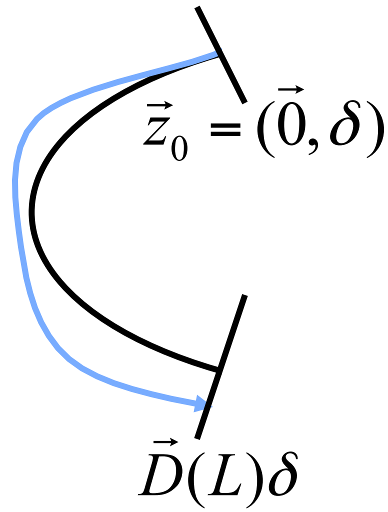
# Dispersion Integral for One-pass Accelerators (1)



$$x' = a$$

$$a' = -(\kappa^2 + k)x + \kappa\delta$$




$$\Delta\kappa = \delta\kappa$$

$$\boxed{D(s) = \sqrt{\beta(s)}\int_0^s \kappa(\hat{s})\sqrt{\beta(\hat{s})}\sin(\psi(s) - \psi(\hat{s}))d\hat{s}}$$

# Variation of Constants (repeat)



$$\vec{z}\,' = \vec{f}(\vec{z},s)$$

$$\vec{z}\,' = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z},s)$$

Field errors, nonlinear fields, etc can lead to $\Delta\vec{f}(\vec{z},s)$

$$\vec{z}\,'_H = \underline{L}(s)\vec{z}_H \quad\Rightarrow\quad \vec{z}_H(s) = \underline{M}(s)\vec{z}_{H0} \quad \text{with} \quad \underline{M}'(s)\vec{a} = \underline{L}(s)\underline{M}(s)\vec{a}$$

$$\vec{z}(s) = \underline{M}(s)\vec{a}(s) \quad\Rightarrow\quad \vec{z}\,'(s) = \underline{M}'(s)\vec{a} + \underline{M}(s)\vec{a}\,'(s) = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z},s)$$

$$\vec{a}(s) = \vec{z}_0 + \int_0^s \underline{M}^{-1}(\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}),\hat{s})\,d\hat{s}$$

$$\vec{z}(s) = \underline{M}(s)\left\{\vec{z}_0 + \int_0^s \underline{M}^{-1}(\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}),\hat{s})\,d\hat{s}\right\}$$

$$= \vec{z}_H(s) + \int_0^s \underline{M}(s-\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}),\hat{s})\,d\hat{s}$$

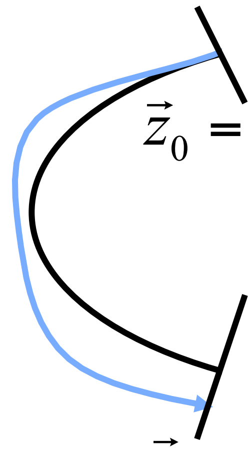
Perturbations are propagated from s to s'

# Dispersion Integral for One-pass Accelerators - 2



$$x' = a$$

$$a' = -(\kappa^2 + k)x + \kappa\delta$$




$$\vec{z} = \underline{M}\vec{z}_0 + \int_0^s \underline{M}(s-\hat{s})\begin{pmatrix} 0 \\ \delta\kappa(\hat{s}) \end{pmatrix} d\hat{s}$$

$$\Rightarrow \quad \vec{D}(s) = \int_0^s \underline{M}(s-\hat{s})\begin{pmatrix} 0 \\ \kappa(\hat{s}) \end{pmatrix} ds'$$

$$\Delta\kappa = \delta\kappa$$

$$\boxed{D(s) = \sqrt{\beta(s)}\int_0^s \kappa(\hat{s})\sqrt{\beta(\hat{s})}\sin(\psi(s) - \psi(\hat{s}))d\hat{s}}$$

# Dispersion correction for One-pass Accelerators



$$x' = a$$

$$a' = -(\kappa^2 + k)x + \kappa\delta$$

$$x(s) = x_0(s) + D(s)\delta$$

$$x_0' = a_o \quad , \quad a_0' = -(\kappa^2 + k)x_0$$

$$D'' = -(\kappa^2 + k)D + \kappa$$

Use quadrupoles for dispersion correction

$$\Delta k \Rightarrow D(s) + \Delta D(s)$$

$$(D + \Delta D)'' = -(\kappa^2 + k)(D + \Delta D) + \kappa - \Delta k \cdot D$$

$$\Delta D'' = -(\kappa^2 + k)\Delta D - \Delta k \cdot D$$

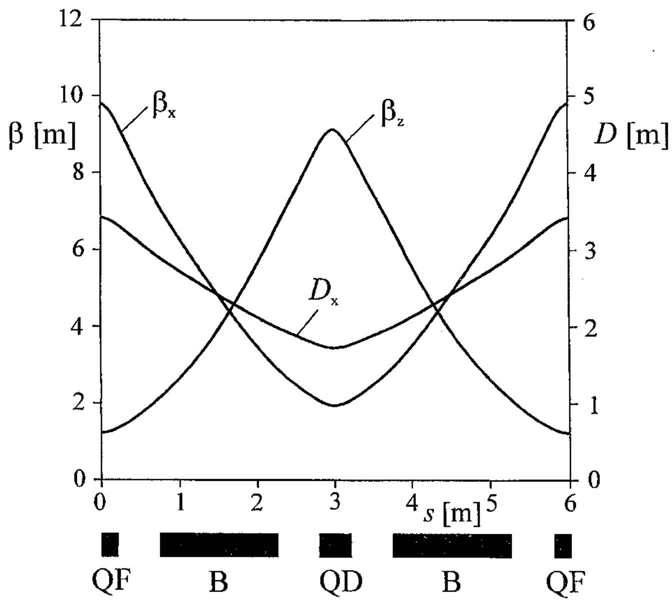
$$\Delta\kappa = \Delta k \cdot D$$

$$\boxed{\Delta D(s) = \sqrt{\beta(s)} \int_0^s \Delta k \cdot D(\hat{s}) \sqrt{\beta(\hat{s})} \sin(\psi(s) - \psi(\hat{s})) d\hat{s}}$$

# The FODO Cell



Alternating gradients allow focusing in both transverse plains. Therefore focusing and defocusing quadrupoles are usually alternated and interleaved with bending magnets.




$$L_{FoDo} \approx 6\text{m}\,, \quad \varphi \approx 22.5^\circ\,, \quad \mu_{FoDo} \approx \tfrac{\pi}{2}$$

$$\overline{\beta} \approx 3.8\text{m}$$

$$\beta_{\max} \approx 10.2\text{m}\,, \quad \beta_{\min} \approx 1.8\text{m}$$

$$\underline{M}_0 = \underline{M}_{FoDo}^N$$

The periodic beta function and dispersion for each FODO is also periodic for an accelerator section that consists of many FODO cells. Often large sections of an accelerator consist of FODOs.

# The Low Beta Insertion