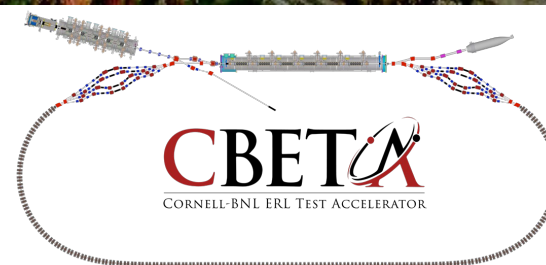
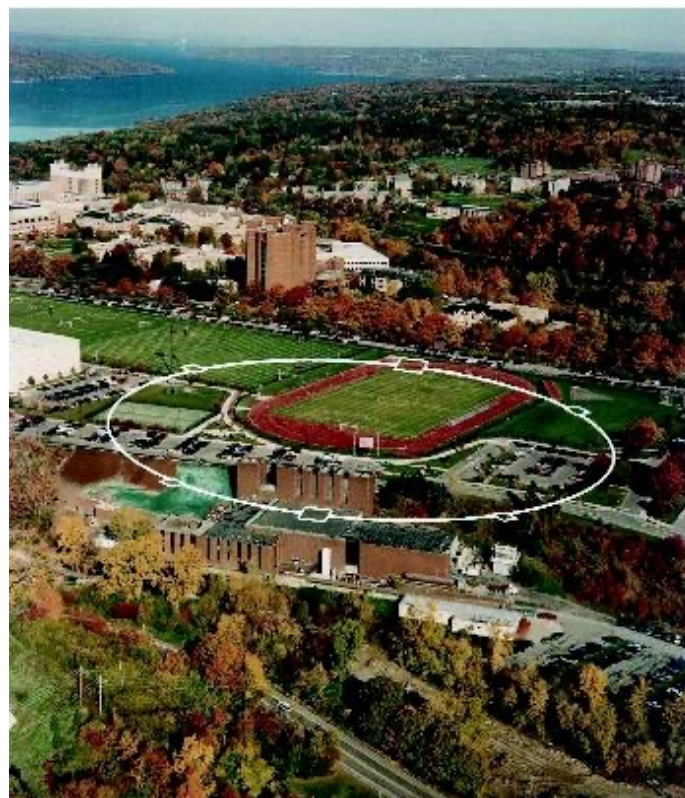


USPAS graduate Accelerator Physics

Content

1. Typical Particle Accelerators
2. Typical Accelerator Components
3. Linear Beam Optics (Circular & Straight)
4. Beam distributions
5. Nonlinear Beam Optics
6. Differential Algebra for Nonlinear Maps
7. Longitudinal Phase Space
8. Synchrotron Radiation
9. Polarization
10. Waveguides and Cavities

Accompanied by homework question and applied accelerator-optics design.



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Class data

Dates:

Monday – Friday June 5 – 9, 2023, 9 - 12 and 2 - 5pm, HW after 7:30pm

Monday – Thursday June 12 – 15, 2023, 9 - 12 and 2 - 5pm, HW after 7:30pm

Friday: summary and final exam, 9am - noon

Expectation:

Collaborate on homework.

Consult TAs for homework and optics design.

Co-instructor for Bmad, design, and simulation: David Sagan dc16@cornell.edu

TAs: Joe Develin jpd285@cornell.edu and Matthew Signorelli mgs255@cornell.edu

Lecture notes, homework, and homework solution will be on the class web page

<https://www.classe.cornell.edu/~hoff/LECTURES/25USPAS/>



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Accelerator optics and simulations

To practice what will be learned, accelerators will be simulated with the programs Tao and Bmad.

You all have accounts to run Bmad, which will be introduced Monday afternoon.

There will be Homework of analytical nature and design / optimization / simulation assignments.



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Images are taken from many sources, including:

The Physics of Particle Accelerators, Klaus Wille, Oxford University Press, 2000, ISBN: 19 850549 3

Particle Accelerator Physics I, Helmut Wiedemann, Springer, 2nd edition, 1999, ISBN 3 540 64671 x

Teilchenbeschleuniger und Ionenoptik, Frank Hinterberger, 1997, Springer, ISBN 3 540 61238 6

Introduction to Ultraviolet and X-ray Free-Electron Lasers, Martin Dohlus, Peter Schmüser, Jörg Rossbach, Springer, 2008

Various web pages, 2003 – 2023



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Literature

Recommended as

Introduction

The Physics of Particle Accelerators: An Introduction, Klaus Wille, Oxford University Press

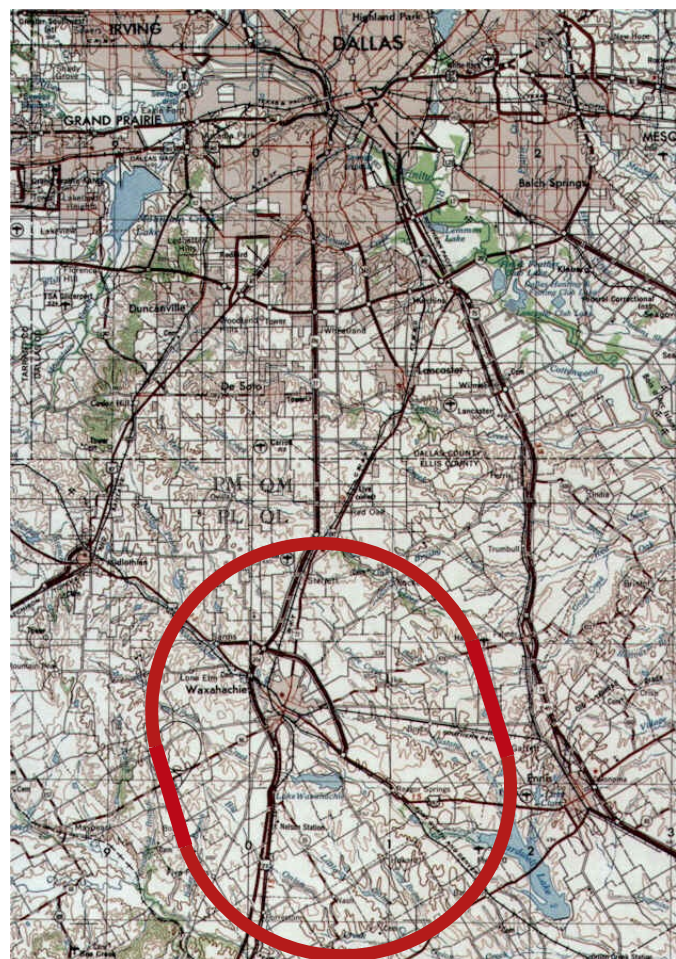
Wide selection of well explained topics

Particle Accelerator Physics, Helmut Wiedemann, Springer, (preferably 3rd edition)

Tremendous overview, with references for derivations and explanations

Handbook of Accelerator Physics and Engineering, Alexander Wu Cao, Maury Tigner, Hans Weise, Frank Zimmermann (3rd edition)

Accelerator techniques through modern history



DALLAS

Superconducting Super Collider

SSC (RIP 1993)



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What is accelerator physics ?

Accelerator Physics has applications in particle accelerators for high energy physics or for x-ray science, in spectrometers, in electron microscopes, and in lithographic devices. These instruments have become so complex that an empirical approach to properties of the particle beams is by no means sufficient and a detailed theoretical understanding is necessary. This course will introduce into theoretical aspects of charged particle beams and into the technology used for their acceleration.

- Physics of beams
- Physics of non-neutral plasmas
- Physics of involved in the technology:
 - Superconductivity in magnets and radiofrequency (RF) devices
 - Surface physics in particle sources, vacuum technology, RF devices
 - Material science in collimators, beam dumps, superconducting materials



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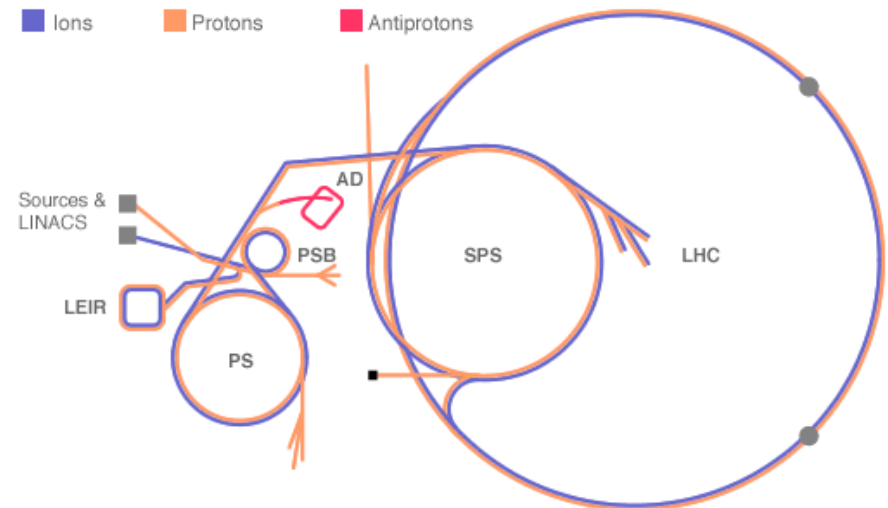
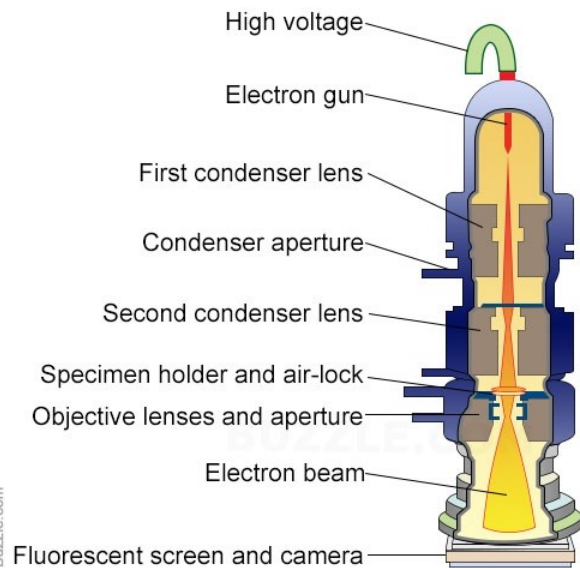


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Particle accelerators, large and small



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Why accelerator physics ?

- Industry

- Food & product safety
- Contraband detection
- Semiconductor fabrication
- Bridge safety

- Medicine

- Tumor detection and treatment.

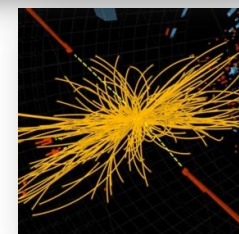
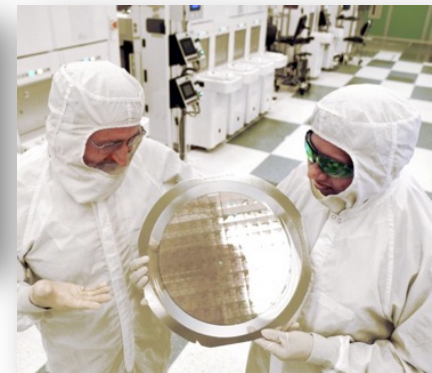
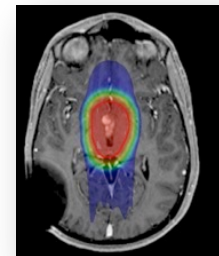
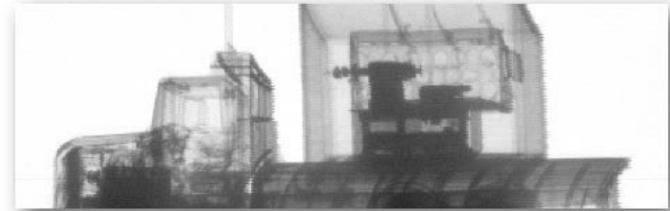
~30,000 industrial and medical accelerators are in use, with annual sales of \$3.5 B and 10% growth per year.

- Research

- X ray sources and colliders for nuclear & particle physics
- Electron microscopes

Since 1943, a Nobel Prize in **Physics** has been awarded to research benefiting from accelerators every 3 years.

Since 1997, the same has been true of **Chemistry**.



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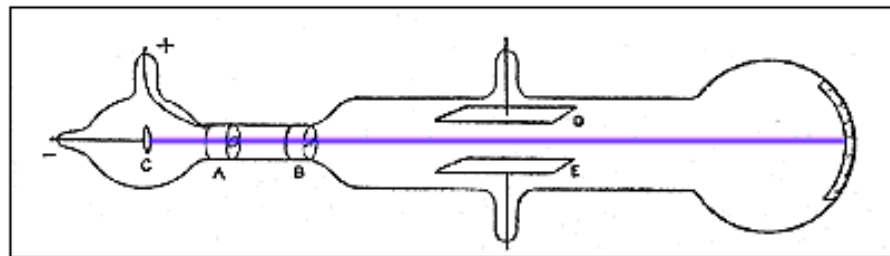
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A short history of accelerators

- 1862: Maxwell theory of electromagnetism
- 1887: Hertz discovery of the electromagnetic wave
- 1886: Goldstein discovers positively charged rays (ion beams)
- 1894: Lenard extracts cathode rays (with a 2.65μm Al Lenard window)
- 1897: JJ Thomson shows that cathode rays are particles since they followed the classical Lorentz force $m\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})$ in an electromagnetic field
- 1926: GP Thomson shows that the electron is a wave
(1929-1930 in Cornell, NP in 1937)



NP 1905, Philipp E.A. von Lenard



NP 1906

Joseph J. Thomson



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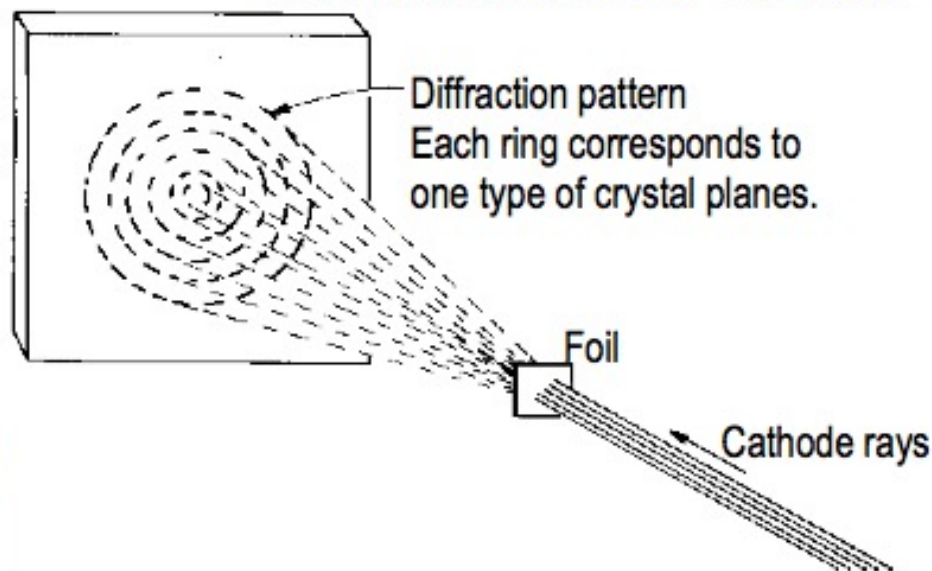
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Discoveries with accelerated beams ...



