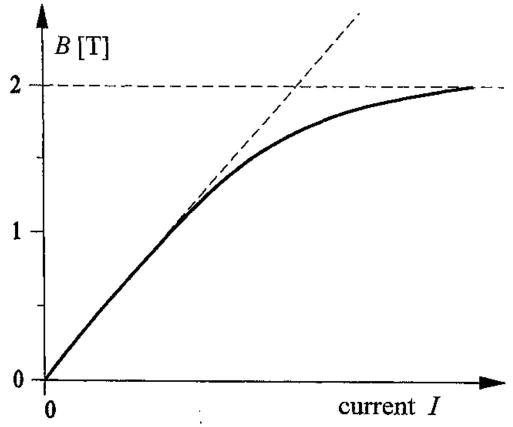
$$2nI = \oint \vec{H}\cdot d\vec{s} = H_{Fe} l_{Fe} + H_0 2a$$

$$= \tfrac{1}{\mu_r} H_0 l_{Fe} + H_0 2a \approx H_0 2a$$

$$\boxed{B_0 = \mu_0 \frac{nI}{a}}$$

Dipole strength: $\frac{1}{\rho} = \frac{q\mu_0}{p}\frac{nI}{a}$

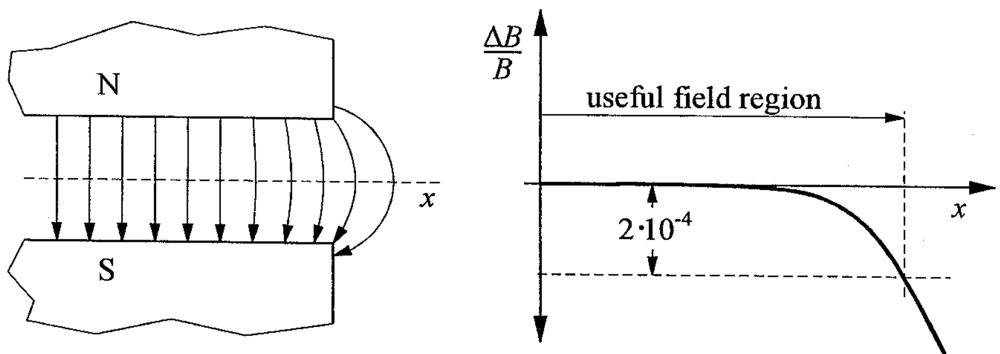
Dipole Fields

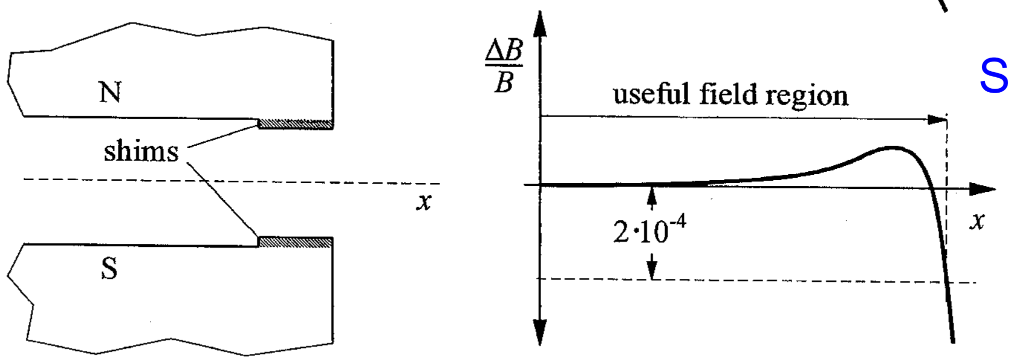


B = 2 T: Typical limit, since the field becomes dominated by the coils, not the iron. Limiting j for Cu is about 100A/mm^2

