USPAS summer 2025, Grad Accelerator Physics

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Homework #3

Exercise 1 (Variation of constants)
Find the time evolution of the driven harmonic oscillator that satisfies the differential equation

$$\frac{d^2}{dt^2}x = -\omega^2 x + \epsilon \cos(\omega_0 t) \tag{1}$$

with the variation of constants method covered in class. What is the matrix $\underline{L}$, what is the perturbation vector $\Delta\vec{f}$, what is the transfer matrix $\vec{M}$, what is the solution $x(t)$ for initial condition x_0 and v_0, where $v = dx/dt$.

Exercise 2 (FODO Twiss parameters)
(a) Find the Twiss parameters of the phase space ellipse that is periodic for a FODO Cell in thin lens approximation, i.e. all particles that enter a FODO cell on this phase space ellipse exit the cell on the same ellipse. The thin lens matrices of drift and quadrupole are

$$\underline{Q}(k) = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix} , \quad \underline{D}(L) = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} .$$

The thin lense matrix for the bend of length L and angle ϕ is $\underline{B}(\phi, L)$, and it creates the disperson $\vec{D}_B$ in the center of the bend with

$$\underline{B}(\phi, L) = \begin{pmatrix} 1 & 0 \\ -\phi^2/L & 1 \end{pmatrix} , \quad \vec{D}_B = \begin{pmatrix} 0 \\ \phi \end{pmatrix} .$$

Neglect the weak focusing term ϕ^2/L of the bend, because it is usually much weaker than the quadrupole focusing. Let the focusing and defocusing quadrupole have the strength k and $-k$. Furthermore, let the cell start with half a focusing quadrupole, and let the distance between quadrupoles be $L/2$ so that the transport matrix of the cell is given by

$$\underline{M} = \underline{Q}(\frac{k}{2})\underline{D}(\frac{L}{2})\underline{Q}(-k)\underline{D}(\frac{L}{2})\underline{Q}(\frac{k}{2}) . \tag{2}$$

(b) Characterize how this periodic phase space ellipse changes along the FODO cell by drawing ellipses in phase space at various points along the cell. Do this for the horizontal and the vertical plane separately.
(c) Compute the periodic dispersion (η, η') for this FODO cell, when the length L between the qudrupoles is occupied by a bend. Use the thin lens approximation for matrix and disperson of this bend.
(d) For what betatron phase advance (in degree) along the FODO is the maximum beta function in the FODO the smallest?

Exercise 3 (Symmetric and Asymmetric 4Bumps)
Given a FODO lattice which has the periodic Twiss parameters $\beta_x = \beta_y = 10m$, $\alpha_x = \alpha_y = 0$ at its exit.
(a) If you want to construct a symmetric arrangement of six quadrupoles to design an interaction region with a horizontal beta function of 0.5m and a vertical beta function of 0.05m in its center. How would the transport matrix from the FODO to the interaction point have to lock like?
(b) Why are six quadrupoles with at fixed locations not sufficient to adjust the two beta functions?
(c) Assume there is also a symmetric arrangement of four horizontal corrector coils and that the Twiss parameters at their places are known. Specify the relative strength of these coils so that a closed bump is created that only changes the orbit position at the low beta point, but not orbit angle.
(d) Specify the relative strength of these coils so that a closed bump is created that only changes the orbit slope at the low beta point, but not the orbit position.

1 Lattice Design #3

Report your results for the following exercises of the Ring Design Tutorial exercises section 2.3 number 1, 2, 5, and 6.

1 - Reverse dispersion suppressor: Construct the reverse dispersion suppressor, optimizing the last two quadrupole strengths for $\eta_z = 0$ and $\eta'_x = 0$ at the end for FoDo cells of 90∘ phase advance. How do the two quadrupole values for the reverse dispersion suppressor compare to those obtained for the forward suppressor?

2 - Forward and Reversed Cells: Check that your forward and reverse cells that both start with focusing quads have different periodic beta and alpha functions. Check also that both cell, for the same phase advance of 90 degrees have exactly the same quadrupole strengths. Explain how this can be the correct solution.

3 - Strength of bends: to a good approximation: A dispersion suppressor can be constructed using two arc FODO cells with the first cell having the bend strengths reduced by a factor α and the second cell with the bend strengths reduced by a factor $1-\alpha$. In the case of a 90° FoDo cells, we showed in class that $\alpha = 0.5$, and for a 60° FoDo we found $\alpha = 0$. Find the suitable α for a 72° FoDo. Either determine α analytically or by matching the FoDo in your ring design to 72° phase advance and then finding the α that leads to dispersion suppression.