In a powdered, microcrystalline substance there is always some crystal which has the correct angle for constructive interference $2d \cos \alpha = n\lambda$



George P. Thomson
(1892-1975)
1937 Nobel prize
Son of Joseph J. T.

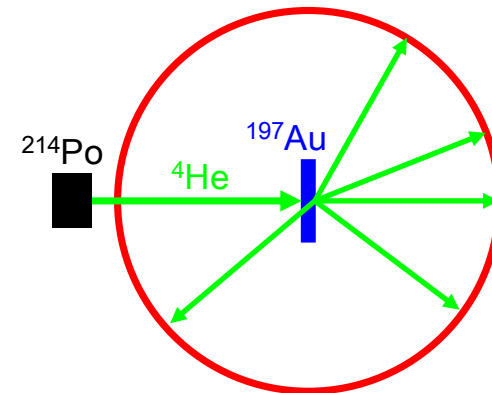
A magnetic field can change the rings, showing the the waves are associated with the electron charge.



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The need for higher energies

- 1911: Rutherford discovers the nucleus with 7.7 MeV ^4He from ^{214}Po alpha decay measuring the elastic cross-section of $^{197}\text{Au} + ^4\text{He} \mapsto ^{197}\text{Au} + ^4\text{He}$.



- 1919: Rutherford produces first nuclear reactions with natural ^4He
 $^{14}\text{N} + ^4\text{He} \mapsto ^{17}\text{O} + \text{p}$
- 1921: Greinacher invents the cascade generator for several 100 keV
- Rutherford is convinced that several 10 MeV are in general needed for nuclear reactions.

1927 E. Rutherford to the Royal Society :“... if it were possible in the laboratory to have a supply of electrons and atoms of matter in general, of which the individual energy of motion is greater even than that of the alfa particle, this would open up an extraordinary new field of investigation....”



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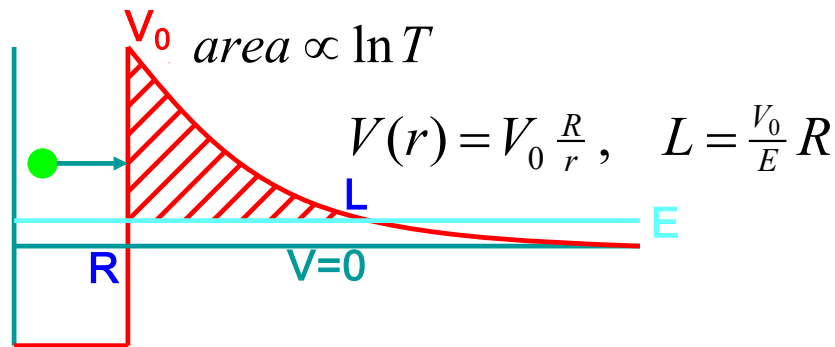
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Tunneling allows low energies

- 1928: Explanation of alpha decay by Gamov as tunneling showed that several 100keV protons might suffice for nuclear reactions

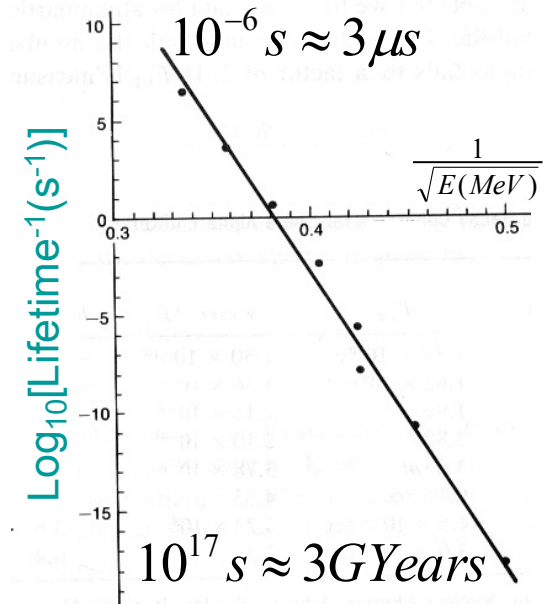
Schroedinger equation: $\frac{\partial^2}{\partial r^2} u(r) = \frac{2m}{\hbar^2} [V(r) - E] u(r), \quad T = \left| \frac{u(L)}{u(0)} \right|^2$

The transmission probability T for an alpha particle traveling from the inside towards the potential well that keeps the nucleus together determines the lifetime for alpha decay.



$$T \approx \exp\left[-2 \int_R^L \frac{\sqrt{2m[V(r)-E]}}{\hbar} dr\right]$$

$$\ln T \approx A - \frac{C}{\sqrt{E}}$$



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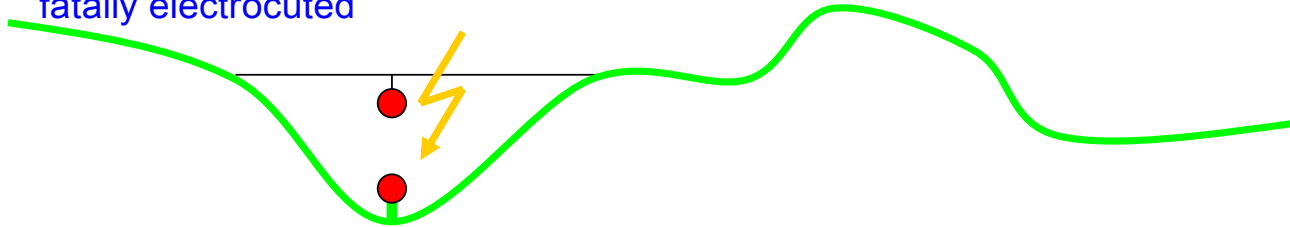
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Accelerators at Cornell

- 1932: Brasch and Lange use potential from lightening, in the Swiss Alps, C. Urban is fatally electrocuted



- 1934: Livingston builds the first Cyclotron outside Berkely (2MeV protons), Cornell RF B54
- 1949: Wilson et al. at Cornell are first to store beam in a synchtotron (later 300MeV, magnet of 80 Tons)
- 1954: Wilson et al. build first synchrotron with strong focusing for 1.1MeV electrons at Cornell, 4cm beam pipe height, only 16 Tons of magnets.
- 1979: 5GeV electron positron collider CESR (designed for 8GeV)
- 1990's CESR collider optimization for CLEO's world leading luminosity for a decade
- 2019 CBETA is the world's first multi-turn Energy Recovery Linac

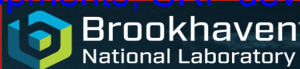
Today:

CESR operation and optimization for CHSS

Accelerator simulations, inc.. Machine learning, EIC design with Brookhaven National Lab
e-source developments, SRF developments



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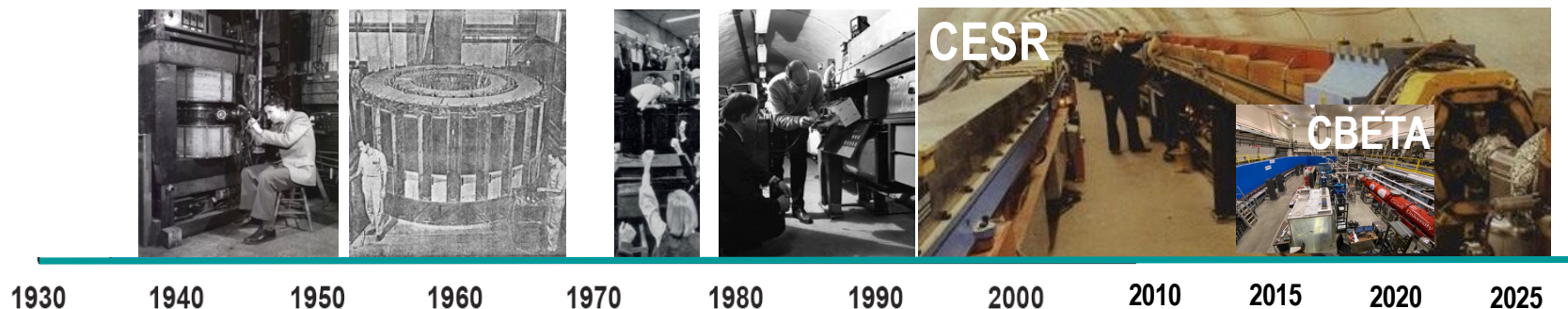


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Accelerators at Cornell



Welcome to graduate accelerator physics

Superconducting acceleration.

Bright electron sources. World record high current, low emittance.

Energy Recovery Linacs. The new accelerator paradigm.

Home of Bmad and SciBmad – versatile accelerator simulations



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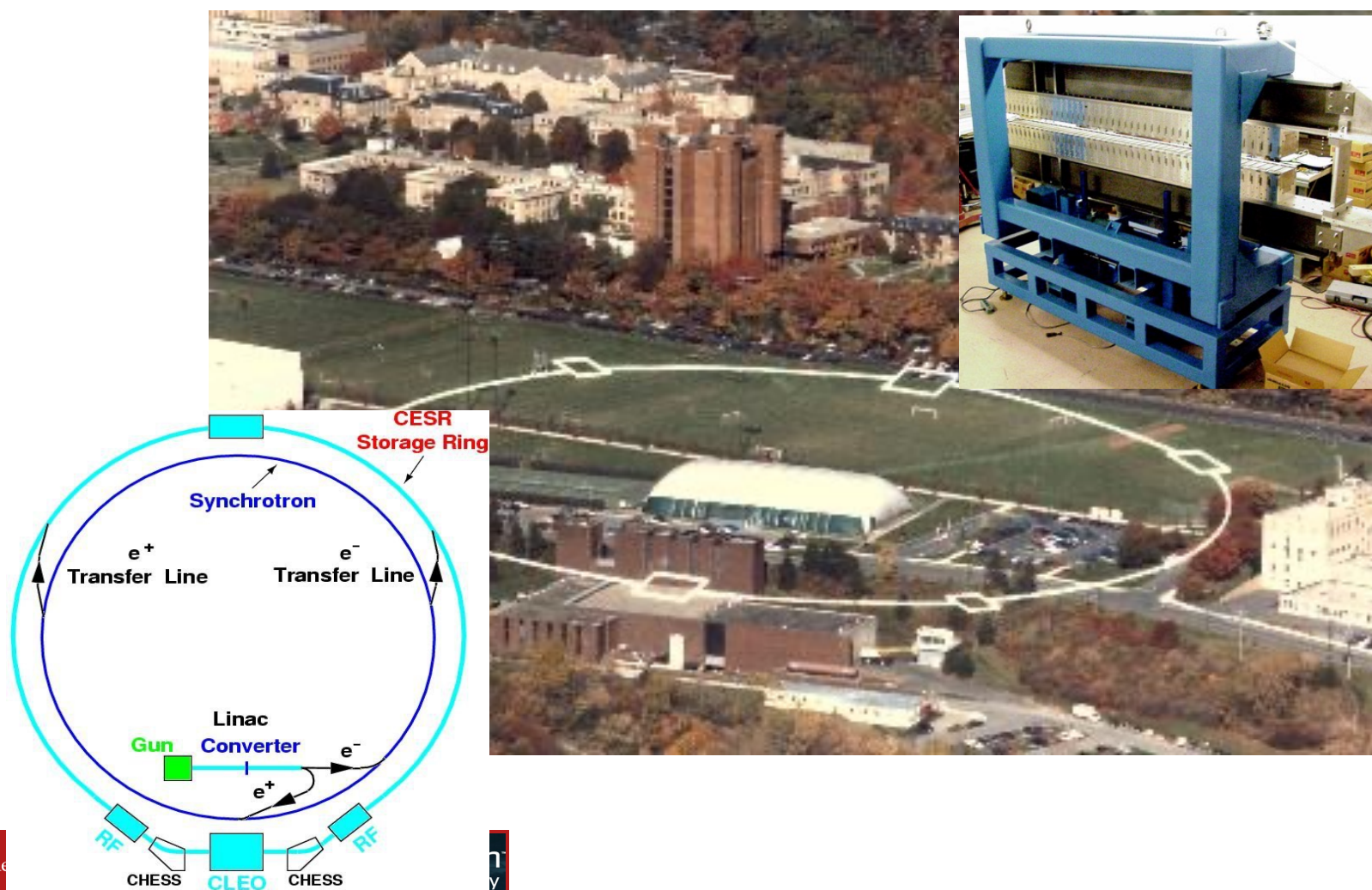
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Cornell's synchrotron and storage ring



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Early construction



Tunnel digging
(as of 1966)

... out



The largest single NSF investment at
up to date at the time.



