USPAS summer 2025, Grad Accelerator Physics

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Homework #5

1 - Inverse of symplectic matrices: Derive a formula for the inverse of a symplectic matrix, i.e. one that satisfied $\underline{M}J\underline{M}^T = J$. And prove that for all symplectic matrices, $\underline{M}^T$ is also symplectic.

2 - Time, energy, and symplecticity: Let the linearized particle transport from initial phase space coordinates $\vec{z}_i$ to final phase space coordinates $\vec{z}_f$ be:

$$\begin{pmatrix} x_f \\ a_f \\ \tau_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & 0 & D_x \\ M_{21} & M_{22} & 0 & D_a \\ T_x & T_a & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ a_i \\ \tau_i \\ \delta_i \end{pmatrix} . \tag{1}$$

The zeros in the matrix show that the particle motion is independent of the starting time and that the energy is independent of the starting conditions.
(a) Describe the meaning of the coefficients D_x, D_a, T_x, and T_a.
(b) Use the fact that this 4×4 matrix is symmplectic to show that the top left-hand 2×2 sub-matrix is symplectic.
(c) Show how T_x and T_a can be computed when this top left-hand sub-matrix and the dispersion D_x and its slope D_a are known.

3 - Time of flight spectrometer: A time of flight spectrometer takes all particles that come from a collision point regardless of their initial slopes x' and y' and transports them to a point in a detector plane by a time-independent magnetic field. The time of flight should depend only on the energy, not on the initial position or the initial angle of the particles in the collision plane. Write the most general form that the transport matrix from the collision plane to the detector plane can have, assuming midplane symmetry.

4 - Path length change by corrector magnets: A corrector magnet is located in a ring where the one-turn matrix is $\underline{M}$ and the dispersion is $\underline{D}$. How much does the length of the periodic orbit change when the deflection in this corrector magnet is changed by an angle θ. Show this the change in τ is $\Delta\tau = \theta\eta_x$, with the periodic disperion at the location of the corrector.

5 - Path length and momentum coordinates: If not the time of flight $\tau = (t_0 - t)\frac{E_0}{P_0}$ and the relative energy change $\delta = \frac{\Delta E}{E}$ had been chosen as phase space variables, but the deviation in path length Δl and the relative momentum deviation $\frac{\Delta P}{P}$, how would the transport matrix look like and how could it be computed from the transport matrix in equation 1?

6 - Combine function magnet: Derive that the linear transport matrix for a combined function bend of curvature $\kappa = 1/\rho$ and focusing strength k has the form

$$\underline{M} = \begin{pmatrix} \underline{M}_x & \underline{0} & \vec{0} & \vec{D} \\ \underline{0} & \underline{M}_y & \vec{0} & \vec{0} \\ \vec{T}^T & \vec{0}^T & 1 & M_{56} \\ \vec{0}^T & \vec{0}^T & 0 & 1 \end{pmatrix}$$

with

$$\underline{M}_x = \begin{pmatrix} \cos(\sqrt{K}s) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}s) \\ -\sqrt{K}\sin(\sqrt{K}) & \cos\sqrt{K}s) \end{pmatrix} ,$$

$$\underline{M}_y = \begin{pmatrix} \cosh(\sqrt{k}s) & \frac{1}{\sqrt{k}}\sinh(\sqrt{k}s) \\ \sqrt{k}\sinh(\sqrt{k}s) & \cosh\sqrt{k}s) \end{pmatrix} ,$$

$$\vec{D} = \begin{pmatrix} \frac{\kappa}{\beta^2 K}[1-\cos(\sqrt{K}s)] \\ \frac{\kappa}{\beta^2\sqrt{K}}\sin(\sqrt{K}s) \end{pmatrix} , \quad \vec{T} = \begin{pmatrix} -\frac{\kappa}{\beta^2\sqrt{K}}\sin(\sqrt{K}s) \\ \frac{\kappa}{\beta^2 K}[\cos(\sqrt{K}s) - 1] \end{pmatrix} ,$$

$$M_{56} = \frac{\kappa^2}{\beta^2\sqrt{K}^3}[\sin(\sqrt{K}) - \sqrt{K}s] .$$

Verify that this matrix satisfies the sympathetic condition.