USPAS summer 2025, Grad Accelerator Physics

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Homework #4

Exercise (Complex potentials) When the coordinates $w = x + iy$ and $\bar{w} = x - iy$ are used, the Laplace operator has been derived to be $\vec{\nabla}^2 = 4\partial_w \partial_{\bar{w}} + \partial_z^2$.
(a) Check that this is correct.
(b) The static magnetic field in a charge free space is given by $\vec{B} = -\vec{\nabla}\psi$. Writing the magnetic field in x and y direction in complex notation as $B = B_x + iB_y$, derive a formula that expresses B and B_z in terms of $\Psi(w, \bar{w}, z)$ and only ∂_w, $\partial_{\bar{w}}$, and ∂_z.
(c) Given the vector potential in complex notation as $A = A_x + iA_y$ and A_z, derive a formula that expresses B and B_z given by $\vec{B} = \nabla \times \vec{A}$, again only using ∂_w, $\partial_{\bar{w}}$, and ∂_z and A, A_z.

Exercise (Rotational field symmetries)

1. The field in a bending magnet has usually two symmetries: Midplane symmetry since the upper and lower part of the magnet are built identically, and a mirror symmetry with respect to the vertical plane, since each pole is build with right/left symmetry when viewed along the beam pipe. Which multipoles, in addition to the main dipole component, satisfy this symmetry and can therefore be associated with such a bending magnet.

2. Similarly, a focusing magnet has C_2 and midplane symmetry. Which multipoles, in addition to the main quadrupole term, satisfy this symmetry and can therefore appear when such a magnet is built.

3. Generalize your observation to a magnet which is built with exact C_n symmetry and midplane symmetry. Which multipole terms can the field have?

Exercise (Solenoid) Consider a box shape solenoid field. On the central axis, the solenoid field is given by

$$\vec{B}(z) = \begin{cases} B_0 \vec{e}_z & \text{for } z \in [0, L] \\ 0 & \text{else} \end{cases} \tag{1}$$

(a) A particle flies into the solenoid parallel to the central axis with a horizontal distance x_0. Describe its trajectory after the solenoid.
(b) If it touches the central axis somewhere after the solenoid, where would that be? How does the focal length depend on the field B_0 and the length L?
(c) Show that the magnetic field $\vec{B} = \{\frac{x}{2}\Psi_0'' , \frac{y}{2}\Psi_0'' , -\Psi_0'\}$ can be derived from the vector potential $\vec{A} = \{\frac{y}{2}\Psi_0' , -\frac{x}{2}\Psi_0' , 0\}$, where Ψ_0 is a function of z.
(d) Show that during this motion, the particle's angular momentum around the z-axis is not conserved. Also show that the z component of its canonical angular momentum $L_z = \left\{\vec{r} \times (\vec{p} + e\vec{A})\right\}_z$ is conserved. To do this, you can show that $\frac{dL_z}{dt} = 0$.
(e) Given a proton beam of $E_k = 5\text{keV}$, how many turns of a 100A current is approximately needed for a 10cm coil to have a 1 meter focal length.
(f) Show that within the solenoid the particles perform helical motion of radius $\frac{x_0}{2}$ around the axis $x = \frac{x_0}{2}$.

Exercise (Multipoles)

1. Describe the magnetic field and the magnetic scalar potential in a duodecapole ?

2. How strong is a duodecapole for which the distance from the central axis to the iron pole is given by a and around each pole is a winding of n wires each having a current I ?

3. Show what fields are created when a n pole is shifted by a distance Δ in the transverse direction. For example, show that a shifted sextupole has a quadrupole field.

1 Lattice Design #3

Report your results for the following exercises of the Ring Design Tutorial exercises section 2.3 number 1, 2, 5, and 6.

1 - Reverse dispersion suppressor: Construct the reverse dispersion suppressor, optimizing the last two quadrupole strengths for $\eta_z = 0$ and $\eta_x' = 0$ at the end for FoDo cells of 90∘ phase advance. How do the two quadrupole values for the reverse dispersion suppressor compare to those obtained for the forward suppressor?

2 - Forward and Reversed Cells: Check that your forward and reverse cells that both start with focusing quads have different periodic beta and alpha functions. Check also that both cell, for the same phase advance of 90 degrees have exactly the same quadrupole strengths. Explain how this can be the correct solution.

3 - Strength of bends: to a good approximation: A dispersion suppressor can be constructed using two arc FODO cells with the first cell having the bend strengths reduced by a factor α and the second cell with the bend strengths reduced by a factor $1-\alpha$. In the case of a 90° FoDo cells, we showed in class that $\alpha = 0.5$, and for a 60° FoDo we found $\alpha = 0$. Find the suitable α for a 72° FoDo. Either determine α analytically or by matching the FoDo in your ring design to 72° phase advance and then finding the α that leads to dispersion suppression.