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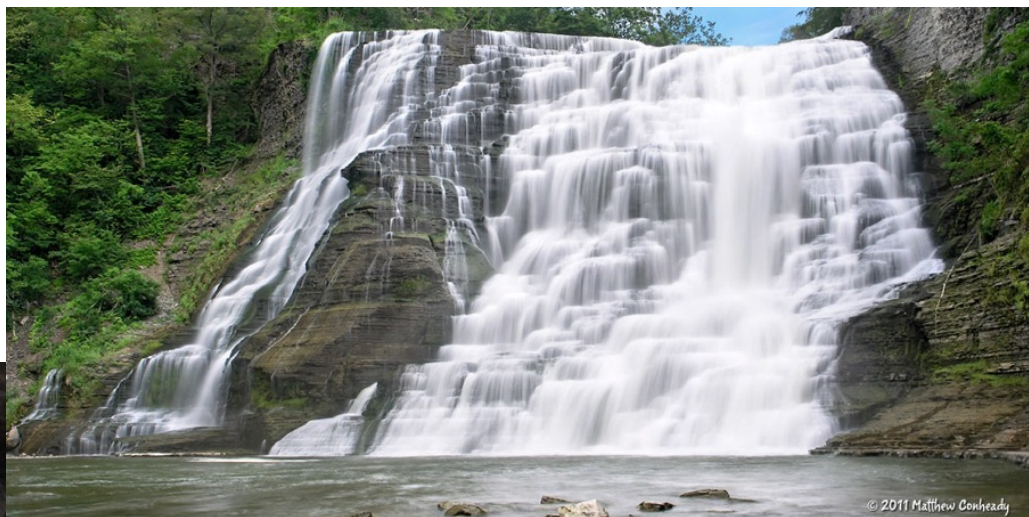
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Long tradition of Cornell tunnels

Tunnel digging
(as of 1830s)



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At Ithaca falls, constructed by Ezra Cornell.

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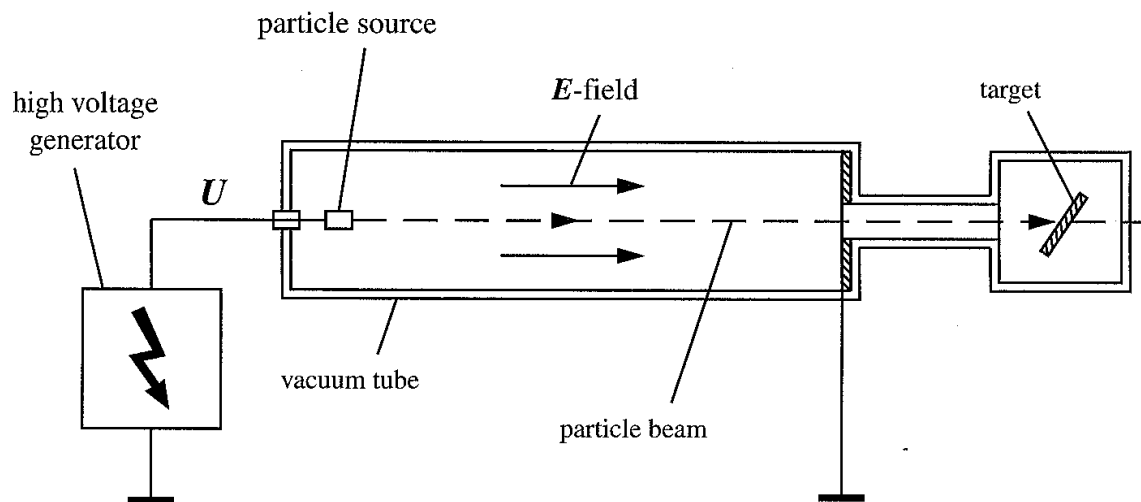
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Three historic lines of accelerators

Direct Voltage Accelerators

Resonant Accelerators Transformer Accelerator



Voltage 1MV
Charge Ze
Energy Z MeV

The energy limit is given by the maximum possible voltage. At the limiting voltage, electrons and ions are accelerated to such large energies that they hit the surface and produce new ions. An avalanche of charge carries causes a large current and therefore a breakdown of the voltage.



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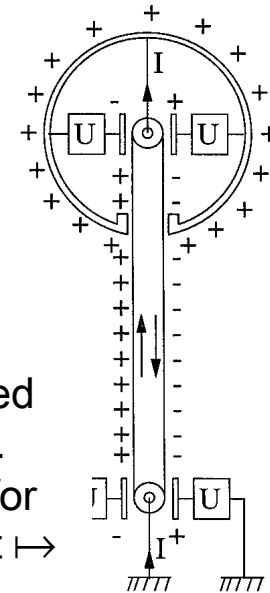
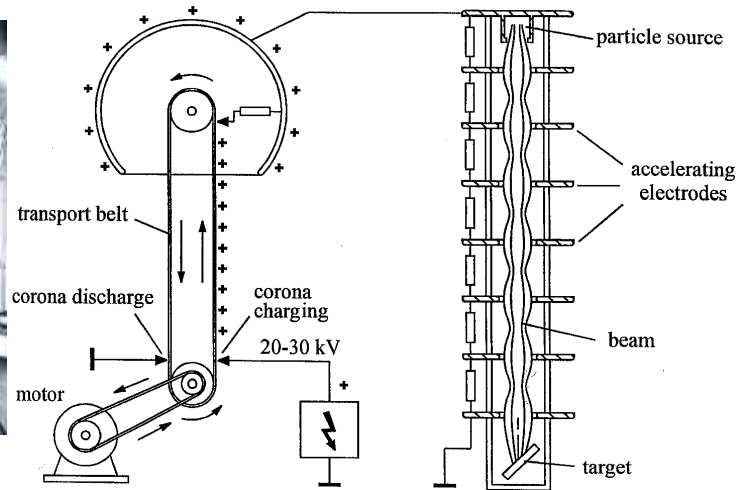
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The Van de Graaff Accelerator

- 1930: van de Graaff builds the first 1.5MV high voltage



Van de
Graaff



- Today Pelletrons (with chains) or Laddertron (with stripes) that are charged by influence are commercially available.
- Used as injectors, for electron cooling, for medical and technical n-source via $d + t \rightarrow n + \alpha$

Up to 17.5 MV with insulating gas (1MPa



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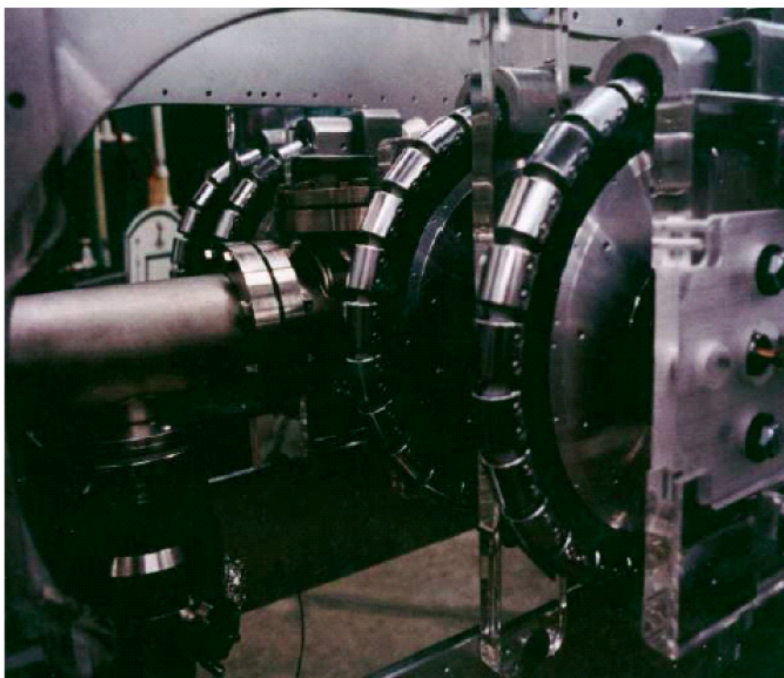
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The Pelletron

- A van de Graaff generator where the conveyer belt is replaced by a chain.



Comb and conveyer
belt of a van de Graaf



Pelletron chain from NEC



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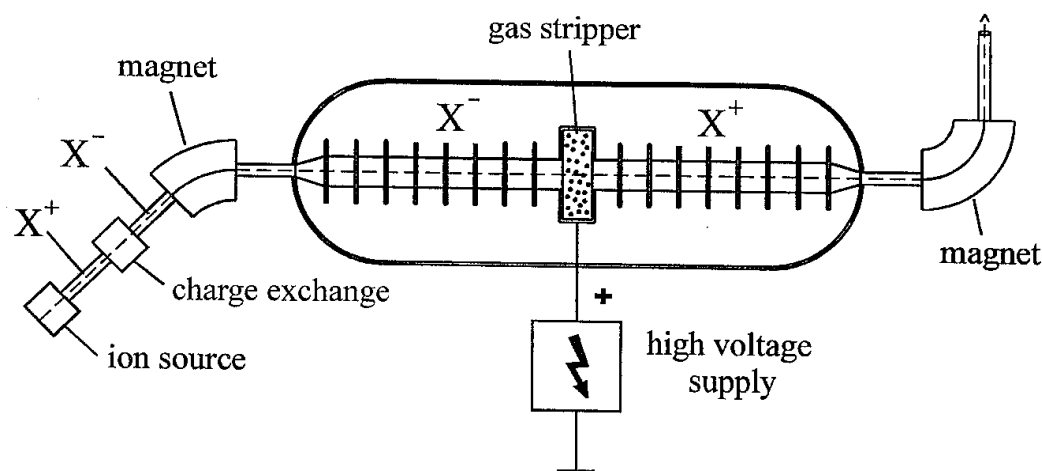
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The tandem (Van de Graaf) Accelerator

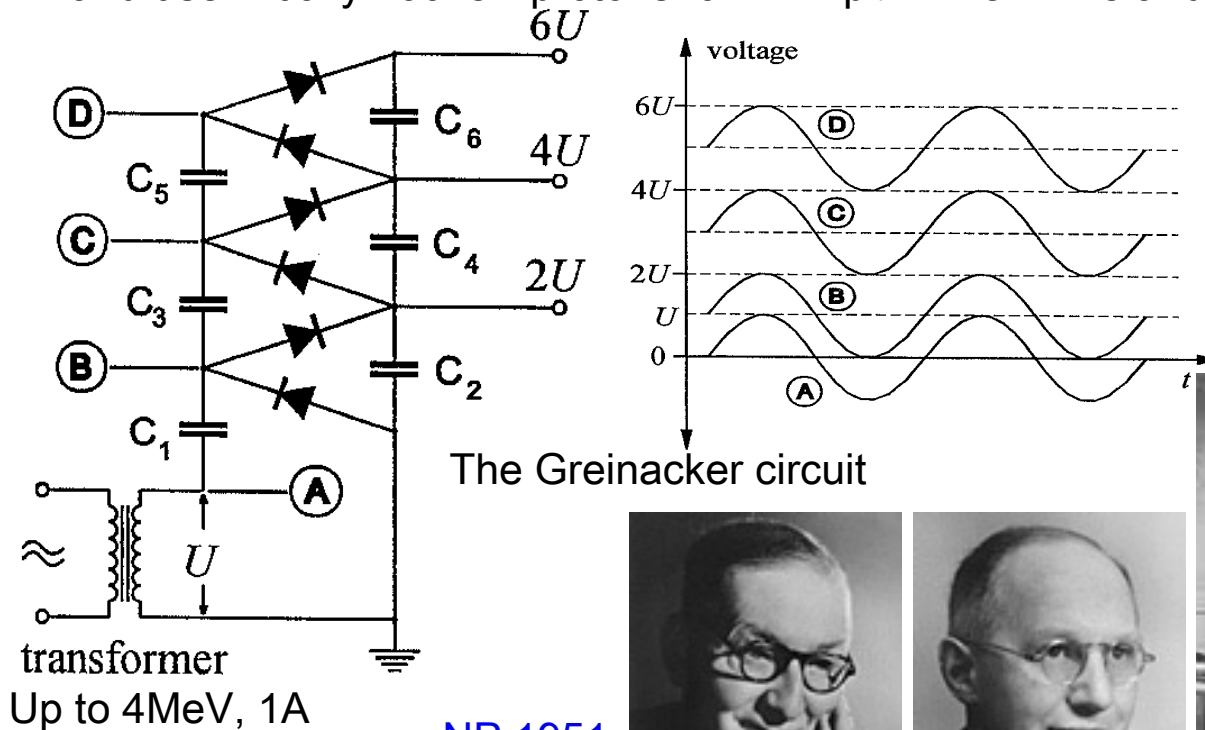


- Two Van de Graaffs, one + one – Daresbury Laboratory (UK), 20 MV
Decommissioned
- The Tandem Van de Graaff, highest energy 35MeV



The Cockcroft-Walton Accelerator

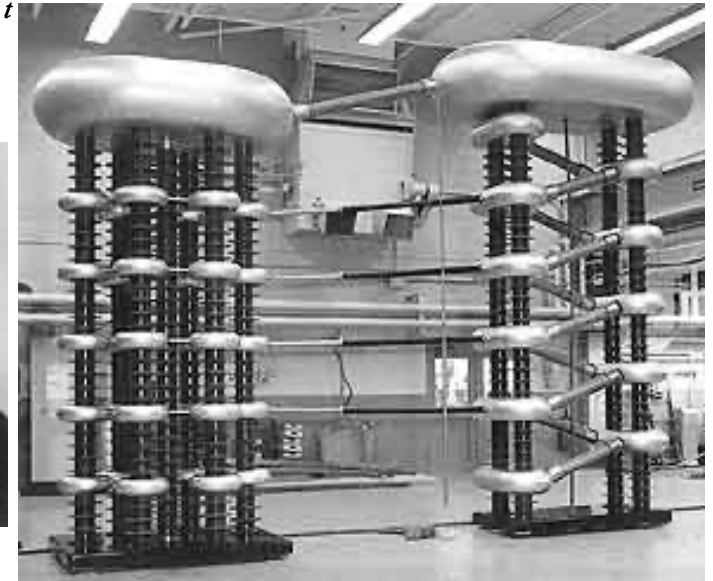
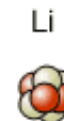
1932: Cockcroft and Walton 1932: 700keV cascade generator (planned for 800keV)
and use initially 400keV protons for ${}^7\text{Li} + p \mapsto {}^4\text{He} + {}^4\text{He}$ and ${}^7\text{Li} + p \mapsto {}^7\text{Be} + n$