B < 1.5 T: Typically used region

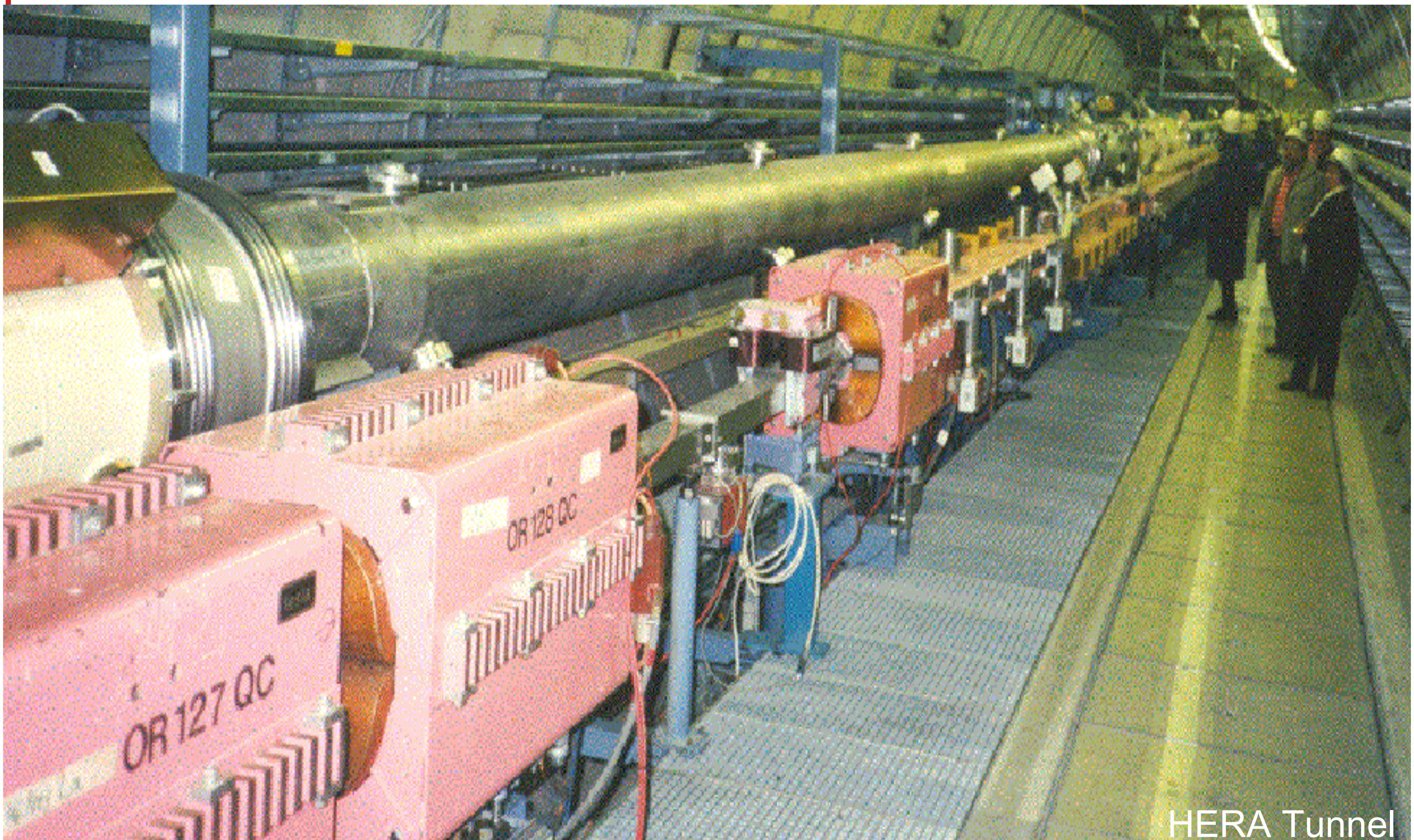
B < 1 T: Region in which $B_0 = \mu_0 \frac{nI}{a}$





Shims reduce the space that is open to the beam, but they also reduce the fringe field region.

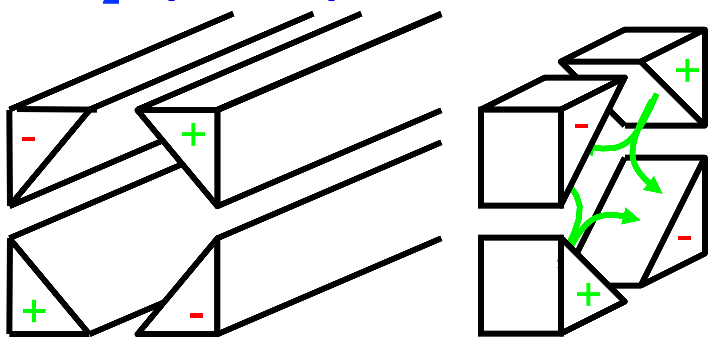
Where is the vertical Dipole?

