USPAS summer 2023, Grad Accelerator Physics

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Homework #3

Midplane symmetry of Hamiltonians: What property does the Hamilton function $H(x, a, y, b, \tau, \delta, s)$ have when the motion is mid-plane symmetric?

Time, energy, and symplecticity: Let the linearized particle transport from initial phase space coordinates $\vec{z}_i$ to final phase space coordinates $\vec{z}_f$ be:

$$\begin{pmatrix} x_f \\ a_f \\ \tau_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & 0 & D_x \\ M_{21} & M_{22} & 0 & D_a \\ T_x & T_a & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ a_i \\ \tau_i \\ \delta_i \end{pmatrix} . \tag{1}$$

The zeros in the matrix show that the particle motion is independent of the starting time and that the energy is independent of the starting conditions.
(a) Describe the meaning of the coefficients D_x, D_a, T_x, and T_a.
(b) Use the fact that this 4×4 matrix is symmplectic to show that the top left-hand 2×2 sub-matrix is symplectic.
(c) Show how T_x and T_a can be computed when this top left-hand sub-matrix and the dispersion D_x and its slope D_a are known.

Time of flight spectrometer: A time of flight spectrometer takes all particles that come from a collision point regardless of their initial slopes x' and y' and transports them to a point in a detector plane. The time of flight should depend only on the energy, not on the initial position or the initial angle of the particles in the collision plane. Write the most general form that the transport matrix from the collision plane to the detector plane can have.

Path length and momentum coordinates: If not the time of flight $\tau = (t_0 - t)\frac{E_0}{P_0}$ and the relative energy change $\delta = \frac{\Delta E}{E}$ had been chosen as phase space variables, but the deviation in path length Δl and the relative momentum deviation $\frac{\Delta P}{P}$, how would the transport matrix look like and how could it be computed from the transport matrix in equation 1?

Solenoid matrix: Determine the transport matrix for a solenoid in the sharp cutoff limit. It has the length L, and the longitudinal field strength on axis is B_z for $x \in [0, L]$ and 0 outside this region. As solenoid strength you can use the parameter $g = \frac{qB_z}{2p}$.

Phase space distribution: **(a)** Given the Twiss parameters α, β, γ: specify the transformation from the amplitude and phase variables J and Ψ to the Cartesian phase space variables x and x'.
(b) Specify the inverse transformation.
(c) Given the Gaussian beam distribution in amplitude and phase variables, $\rho(J, \phi) = \frac{1}{2\pi\epsilon} e^{-\frac{J}{\epsilon}}$. What is the projection $\rho(x)$ of this distribution on the x axis. Check that the rms width of this distribution leads to $\sqrt{< x^2 >} = \sqrt{\beta\epsilon}$.

Periodic Twiss parameters: **a)** Use the transport matrix from s_0 to s written in terms of Twiss parameters at s_0 and s to show that the one turn matrix of a ring at s can be written as

$$\underline{M} = \underline{1} \cos\mu + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \sin\mu \tag{2}$$

when α, β, and γ are the Twiss parameters that are periodic with the length L of the ring and $\mu = \Psi(L) - \Psi(0)$ is the one turn phase advance.
b) Show that the matrix before $\sin\mu$ in this equation has a characteristic of the complex i in that squaring it leads to $-\underline{1}$.
c) Use this to compute $\underline{M}^n$.

One turn matrix: If the one turn matrix

$$\underline{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{3}$$

is known, specify how the periodic Twiss parameters and the one turn phase advance can be computed. Under what conditions is the one turn phase advance real? What does this mean for the long term motion in phase space which is described by $\underline{M}^n \vec{z}_0$ for large n.

Propagation of Twiss parameters: Characterize Twiss parameters by $\{\beta(s), \alpha(s), \psi(s)\}$. Imagine two sections of a beam line where the first section transports Twiss parameters $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_1, \alpha_1, \psi_1\}$ and the second transports $\{\beta_1, \alpha_1, 0\}$ to $\{\beta_2, \alpha_2, \psi_2\}$. Show that the total beam-line transports $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_2, \alpha_2, \psi_1 + \psi_2\}$.

1 Lattice Design #3

Dispersion creator: Construct a reverse dispersion creator that uses 2 reverse arc FoDo's with 1/2 strength dipoles to match zero dispersion to the periodic dispersion of a reverse arc cell. Do you need to fit quadrupole strengths, or can you use some of the quadrupole strength you computed yesterday?

Straight section to dispersion creator matching: Match a straight FoDo to the reversed dispersion creator. Do you need to fit quadrupole strengths, or can you use some of the quadrupole strength you computed yesterday?

Forward sextant: Assemble a reverse sextant from four periodic straight FoDos, matching to a reverse dispersion creator, through the reverse dispersion creator, through 20 reverse arc FoDos, through a reverse dispersion suppressor, matching to the straight FoDo, followed by 4 straight FoDos. Plot betas and dispersion along the sextant when starting the line with the periodic Twiss functions of the straight FODO. How do the beta function along the sextant change when you specify the geometry as closed?