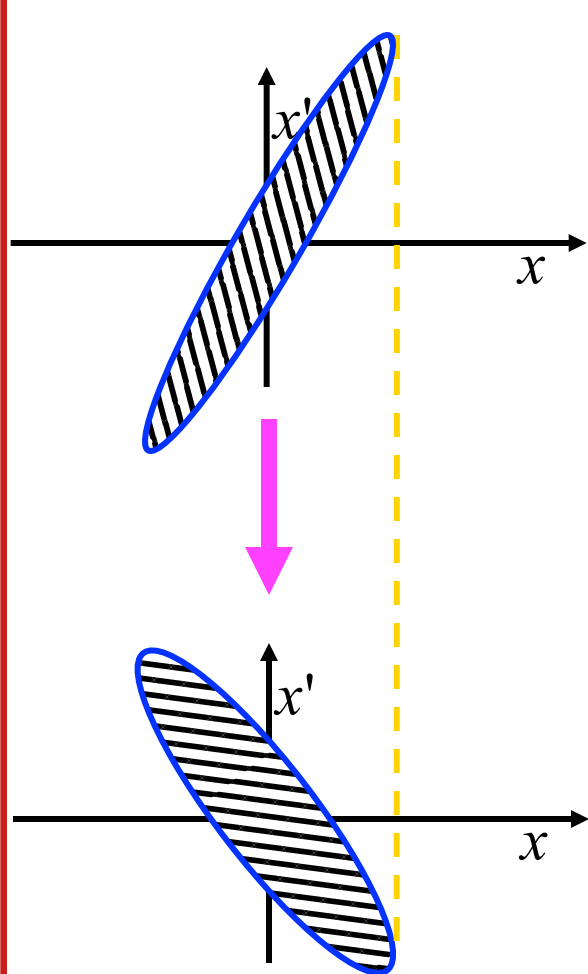




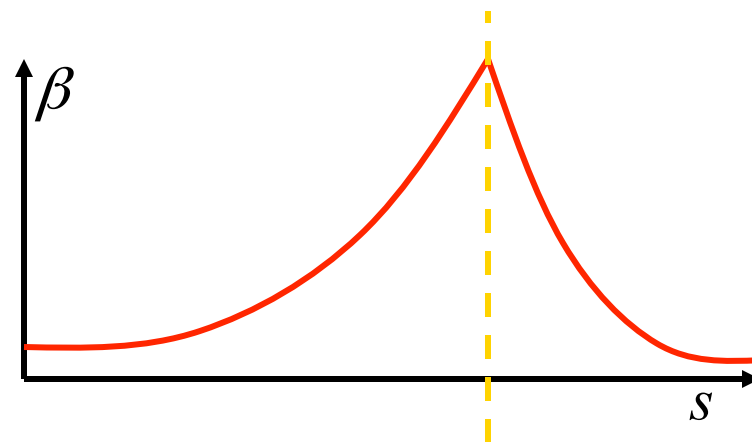
Twiss Parameters after a thin Quadrupole

CHESS & LEPP



$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix} \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} \begin{pmatrix} 1 & -k \\ 0 & 1 \end{pmatrix}$$

$$\alpha = \alpha_0 + k\beta_0$$





From Twiss to Transport Matrix

$$\begin{pmatrix} x_0 \\ x_0' \end{pmatrix} = \sqrt{2J} \begin{pmatrix} \sqrt{\beta_0} & 0 \\ -\frac{\alpha_0}{\sqrt{\beta_0}} & \frac{1}{\sqrt{\beta_0}} \end{pmatrix} \begin{pmatrix} \sin(\phi_0) \\ \cos(\phi_0) \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \sqrt{2J} \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \sin(\psi(s) + \phi_0) \\ \cos(\psi(s) + \phi_0) \end{pmatrix}$$

$$= \sqrt{2J} \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \cos\psi(s) & \sin\psi(s) \\ -\sin\psi(s) & \cos\psi(s) \end{pmatrix} \begin{pmatrix} \sin\phi_0 \\ \cos\phi_0 \end{pmatrix}$$