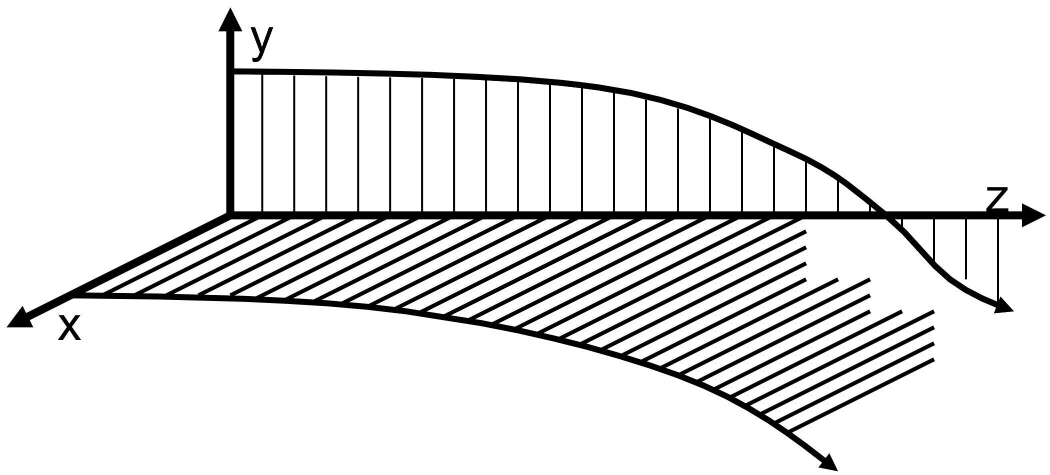


Multipoles in Accelerators   ν=2: Quadrupoles

$$\psi = \Psi_2 \,\mathrm{Im}\{(x-iy)^2\} = -\Psi_2 \cdot 2xy \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_2 \, 2\begin{pmatrix} y \\ x \end{pmatrix}$$

C_2 Symmetry

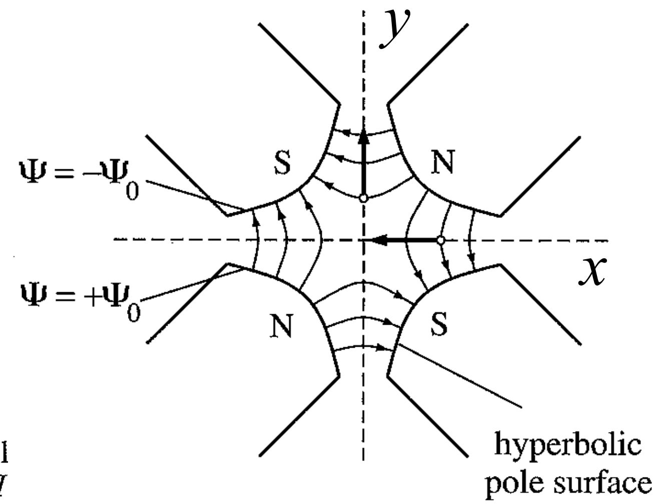


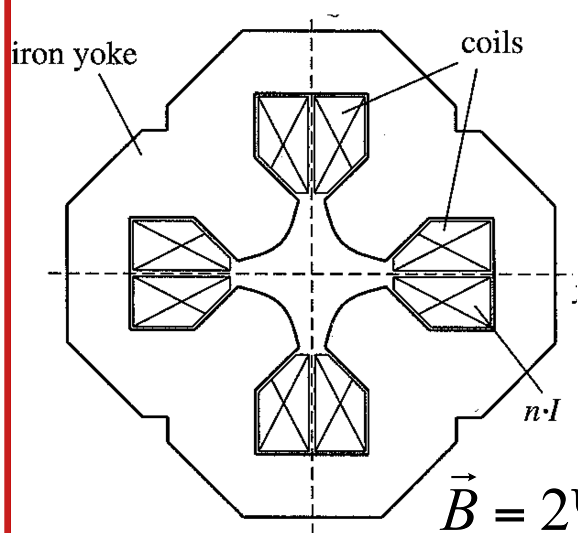


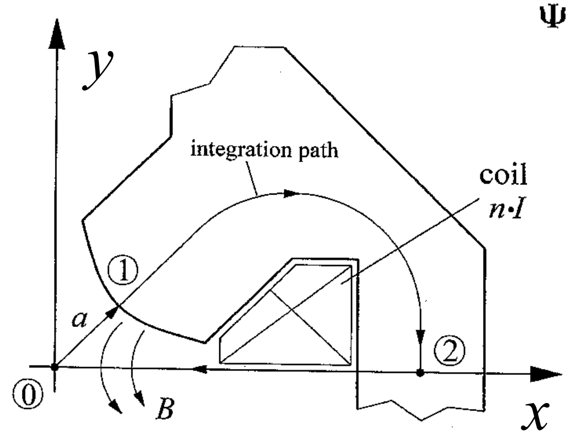
In a quadrupole particles are focused in one plane and defocused in the other plane. Other modes of strong focusing are not possible.