NP 1951

Sir John D Cockroft

Ernest T S Walton



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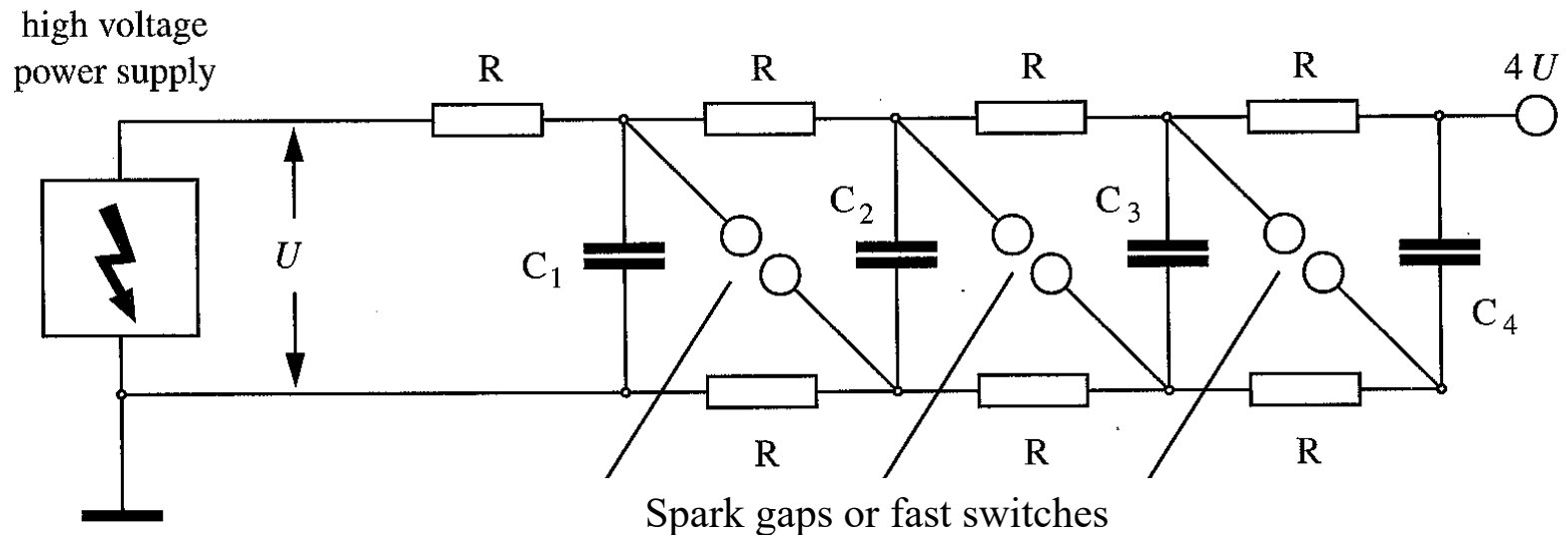
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The Marx Generator

- 1932: Marx Generator achieves 6MV at General Electrics



After capacitors of around 2 μ F are filled to about 20kV, the spark gaps or switches close as fast as 40ns, allowing up to 500kA.

Today: The Z-machine (Physics Today July 2003) for z-pinch initial confinement fusion has 40TW for 100ns from 36 Marx generators



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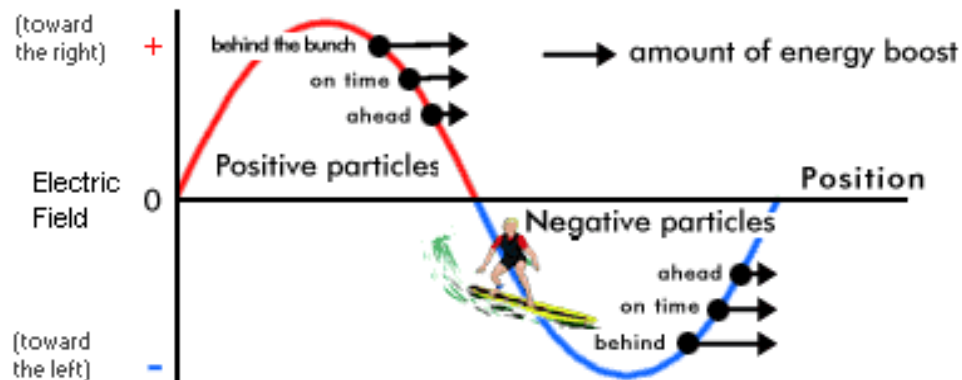
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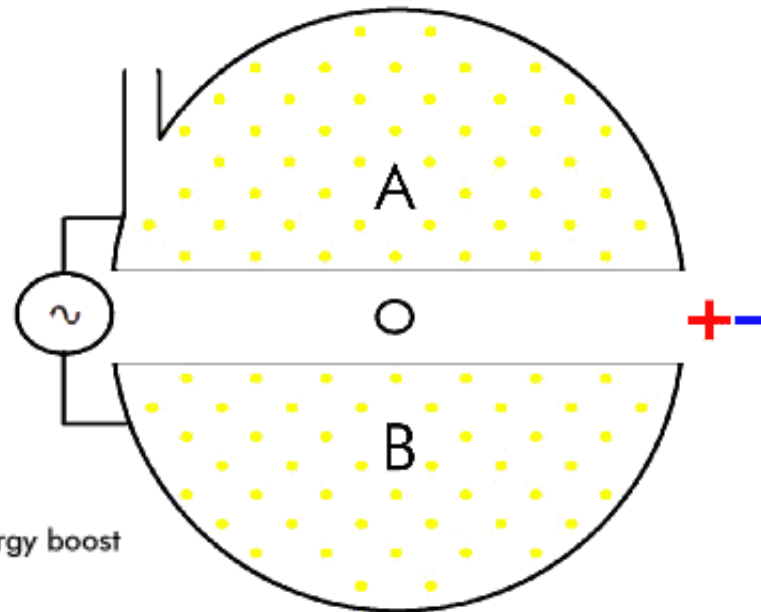
Three historic lines of accelerators

Direct Voltage Accelerators



Resonant Accelerators

Transformer Accelerator



Particles must have the correct phase relation to the accelerating voltage.



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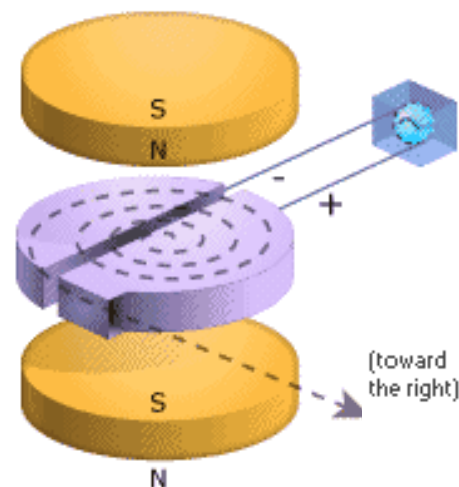
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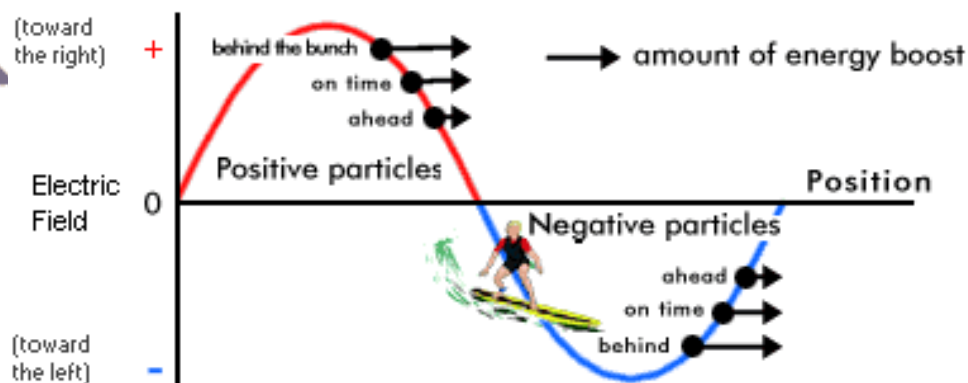
The Cyclotron



NP 1939

Ernest O Lawrence

- 1930: Lawrence proposes the Cyclotron
(before he develops a workable color TV screen)
- 1932: Lawrence and Livingston use a cyclotron for 1.25MeV protons and mention longitudinal (phase) focusing



- 1934: Livingston builds the first Cyclotron away from Berkely (2MeV protons) at Cornell (in room B54)



M Stanley Livingston



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The Cyclotron frequency

$$F_r = m_0 \gamma \omega_z v = q v B_z$$

$$\omega_z = \frac{q}{m_0 \gamma} B_z = \text{const}$$

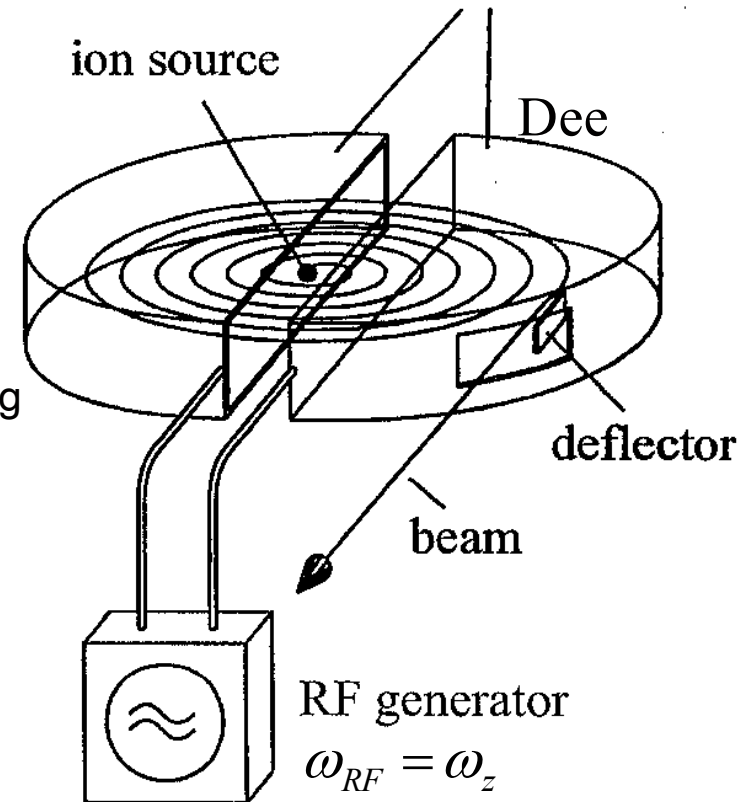
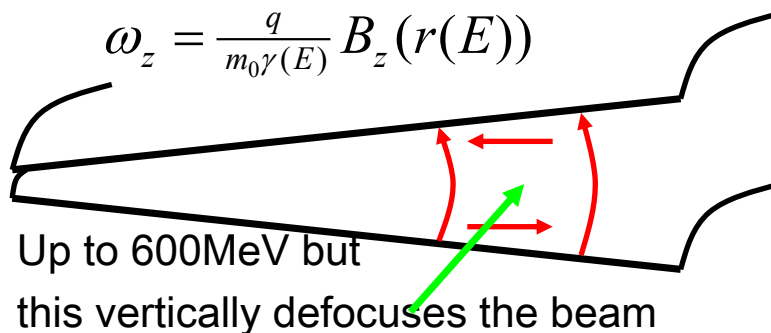
Condition: Non-relativistic particles.
Therefore, not for electrons.