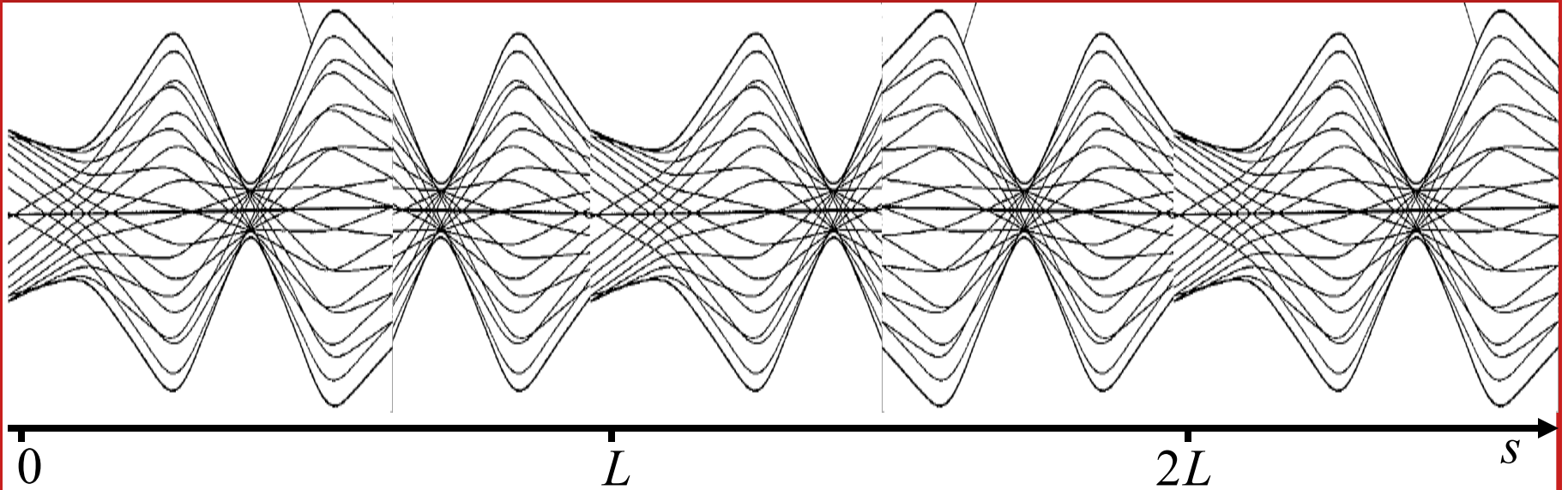
$$\underline{M}(s) = \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \cos\psi(s) & \sin\psi(s) \\ -\sin\psi(s) & \cos\psi(s) \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\beta_0}} & 0 \\ \frac{\alpha_0}{\sqrt{\beta_0}} & \sqrt{\beta_0} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}} [\cos\psi + \alpha_0 \sin\psi] & \sqrt{\beta_0 \beta} \sin\psi \\ \sqrt{\frac{1}{\beta_0 \beta}} [(\alpha_0 - \alpha) \cos\psi - (1 + \alpha_0 \alpha) \sin\psi] & \sqrt{\frac{\beta_0}{\beta}} [\cos\psi - \alpha \sin\psi] \end{pmatrix}$$



Periodic sections in an accelerator

CHESS & LEPP



$$\vec{z}(s) = \underline{M}(s,0)\vec{z}(0)$$

$$\vec{z}(L) = \underline{M}(L,0)\vec{z}(0)$$

$$\vec{z}(s+L) = \underline{M}_0(s)\vec{z}(s) \quad , \quad \underline{M}_0 = \underline{M}(s+L,s)$$

$$\vec{z}(s+nL) = \underline{M}_0^n(s)\vec{z}(s)$$