Quadrupole Fields

$$\psi = -\Psi_2 \cdot 2xy \quad \Rightarrow \quad \text{Equipotential: } x = \frac{\text{const.}}{y}$$







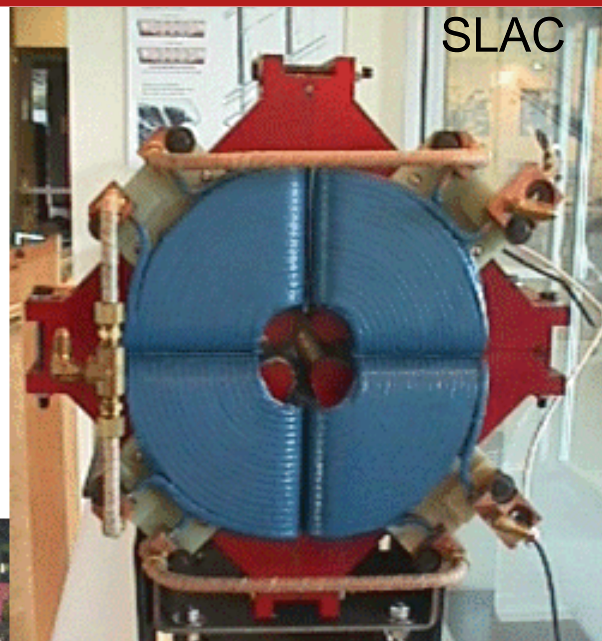
$$\vec{B} = 2\Psi_2 \begin{pmatrix} y \\ x \end{pmatrix} \quad \Rightarrow \quad \vec{B}(0 \mapsto 1) = 2\Psi_2 r\, \vec{e}_r$$

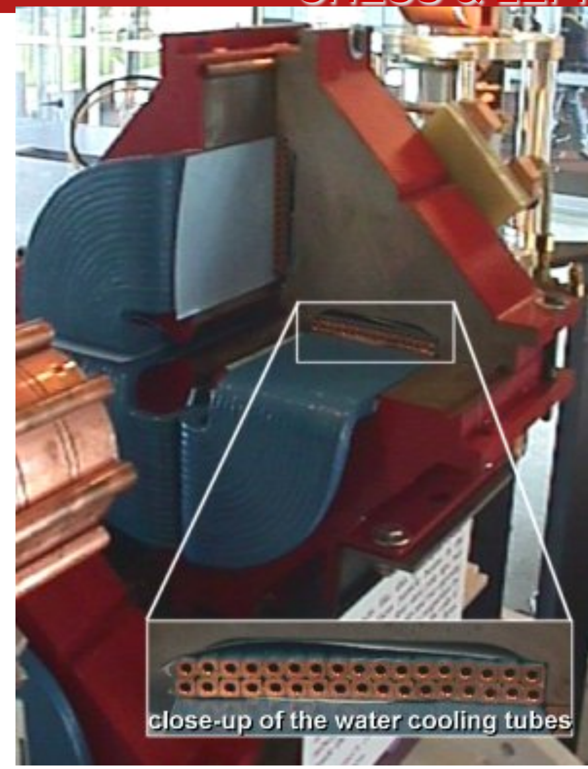
$$nI = \oint \vec{H} \cdot d\vec{s} \approx \int_0^a H_r dr = \Psi_2 \frac{a^2}{\mu_0}$$

Quadrupole strength:

$$k_1 = \frac{q}{p} \partial_x B_y \Big|_0 = 2\frac{q}{p} \Psi_2 = \frac{q\mu_0}{p} \frac{2nI}{a^2}$$

Real Quadrupoles


SLAC



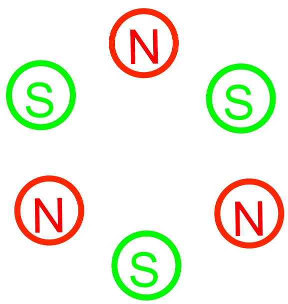

PETRA Tunnel

The coils show that this is an upright quadrupole not a rotated or skew quadrupole.

Multipoles in Accelerators ν=3: Sextupoles

$$\psi = \Psi_3 \,\mathrm{Im}\{(x - iy)^3\} = \Psi_3 \cdot (y^3 - 3x^2 y) \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_3 \, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$$

C_3 Symmetry



i) Sextupole fields hardly influence the particles close to the center, where one can linearize in x and y.

ii) In linear approximation a by Δx shifted sextupole has a quadrupole field.

iii) When Δx depends on the energy, one can build an energy dependent quadrupole.