- The synchrocyclotron:
Acceleration of bunches with decreasing

$$\omega_z(E) = \frac{q}{m_0 \gamma(E)} B_z$$

- The isocyclotron with constant

$$\omega_z = \frac{q}{m_0 \gamma(E)} B_z(r(E))$$



- 1938: Thomas proposes strong (transverse) focusing for a cyclotron



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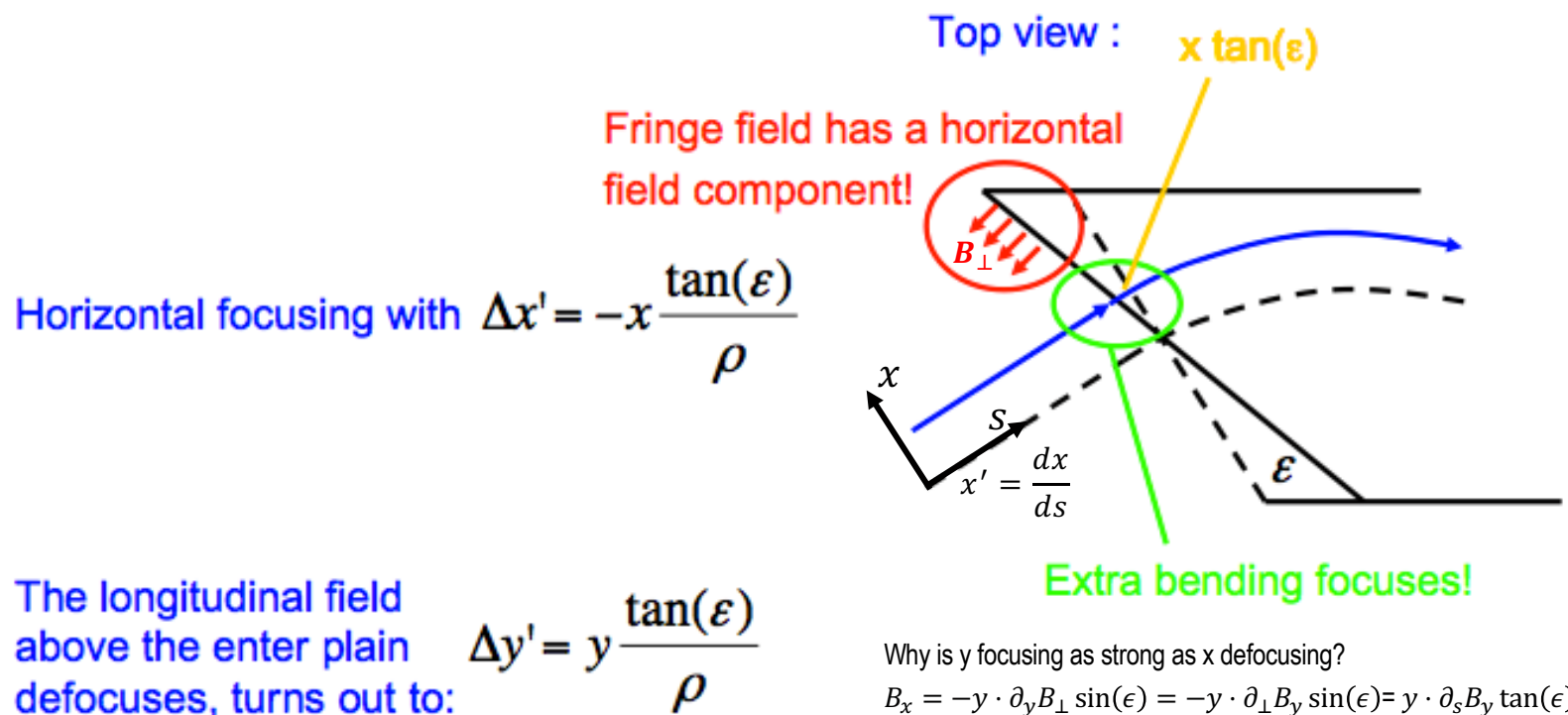
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Edge focusing



Quadrupole effect: focusing in x and defocusing in y or
defocusing in x and focusing in y.



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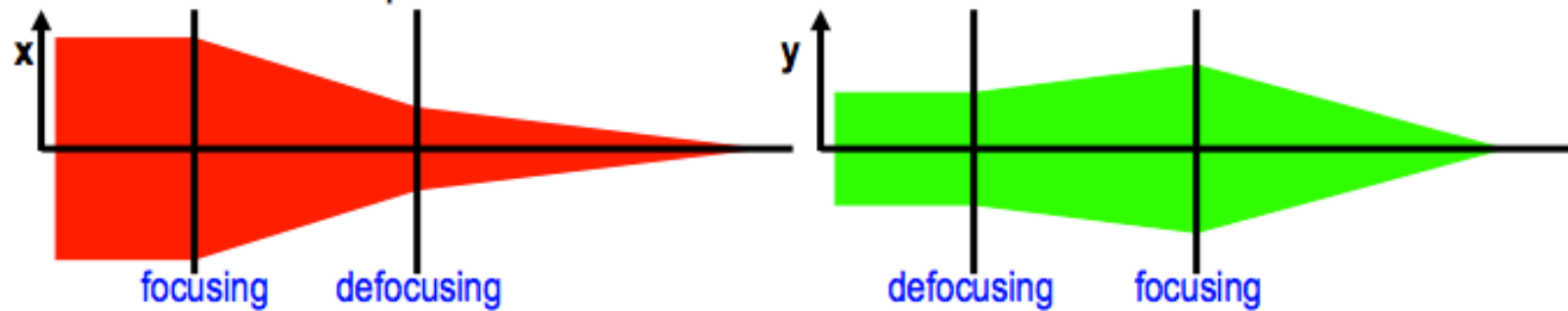
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Quadrupole Focusing

Transverse fields defocus in one plane if they focus in the other plane.
But two successive elements, one focusing the other defocusing,
can focus in both planes:



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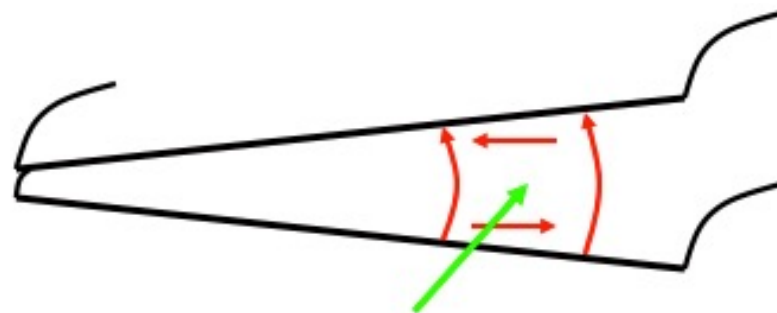
The Isocyclotron

- The isocyclotron with constant

$$\omega_z = \frac{q}{m_0 \gamma(E)} B_z(r(E))$$

Up to 600MeV but
this vertically defocuses the beam.

Edge focusing is therefore used.



First Medical Accelerators

- 1939: Lawrence uses 60' cyclotron for 9MeV protons, 19MeV deuterons, and 35MeV ^4He . First tests of tumor therapy with neutrons via $d + t \mapsto n + \alpha$
With 200-800keV d to get 10MeV neutrons.



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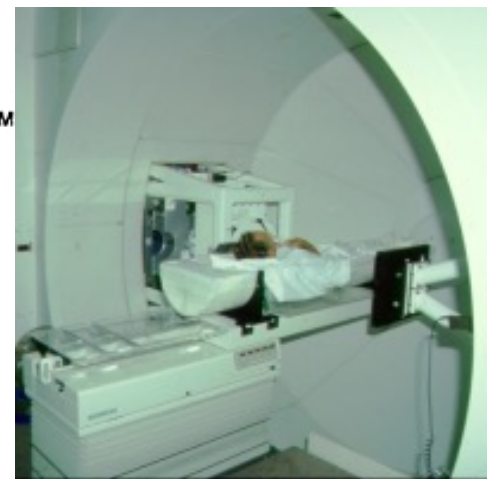
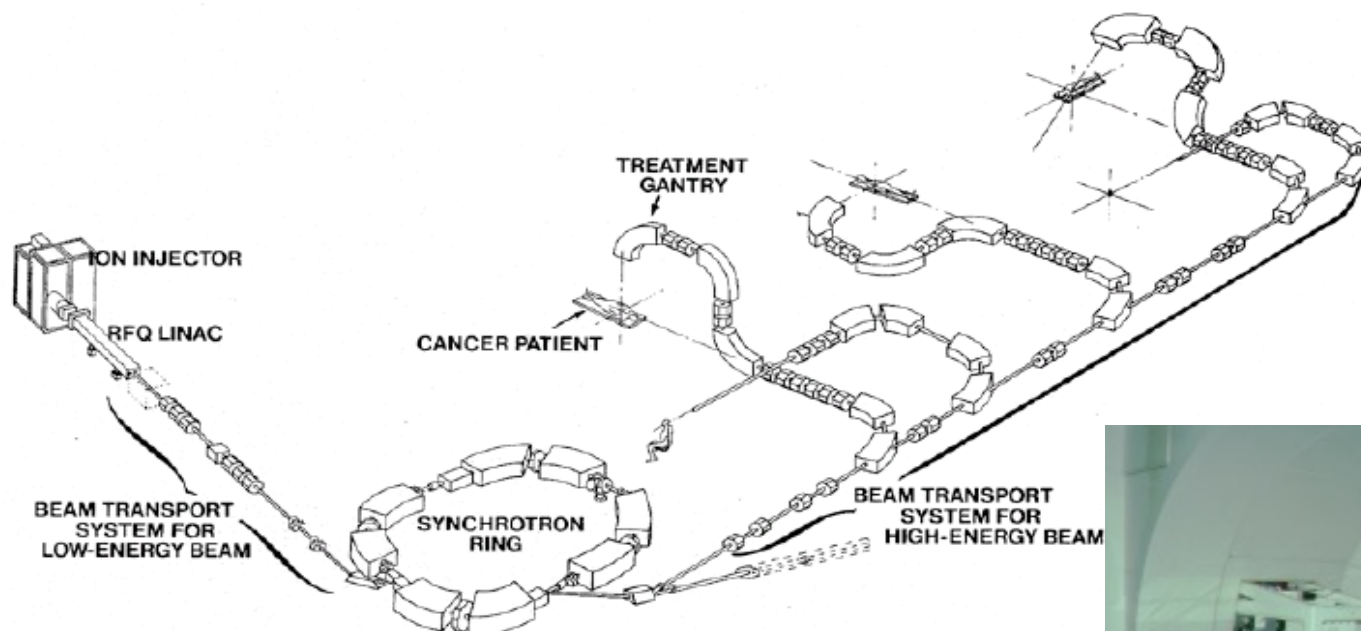
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Modern Nuclear Radiation Therapy

The Loma Linda proton therapy facility



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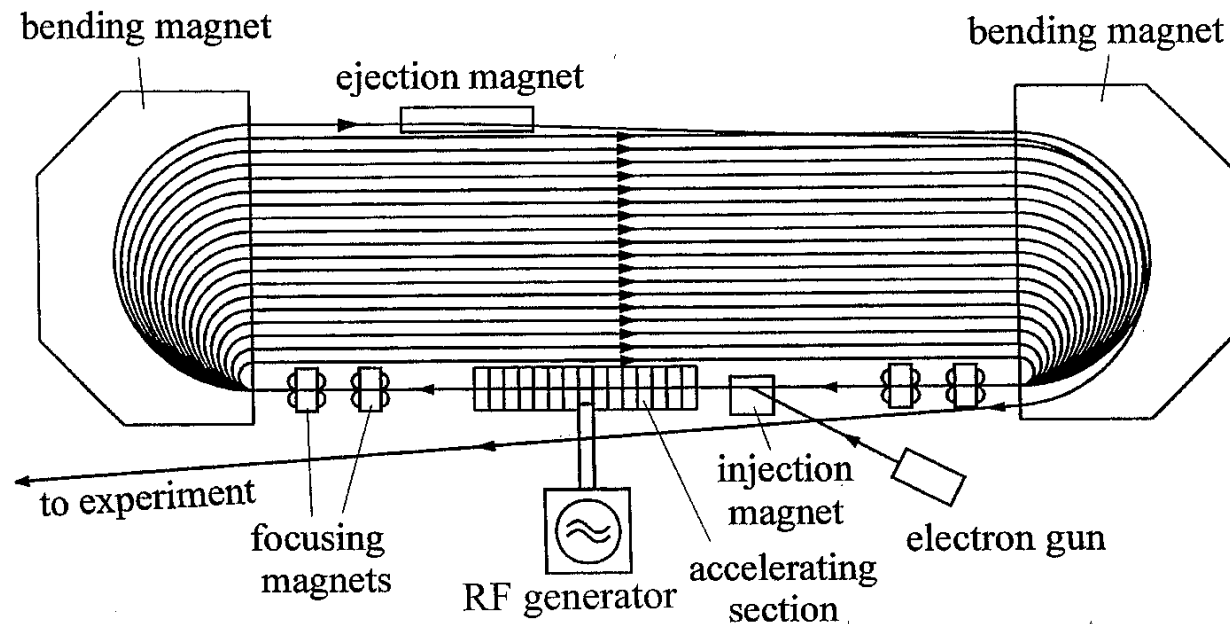
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The Microtron

- Electrons are quickly relativistic and cannot be accelerated in a cyclotron.
- In a microtron the revolution frequency changes, but each electron misses an integer number of RF waves.



- Today: Used for medical applications with one magnet and 20MeV.
- Nuclear physics: MAMI designed for 820MeV as race track microtron.



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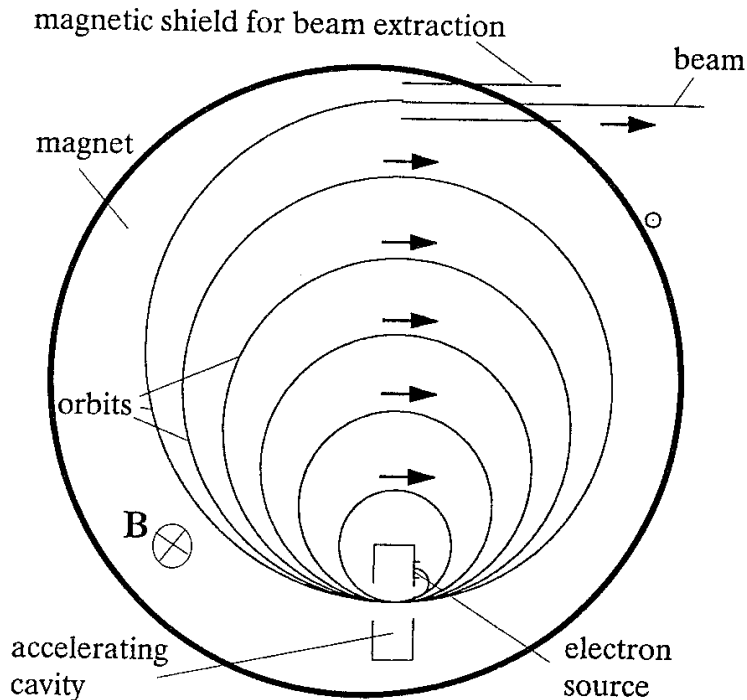
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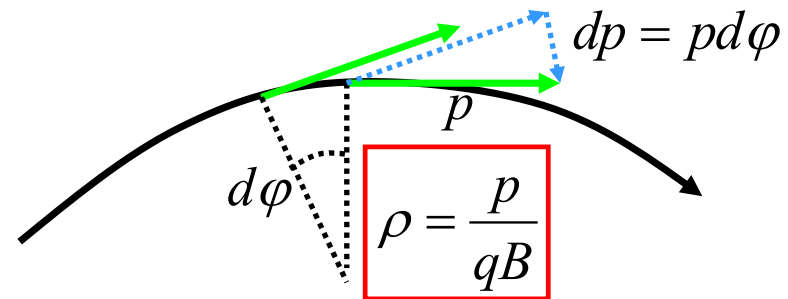
The Microtron Condition

- The extra time that each turn takes must be a multiple of the RF period.



