USPAS summer 2023, Grad Accelerator Physics

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Homework #4

Exercise (FODO Twiss parameters)
(a) Find the Twiss parameters of the phase space ellipse that is periodic for a FODO Cell in thin lens approximation, i.e. all particles that enter a FODO cell on this phase space ellipse exit the cell on the same ellipse. Let the focusing and defocusing quadrupole have the strength k and $-k$. Furthermore, let the cell start with half a focusing quadrupole, and let the distance between quadrupoles be $L/2$ so that the transport matrix of the cell is given by

$$\underline{M} = \underline{Q}(\frac{k}{2})\underline{D}(\frac{L}{2})\underline{Q}(-k)\underline{D}(\frac{L}{2})\underline{Q}(\frac{k}{2}) \ . \tag{1}$$

(b) Characterize how this periodic phase space ellipse changes along the FODO cell by drawing ellipses in phase space at various points along the cell. Do this for the horizontal and the vertical plane separately.
(c) Compute the periodic dispersion (η, η') for this FODO cell, assuming that there is a thin lens dipole with bending angle ϕ in the center between both quadrupoles.
(d) For what betatron phase advance (in degree) along the FODO is the maximum beta function in the FODO the smallest?
(e) Let the FODO have quadrupoles of 1m length, 5cm bore radius, and a pole tip field of 1T for protons of 40GeV. How long does the FODO have to be to obtain 60° phase advance?

Exercise (Symmetric and Asymmetric 4Bumps)
Given a FODO lattice which has the periodic Twiss parameters $\beta_x = \beta_y = 10m$, $\alpha_x = \alpha_y = 0$ at its exit.
(a) If you want to construct a symmetric arrangement of six quadrupoles to design an interaction region with a horizontal beta function of 0.5m and a vertical beta function of 0.05m in its center. How would the transport matrix from the FODO to the interaction point have to lock like?
(b) Why are six quadrupoles with at fixed locations not sufficient to adjust the two beta functions?
(c) Assume there is also a symmetric arrangement of four horizontal corrector

coils and that the Twiss parameters at their places are known. Specify the relative strength of these coils so that a closed bump is created that only changes the orbit position at the low beta point, but not orbit angle.
(d) Specify the relative strength of these coils so that a closed bump is created that only changes the orbit slope at the low beta point, but not the orbit position.

Exercise (Beta Measurement)
Assume you are able to change the strength of all optical elements in a storage ring.
(a) How can you measure β_x and β_y in a quadrupole.
(b) How can you measure the beam position in a sextupole?
(c) Assuming there is a BPM just next to a horizontal correction dipole. How can you measure the horizontal beta function at that position?

Exercise (Beta Beat for Quad Errors in a Ring)
The first quadrupole after an interaction region typically has a very large vertical beta function.
(a) Use thin lens approximation to find β_y in this quadrupole for $\beta_y^* = 1$cm and a distance of 1.5m from quadrupole to interaction point.
(b) If this quadrupole has a focal strength of 0.5m^{-1} and its current changes accidentally by 1%. How many percent of beta-beat ($\Delta\beta/\beta$) would you expect for a tune of $\nu = 0.52$?
(c) What tune shift would you expect? Note that you need to derive the beta beat for a periodic system, because in class the beta beat was derived for a one-pass system.

1 Lattice Design #3

Interaction region: In class you inserted half an interaction region into your lattice. Insert the other half of the interaction region and optimize the betas in the center to $\beta_x^* = 0.6m$ and $\beta_y^* = 0.06m$.

Full Ring: Form the full ring of all 6 sextant, with the full interaction region. Does the ring close? How long is your ring?