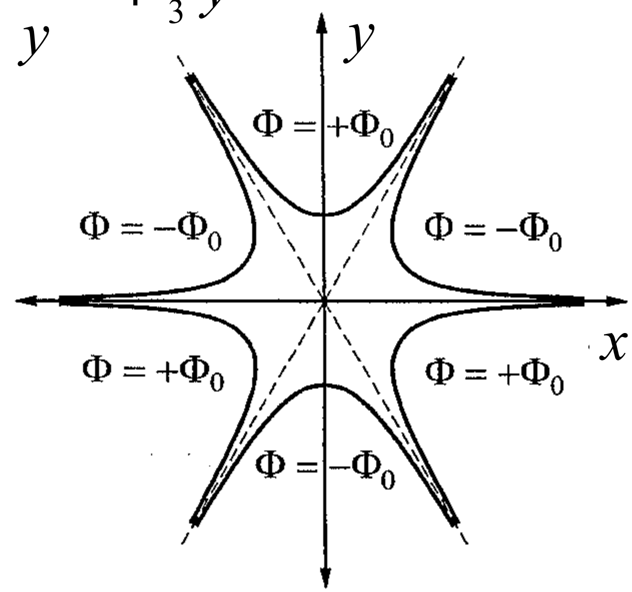
$$\vec{B} = -\vec{\nabla}\psi = \Psi_3 \, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$$

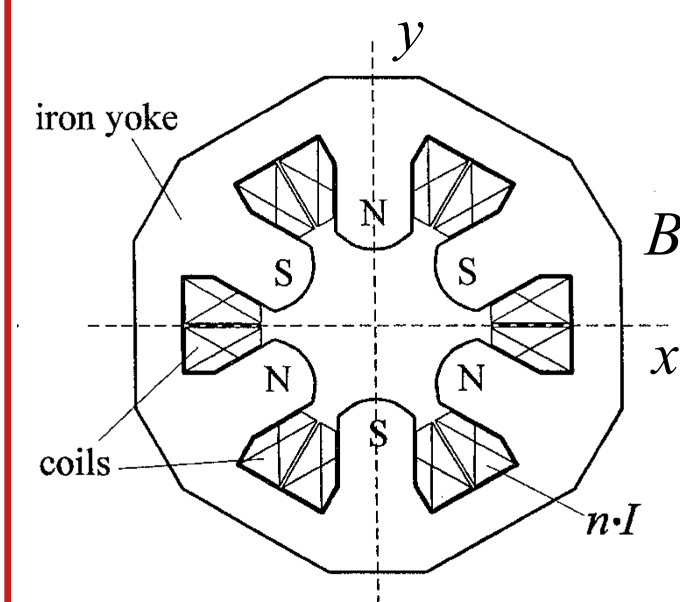
$$x \mapsto \Delta x + x$$

$$\vec{B} \approx \Psi_3 \, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix} + 6\Psi_3 \Delta x \begin{pmatrix} y \\ x \end{pmatrix} + O(\Delta x^2)$$

Sextupole Fields

$$\psi = \Psi_2 \cdot (y^3 - 3x^2 y) \quad \Rightarrow \quad \text{Equipotential:} \quad x = \sqrt{\frac{\text{const.}}{y} + \tfrac{1}{3} y^2}$$