$B=1\text{T}$, $n=1$, and $f_{\text{RF}}=3\text{GHz}$ leads to 4.78MeV
This requires a small linear accelerator.

$$\frac{dp}{dt} = qvB \Rightarrow \rho = \frac{dl}{d\phi} = \frac{vdt}{dp/p} = \frac{p}{qB}$$



$$\Delta t = 2\pi \left(\frac{\rho_{n+1}}{v_{n+1}} - \frac{\rho_n}{v_n} \right)$$

$$= \frac{2\pi}{qB} (m_0 \gamma_{n+1} - m_0 \gamma_n) = \frac{2\pi}{qBc^2} \Delta K$$

$$\Delta K = n \frac{qBc^2}{\omega_{\text{RF}}} \quad \text{for an integer } n$$



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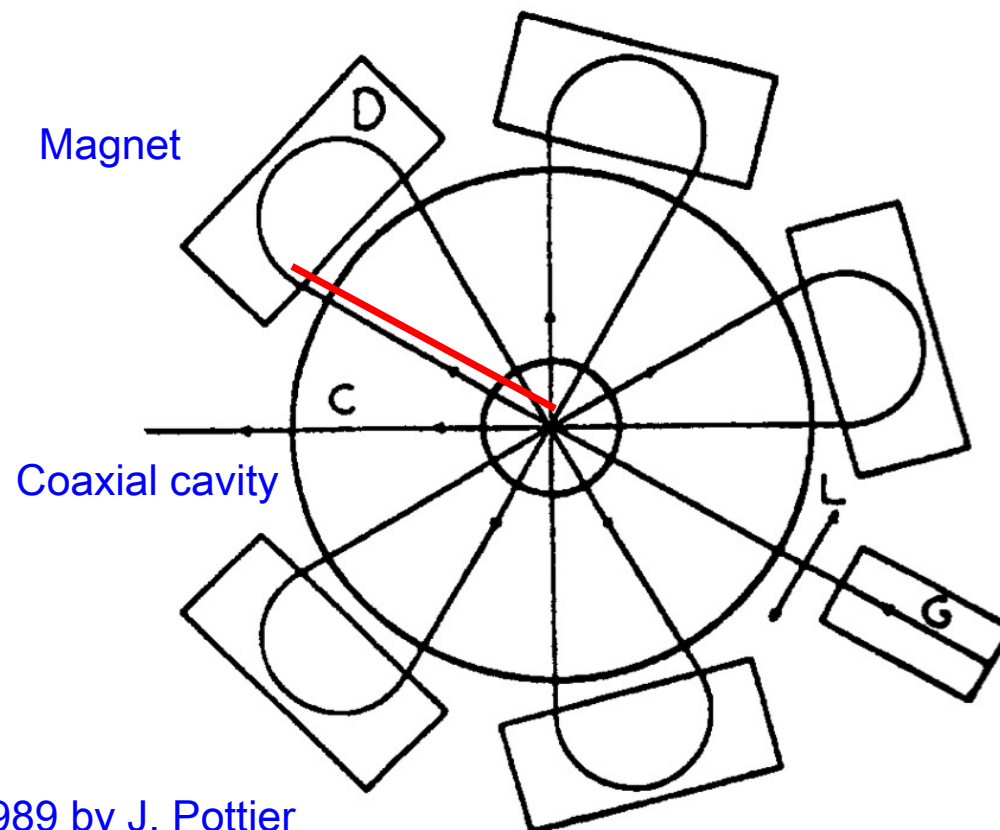
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The Rhodotron



- Invented in 1989 by J. Pottier
(French, CEA)



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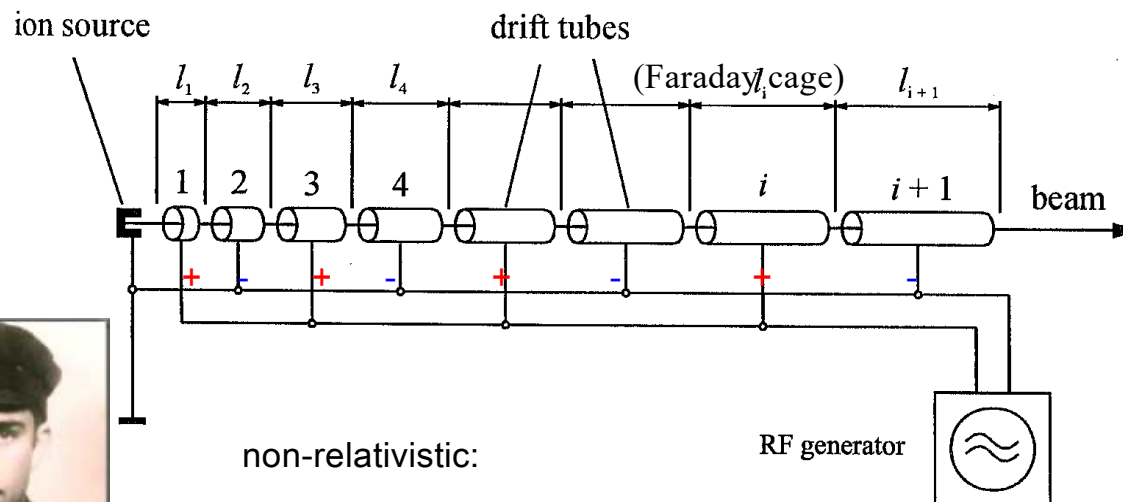
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The Wideroe linear accelerator



Rolf Wideroe

Part of dissertation 1928

Based on ideas by Gustav Ising 1924

(according to E.O. Lawrence Ising is the father of multiple acceleration.)

non-relativistic:

$$K_n = nqU_{\max} \sin \psi_0 = \frac{1}{2} m v_n^2$$

$$l_n = \frac{1}{2} v_n T_{RF} = \frac{1}{2} \beta_n \lambda_{RF} \propto \sqrt{n}$$

Called the π or the $1/2\beta\lambda$ mode



Gustav Ising



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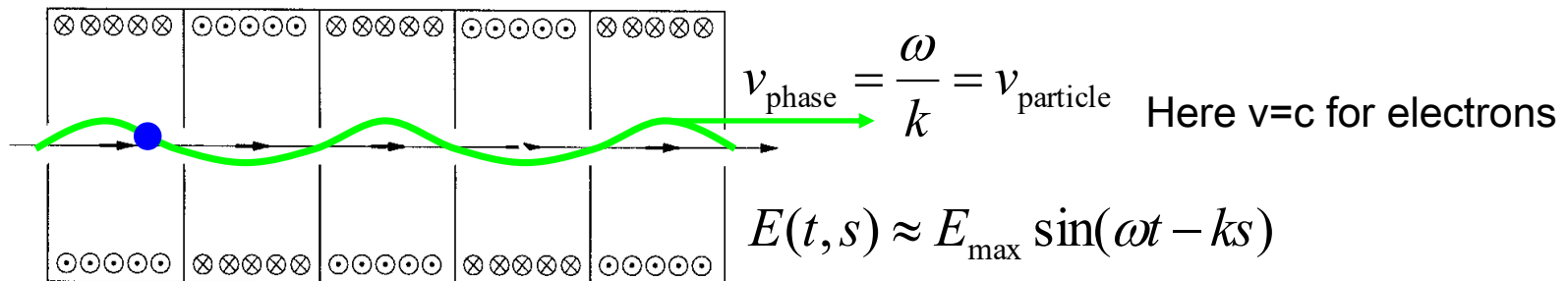
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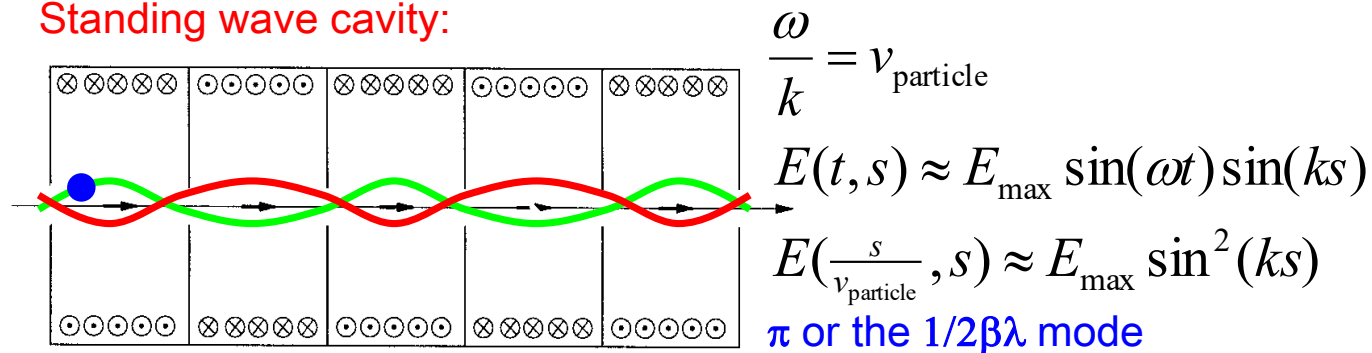
Accelerating Cavities

- 1933: J.W. Beams uses resonant cavities for acceleration

Traveling wave cavity:



Standing wave cavity:



Transit factor (for this example): $\langle E \rangle = \frac{1}{\lambda_{RF}} \int_0^{\lambda_{RF}} E(\frac{s}{v_{\text{particle}}}, s) ds \approx \frac{1}{2} E_{\text{max}}$



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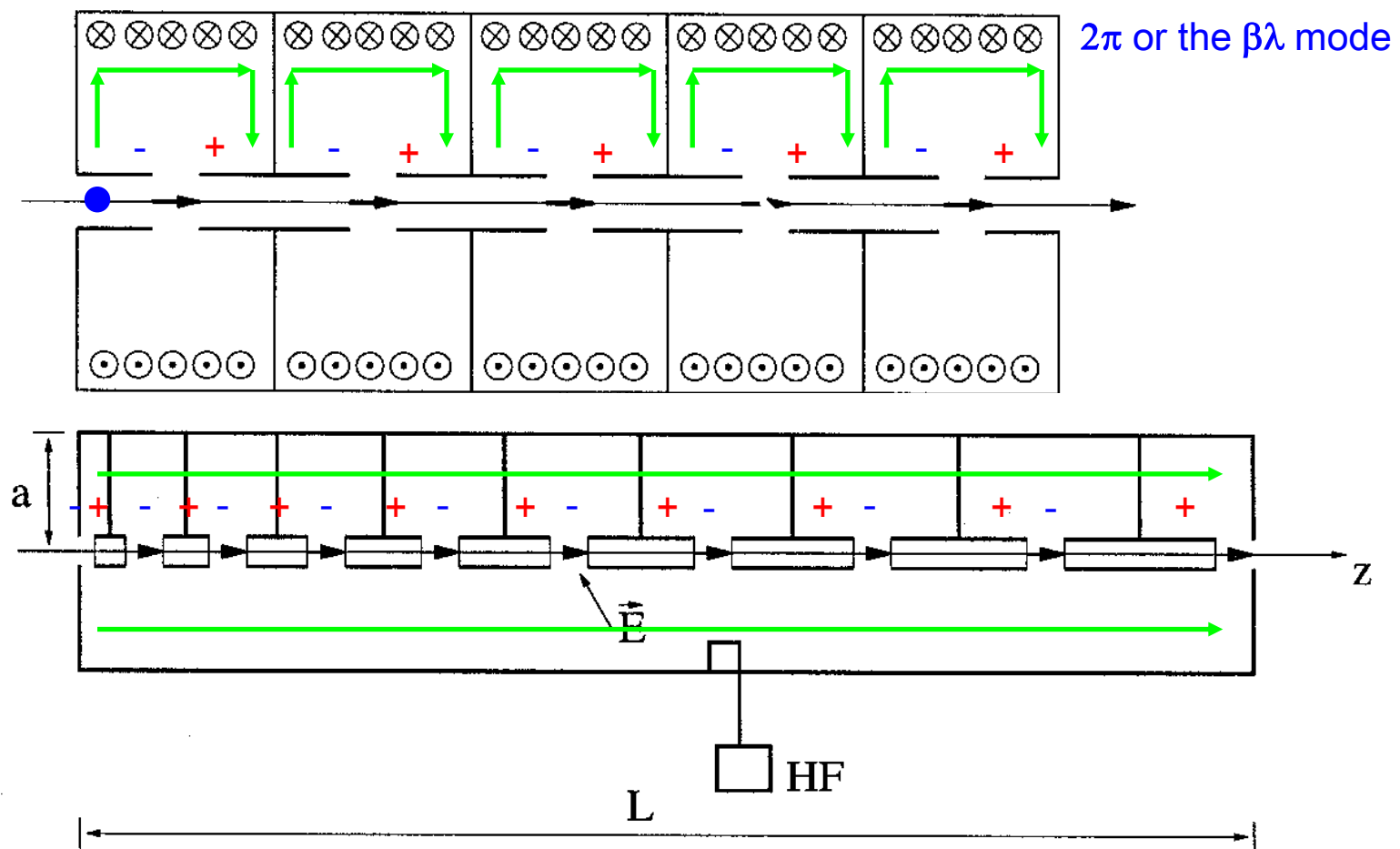
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The Alvarez Linear Accelerator



Needs only one power input coupler and walls do not dissipate energy.