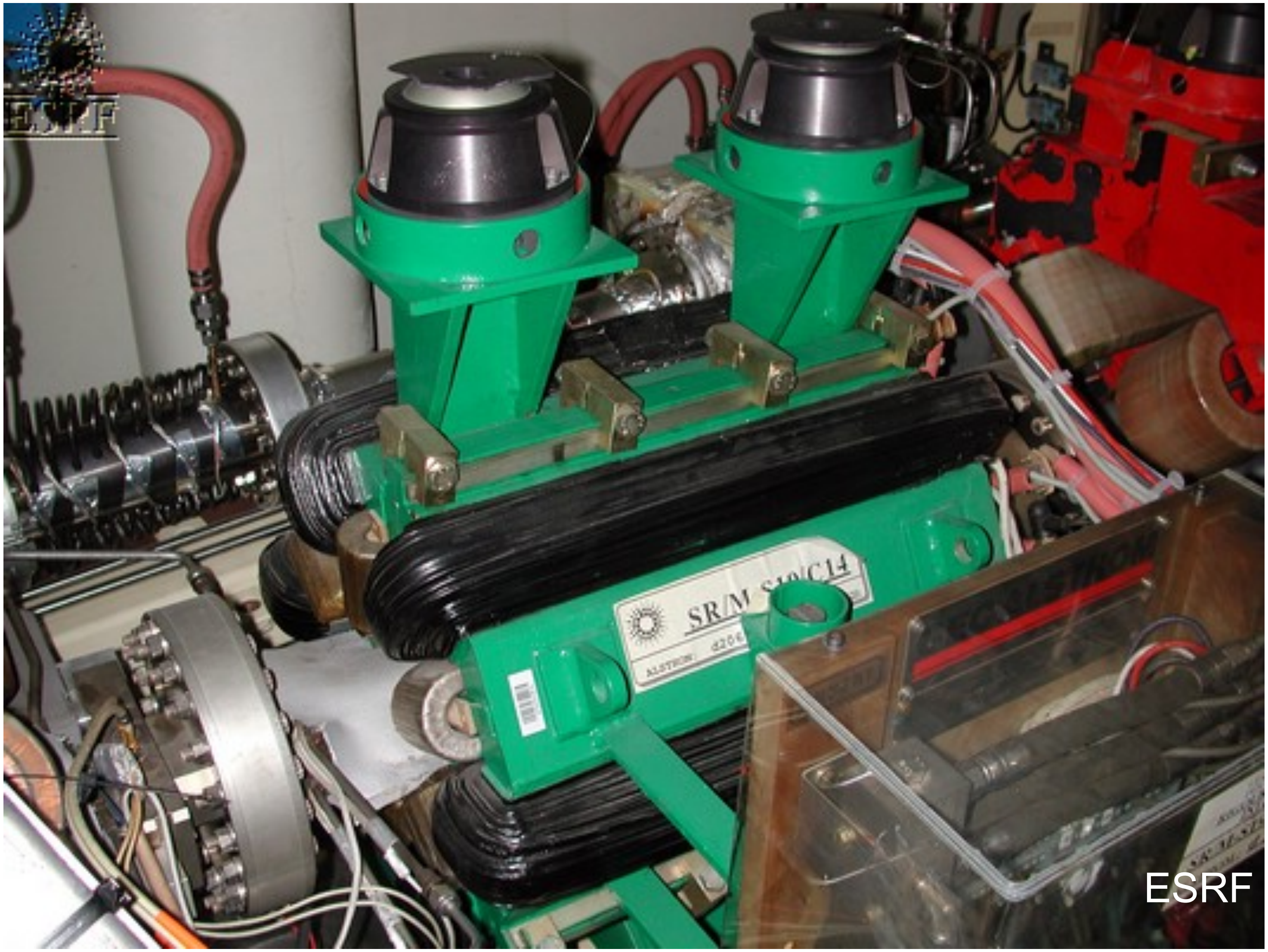
$$B_y\big|_{x=0} = -\Psi_3 3y^2$$

$$nI = \oint \vec{H} \cdot d\vec{s} \approx \int_0^a H_r dr = \Psi_3 \frac{a^3}{\mu_0}$$

Quadrupole strength:

$$k_2 = \frac{q}{p} \partial_x^2 B_y\big|_0 = 6 \frac{q}{p} \Psi_3 = \frac{q\mu_0}{p} \frac{6nI}{a^3}$$