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Alvarez and the end of dinosaurs

The **Alvarez hypothesis** posits that the mass extinction of the non-avian dinosaurs (and many other living things 66 Ma ago) was caused by a large asteroid.

This theory is generally accepted since around 2010.

In 1980, Nobel prize-winning physicist Luis Alvarez and his son, geologist Walter Alvarez discovered that sedimentary layers found all over the world at the Cretaceous–Paleogene boundary (K–Pg boundary) contain a concentration of iridium hundreds times greater than normal.

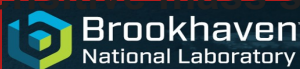
Luis A. knew that **accelerators** in Berkely could be used as ~~sensitive mass spectrometers~~.



Luis Alvarez (left) and his son Walter, at the K-Pg boundary 1981.



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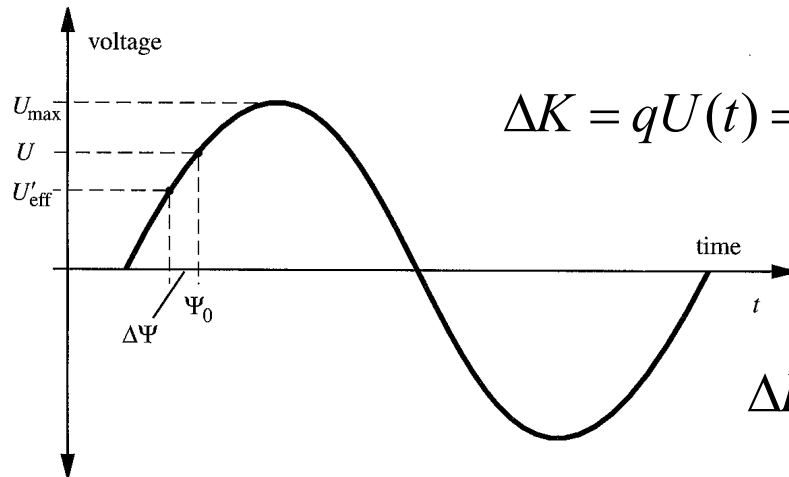
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Phase Focusing

- 1945: Veksler (UDSSR) and McMillan (USA) realize the importance of phase focusing



$$\Delta K = qU(t) = qU_{\max} \sin(\omega(t - t_0) + \psi_0)$$

Longitudinal position in the bunch:

$$\sigma = s - s_0 = -v_0(t - t_0)$$

$$\Delta K(\sigma) = qU_{\max} \sin\left(-\frac{\omega}{v_0}(s - s_0) + \psi_0\right)$$

$$\Delta K(0) > 0 \quad (\text{Acceleration})$$

$$\Delta K(\sigma) < \Delta K(0) \text{ for } \sigma > 0 \Rightarrow \frac{d}{d\sigma} \Delta K(\sigma) < 0 \quad (\text{Phase focusing})$$

$$\left. \begin{array}{l} qU(t) > 0 \\ q \frac{d}{dt} U(t) > 0 \end{array} \right\} \underline{\underline{\psi_0 \in (0, \frac{\pi}{2})}}$$

Phase focusing is required in any RF accelerator.



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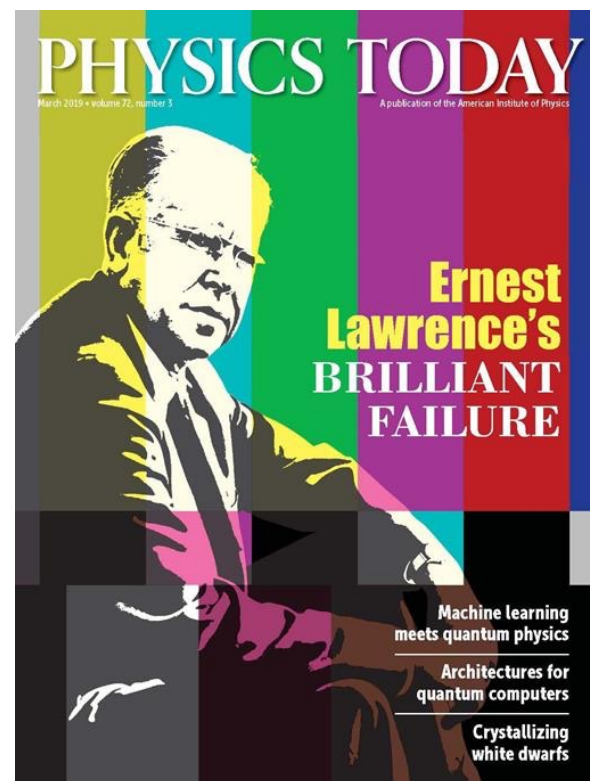
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Lawrence, Alvarz, McMillen and color TV



Ernest Lawrence, Edwin McMillan, and Luis Alvarez (left to right) admire a finished

The three accelerator physicists Lawrence, Alvarez, and McMillan has an important role in color TV development and started the company Chromatic Television.



Physics Today: "No company has employed three Nobel laureates and failed as dramatically as Chromatic Television."



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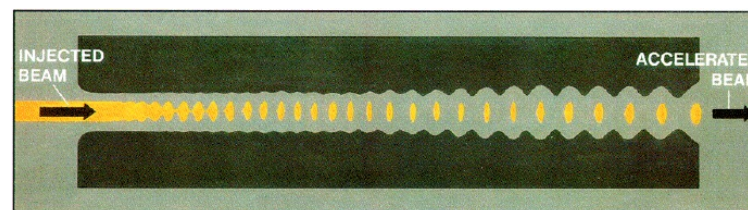
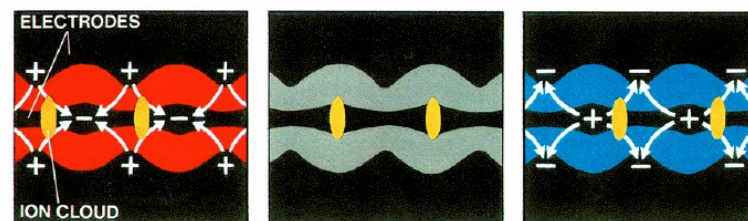
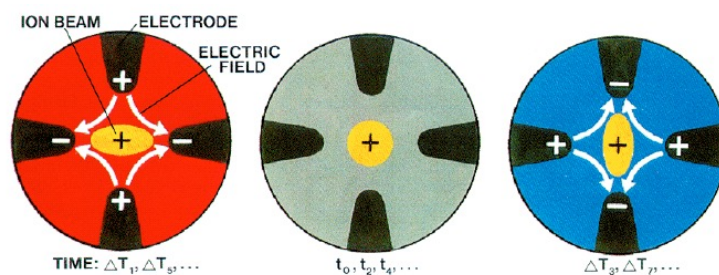
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Radio Frequency Quadrupoles



- 1970: Kapchinskii and Teplyakov invent the RFQ



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Three historic lines of accelerators

Direct Voltage Accelerators Resonant Accelerators Transformer Accelerator

- 1924: Wideroe invents the betatron
- 1940: Kerst and Serber build a betatron for 2.3MeV electrons and understand betatron (transverse) focusing (in 1942: 20MeV)

Betatron:

$R = \text{const}, B = B(t)$

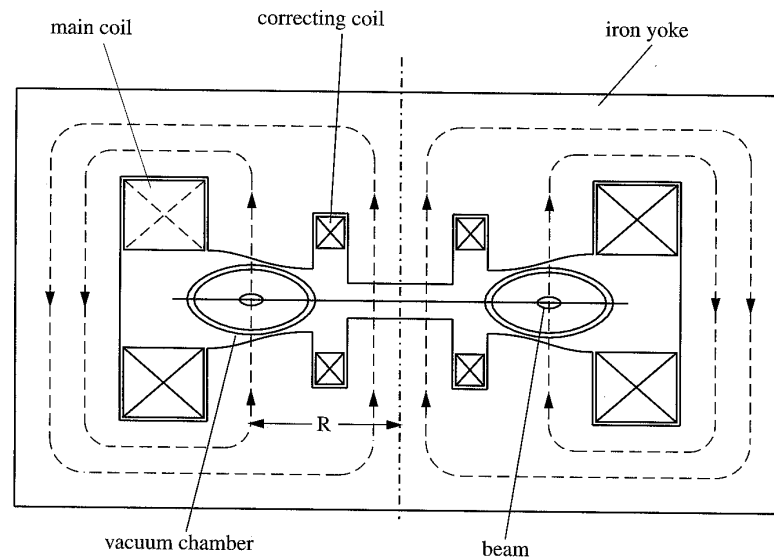
Whereas for a cyclotron:

$R(t), B = \text{const}$

No acceleration section is needed since

$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = - \iint_A \frac{d}{dt} \vec{B} \cdot d\vec{a}$$

Invented by Rolf Wideroe in 1923, 21 years old (5 years before his PhD) – *but not built*



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The Betatron Condition

Condition: $R = \frac{-p_\phi(t)}{qB_z(R,t)} = \text{const.}$ given $\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\iint_A \frac{d}{dt} \vec{B} \cdot d\vec{a}$

$$E_\phi(R,t) = -\frac{1}{2\pi R} \int \frac{d}{dt} B_z(r,t) r dr d\phi = -\frac{R}{2} \left\langle \frac{d}{dt} B_z \right\rangle$$

$$\frac{d}{dt} p_\phi(t) = qE_\phi(R,t) = -q \frac{R}{2} \left\langle \frac{d}{dt} B_z \right\rangle$$

$$p_\phi(t) = p_\phi(0) - q \frac{R}{2} [\langle B_z \rangle(t) - \langle B_z \rangle(0)] = -RqB_z(R,t)$$

$$B_z(R,t) - B_z(R,0) = \frac{1}{2} [\langle B_z \rangle(t) - \langle B_z \rangle(0)]$$

Small deviations from this condition lead to transverse beam oscillations called **betatron oscillations** in all accelerators.

- Today: Betatrons with typically about 20MeV for medical applications



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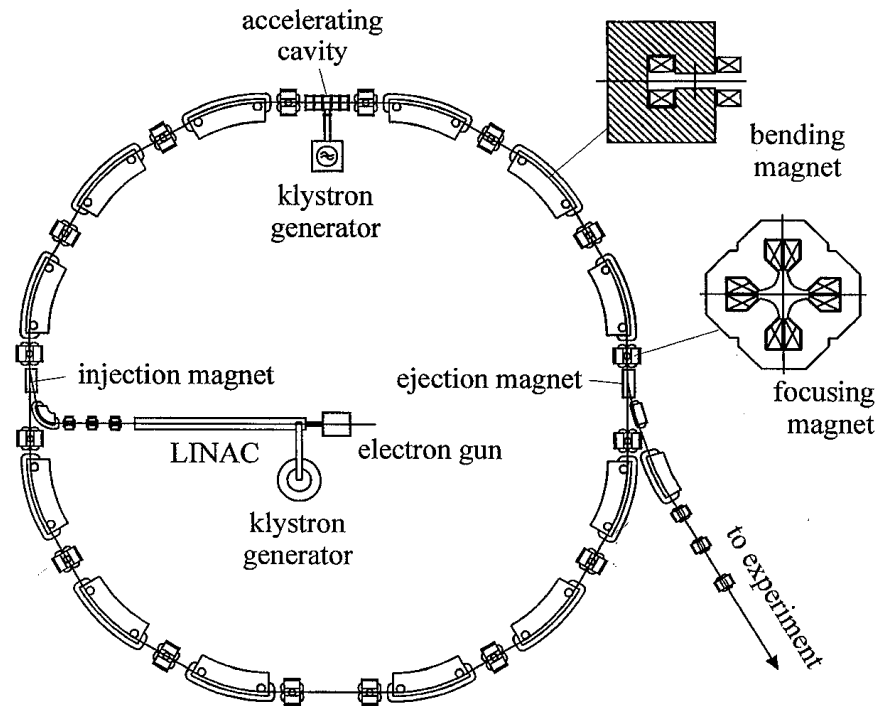
The Synchrotron

- 1945: Veksler (UDSSR) and McMillan (USA) invent the synchrotron
 - 1946: Goward and Barnes build the first synchrotron (using a betatron magnet)
 - 1949: Wilson et al. at Cornell are first to store beam in a synchrotron (later 300MeV, magnet of 80 Tons)
 - 1949: McMillan builds a 320MeV electron synchrotron
- Many smaller magnets instead of one large magnet
- Only one acceleration section is needed, with

$$R = \frac{p(t)}{qB(R,t)} = \text{const.}$$

$$\omega = 2\pi \frac{v_{\text{particle}}}{L} n$$

for an integer n called the harmonic number



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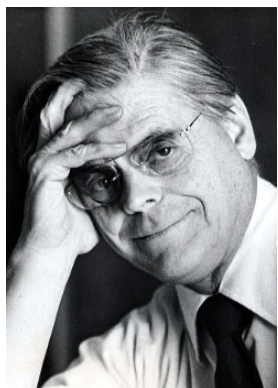
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Robert Wilson, Fermilab ...



Science Ed Center, FNAL (1990)



Robert R Wilson
USA 1914-2000



Wilson Hall, FNAL



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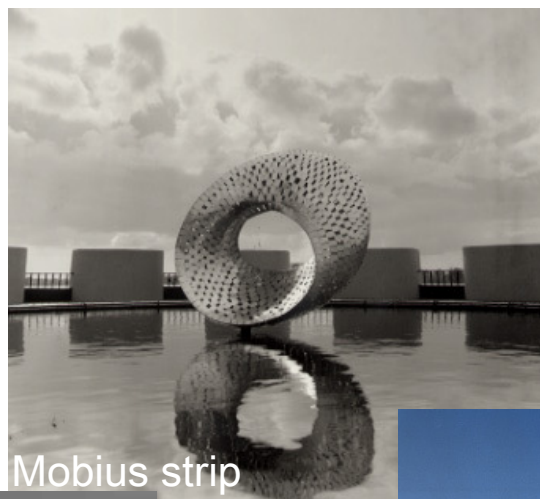
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... art, architecture, and accelerators.