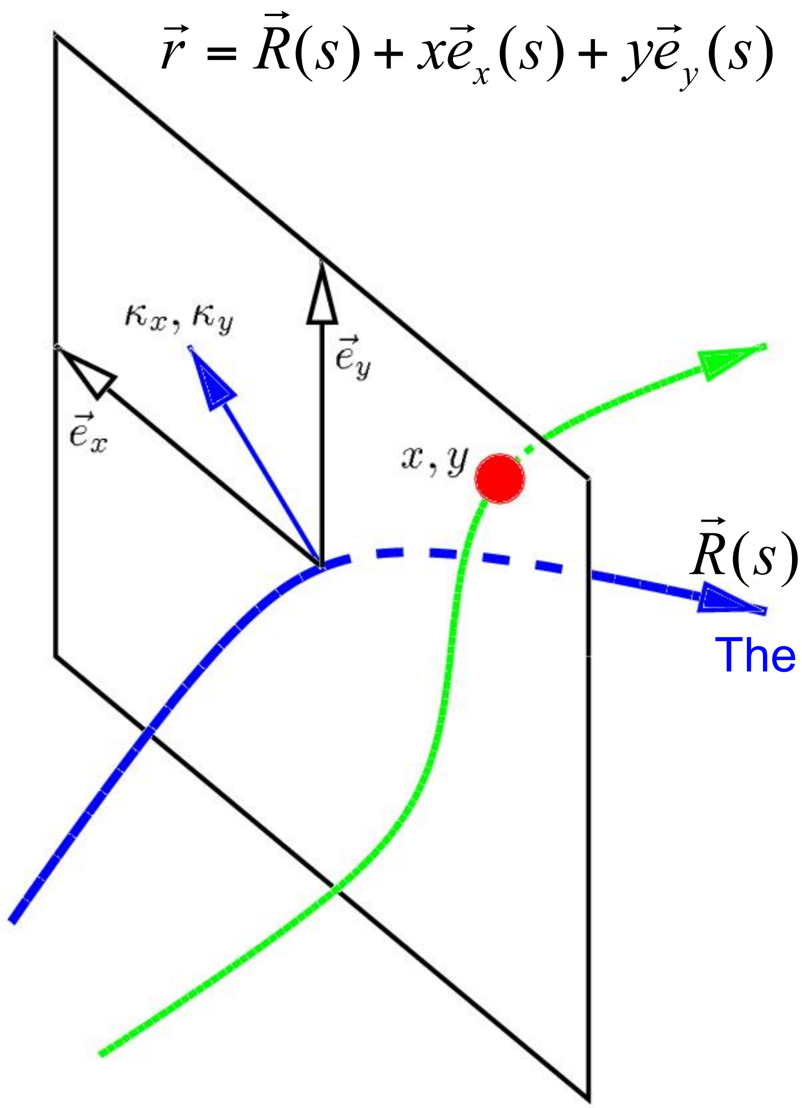
Real Sextupoles



The CESR Tunnel

The comoving Coordinate System

CHESS & LEPP



$$\left|d\vec{R}\right| = ds$$

$$\boxed{\vec{e}_s \equiv \tfrac{d}{ds}\vec{R}(s)}$$

The time dependence of a particle's motion is often not as interesting as the trajectory along the accelerator length "s".

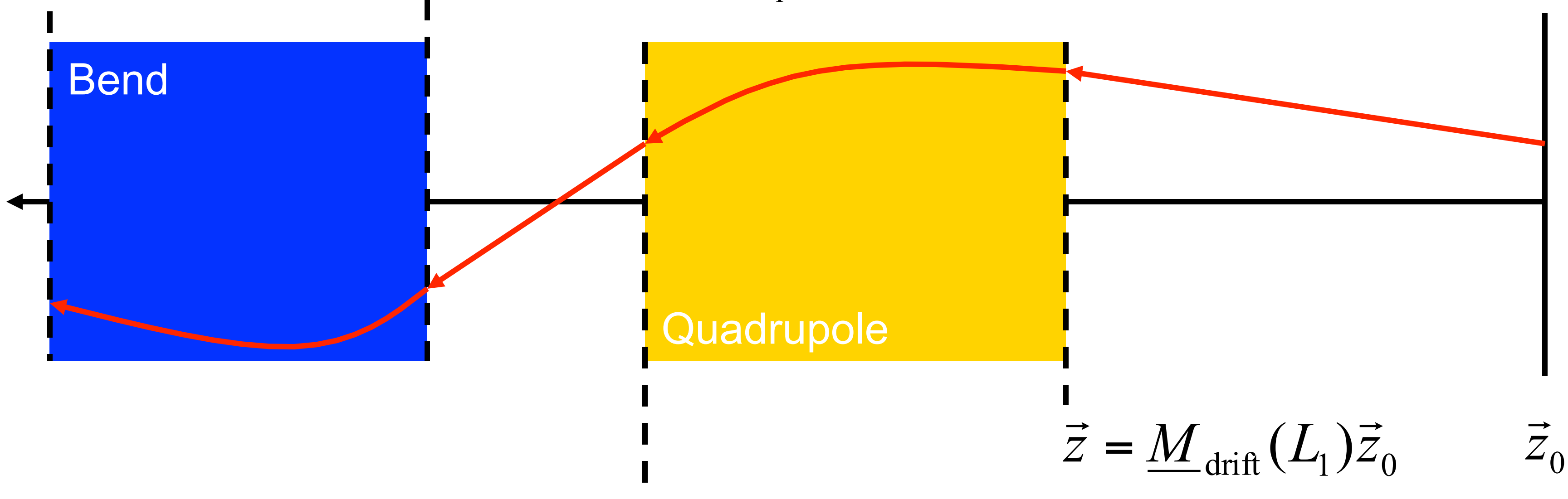
Matrix Solutions

Linear equation of motion: $\vec{z}\,' = \underline{F}(s)\vec{z}$

Matrix solution of the starting condition $\vec{z}(0) = \vec{z}_0$

$\vec{z} = \underline{M}_{\text{bend}}(L_4)\underline{M}_{\text{drift}}(L_3)\underline{M}_{\text{quad}}(L_2)\underline{M}_{\text{drift}}(L_1)\vec{z}_0$

$\vec{z} = \underline{M}_{\text{drift}}(L_3)\underline{M}_{\text{quad}}(L_2)\underline{M}_{\text{drift}}(L_1)\vec{z}_0$



$\vec{z} = \underline{M}_{\text{drift}}(L_1)\vec{z}_0$ $\quad \vec{z}_0$

$\vec{z} = \underline{M}_{\text{quad}}(L_2)\underline{M}_{\text{drift}}(L_1)\vec{z}_0$

The Drift

$$\begin{pmatrix} x' \\ a' \\ y' \\ b' \\ \tau' \\ \delta' \end{pmatrix} = \begin{pmatrix} a \\ 0 \\ b \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\ddot{x} = 0 \quad \Rightarrow \quad x'' = 0 \quad \Rightarrow \quad a = x', a' = 0$$

Linear solution:

$$x(s) = x_0 + x_0' s$$

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \tau \\ \delta \end{pmatrix} = \begin{pmatrix} x_0 + s a_0 \\ a \\ y_0 + s b_0 \\ b_0 \\ \tau_0 \\ \delta_0 \end{pmatrix} = \begin{pmatrix} \begin{matrix} 1 & s \\ 0 & 1 \end{matrix} & \underline{0} & \underline{0} \\ \underline{0} & \begin{matrix} 1 & s \\ 0 & 1 \end{matrix} & \underline{0} \\ \underline{0} & \underline{0} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{pmatrix} \vec{z}_0$$

The 4D Equation of Motion

$$\frac{d^2}{dt^2}\vec{r} = \vec{f}_r(\vec{r}, \tfrac{d}{dt}\vec{r}, t)$$

3 dimensional ODE of 2^{nd} order can be changed to a

6 dimensional ODE of 1^{st} order:

$$\left.\begin{aligned} \frac{d}{dt}\vec{r} &= \frac{1}{m\gamma}\vec{p} = \frac{c}{\sqrt{p^2-(mc)^2}}\vec{p} \\ \frac{d}{dt}\vec{p} &= \vec{F}(\vec{r},\vec{p},t) \end{aligned}\right\} \quad \frac{d}{dt}\vec{Z} = \vec{f}_Z(\vec{Z},t)\,, \quad \vec{Z} = (\vec{r},\vec{p})$$

If the force does not depend on time, as in a typical beam line magnet, the energy is conserved so that one can reduce the dimension to 5. The equation of motion is then autonomous.

Furthermore, the time dependence is often not as interesting as the trajectory along the accelerator length “s”. Using “s” as the independent variable reduces the dimensions to 4. The equation of motion is then no longer autonomous.

$$\frac{d}{ds}\vec{z} = \vec{f}_z(\vec{z},s)\,, \quad \vec{z} = (x,y,p_x,p_y)$$

The 6D Equation of Motion

Usually one prefers to compute the trajectory as a function of "s" along the accelerator even when the energy is not conserved, as when accelerating cavities are in the accelerator.

Then the energy "E" and the time "t" at which a particle arrives at the cavities are important. And the equations become 6 dimensional again:

$$\frac{d}{ds}\vec{z} = \vec{f}_z(\vec{z},s)\,, \quad \vec{z} = (x, y, p_x, p_y, -t, E)$$

But: $\vec{z} = (\vec{r}, \vec{p})$ is an especially suitable variable, since it is a phase space vector so that its equation of motion comes from a Hamiltonian, or by variation principle from a Lagrangian.

$$\delta\int\left[p_x\dot{x} + p_y\dot{y} + p_s\dot{s} - H(\vec{r},\vec{p},t)\right]dt = 0 \quad \Rightarrow \quad \text{Hamiltonian motion}$$

$$\delta\int\left[p_x x' + p_y y' - H\,t' + p_s(x,y,p_x,p_y,t,H)\right]ds = 0 \quad \Rightarrow \quad \text{Hamiltonian motion}$$

The new canonical coordinates are: $\vec{z} = (x, y, p_x, p_y, -t, E) \quad \text{with} \quad \mathrm{E} = \mathrm{H}$

The new Hamiltonian is: $K = -p_s(\vec{z}, s)$