Robert Wilson and Carl Sagan (of COSMOS)

The so-called Mausoleum in Ithaca is locally famous as creative quarter for science popularizer Carl Sagan, located directly opposite Ezra Cornell's tunnel at Ithaca falls.

Built in 1926 for Sphinx Head, the longest-running (secret) honor society at Cornell, which introduced Dragon Day, the name "Big Red", and Slope Day (initially called Spring Fest).

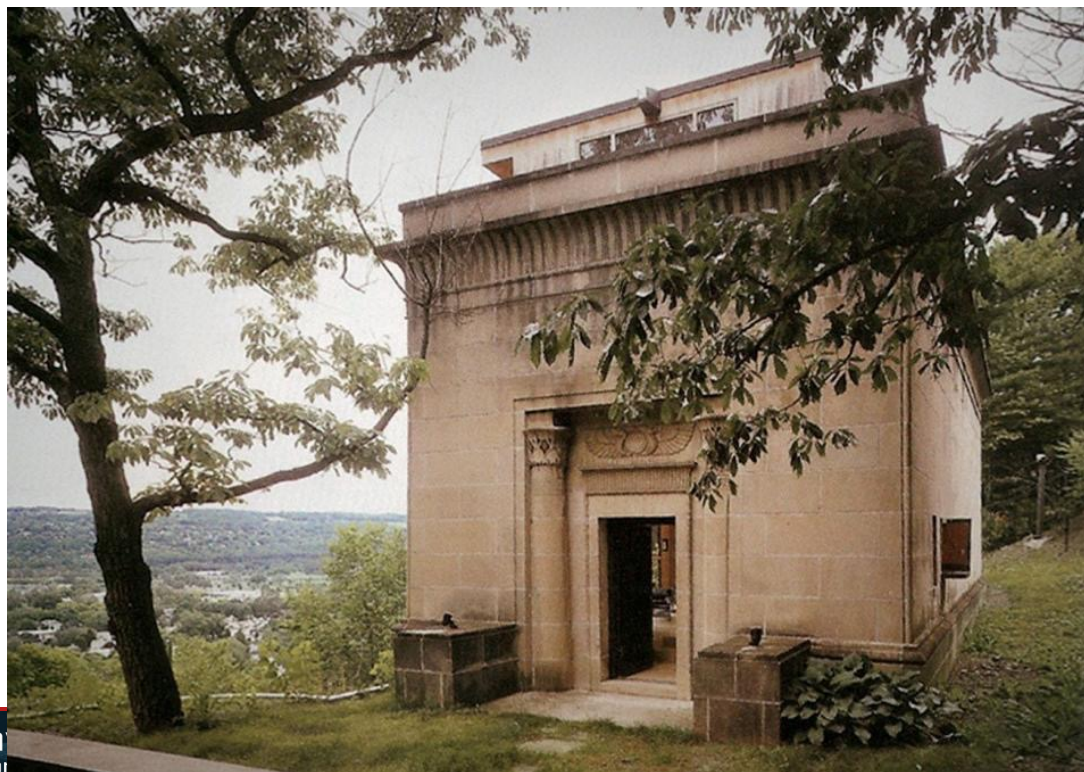
The neighbor Robert Wilson purchased it as art studio in 1978.

He sold it to Carl Sagan in 1981.

His Widow Ann Druyan sold it in 2024.

Being renovated by Pei architects

Note: I.M. Pei's first art museum is at Cornell.



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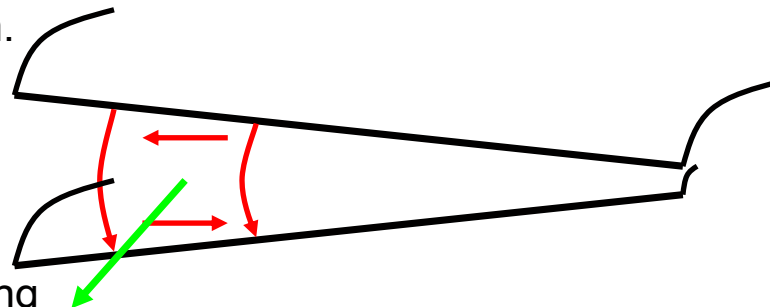
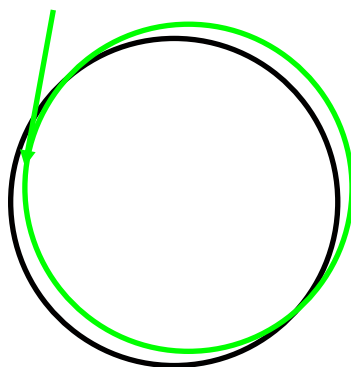
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The weak focusing synchrotron

- 1952: Operation of the Cosmotron, 3.3 GeV proton synchrotron at Brookhaven: Beam pipe height: 15cm.

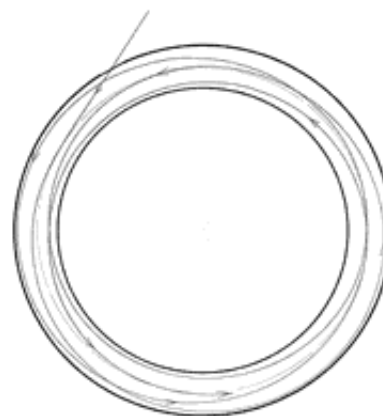
Natural ring focusing:



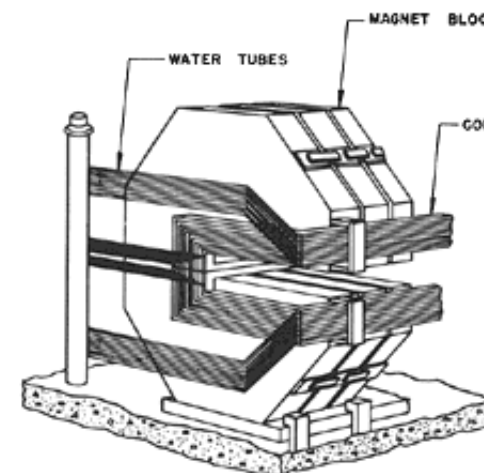
Vertical focusing
+ Horizontal defocusing + ring focusing
Focusing in both planes



The Cosmotron



Weak focusing accelerator



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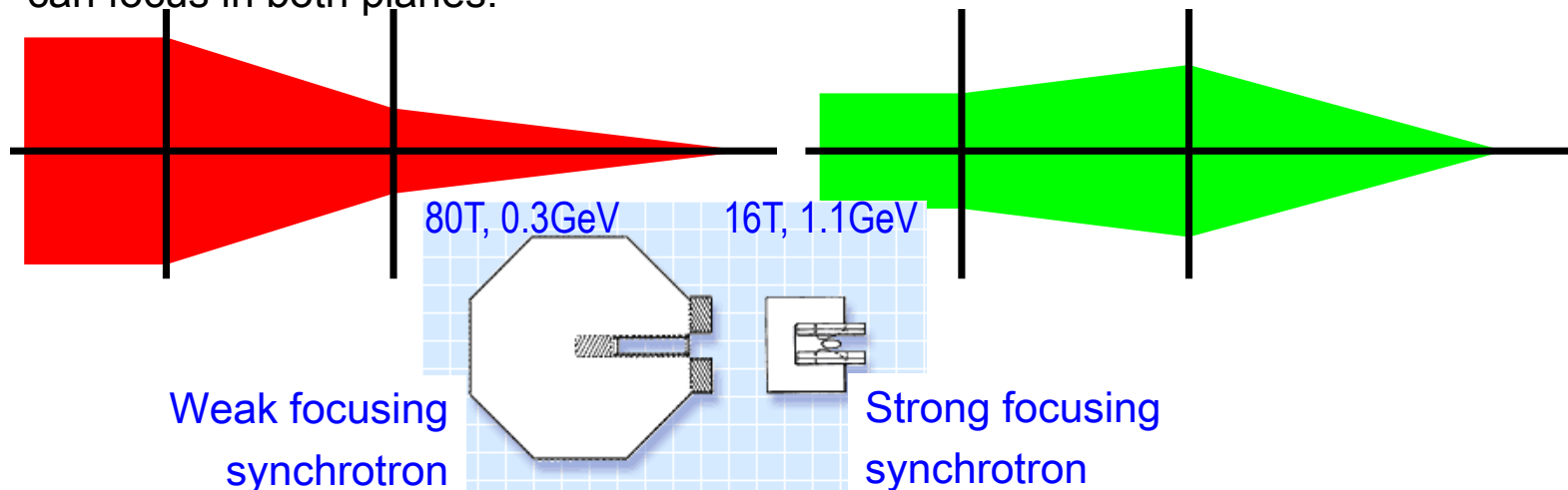
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Strong focusing synchrotrons

- 1952: Courant, Livingston, Snyder publish about strong focusing
- 1954: Wilson et al. build first synchrotron with strong focusing for 1.1MeV electrons at Cornell, 4cm beam pipe height, only 16 Tons of magnets.
- 1959: CERN builds the PS for 28GeV after proposing a 5GeV weak focusing accelerator for the same cost (still in use)

Transverse fields defocus in one plane if they focus in the other plane.
But two successive elements, one focusing the other defocusing, can focus in both planes:



- Today: only strong focusing is used. Due to bad field quality at lower field excitations the injection energy is 20-500MeV from a linac or a microtron.



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Limits of synchrotrons

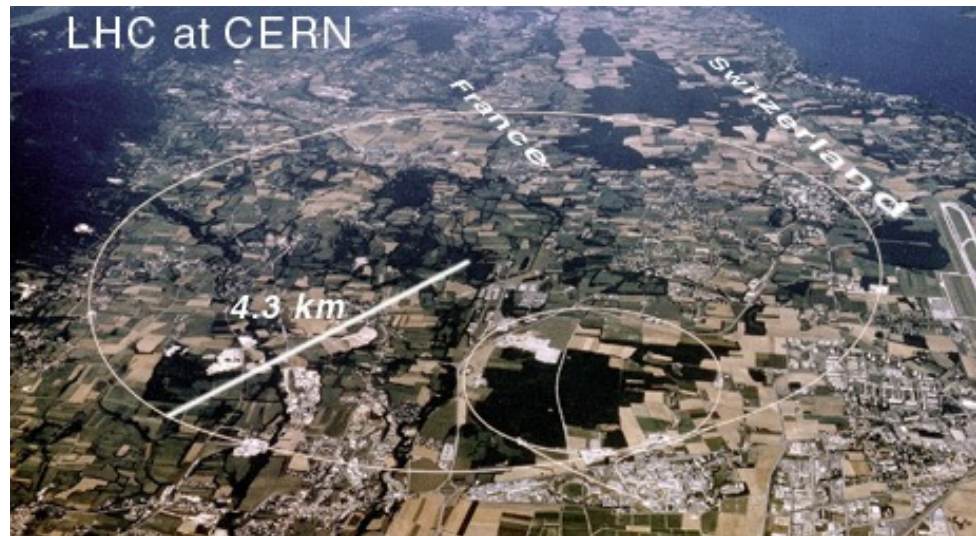
$$\rho = \frac{p}{qB} \Rightarrow \text{The rings become too long}$$

Protons with $p = 20 \text{ TeV}/c$, $B = 6.8 \text{ T}$ would require a 87 km SSC tunnel

Protons with $p = 7 \text{ TeV}/c$, $B = 8.4 \text{ T}$ require CERN's 27 km LHC tunnel

$$P_{\text{radiation}} = \frac{c}{6\pi\epsilon_0} N \frac{q^2}{\rho^2} \gamma^4 \quad \Downarrow$$

Energy needed to compensate
Radiation becomes too large



Electron beam with $p = 0.1 \text{ TeV}/c$ in CERN's 27 km LEP tunnel radiated 20 MW
Each electron lost about 4 GeV per turn, requiring many RF accelerating sections.



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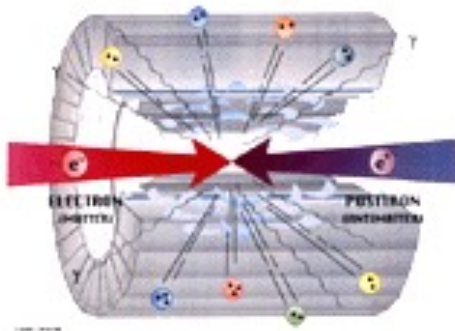
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Colliding beam accelerators

- 1961: First storage ring for electrons and positrons (AdA) in Frascati for 250MeV
- 1972: SPEAR electron positron collider at 4GeV. Discovery of the J/Psi at 3.097GeV by Richter (SPEAR) and Ting (AGS) starts the November revolution and was essential for the quarkmodel and chromodynamics.
- 1979: 5GeV electron positron collider CESR (designed for 8GeV)

Advantage:

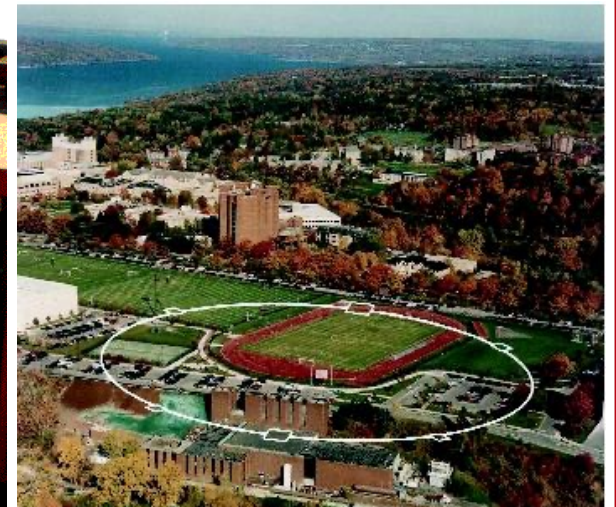
More center of mass energy



Drawback:

Less dense target

The beams therefore must be stored for a long time.



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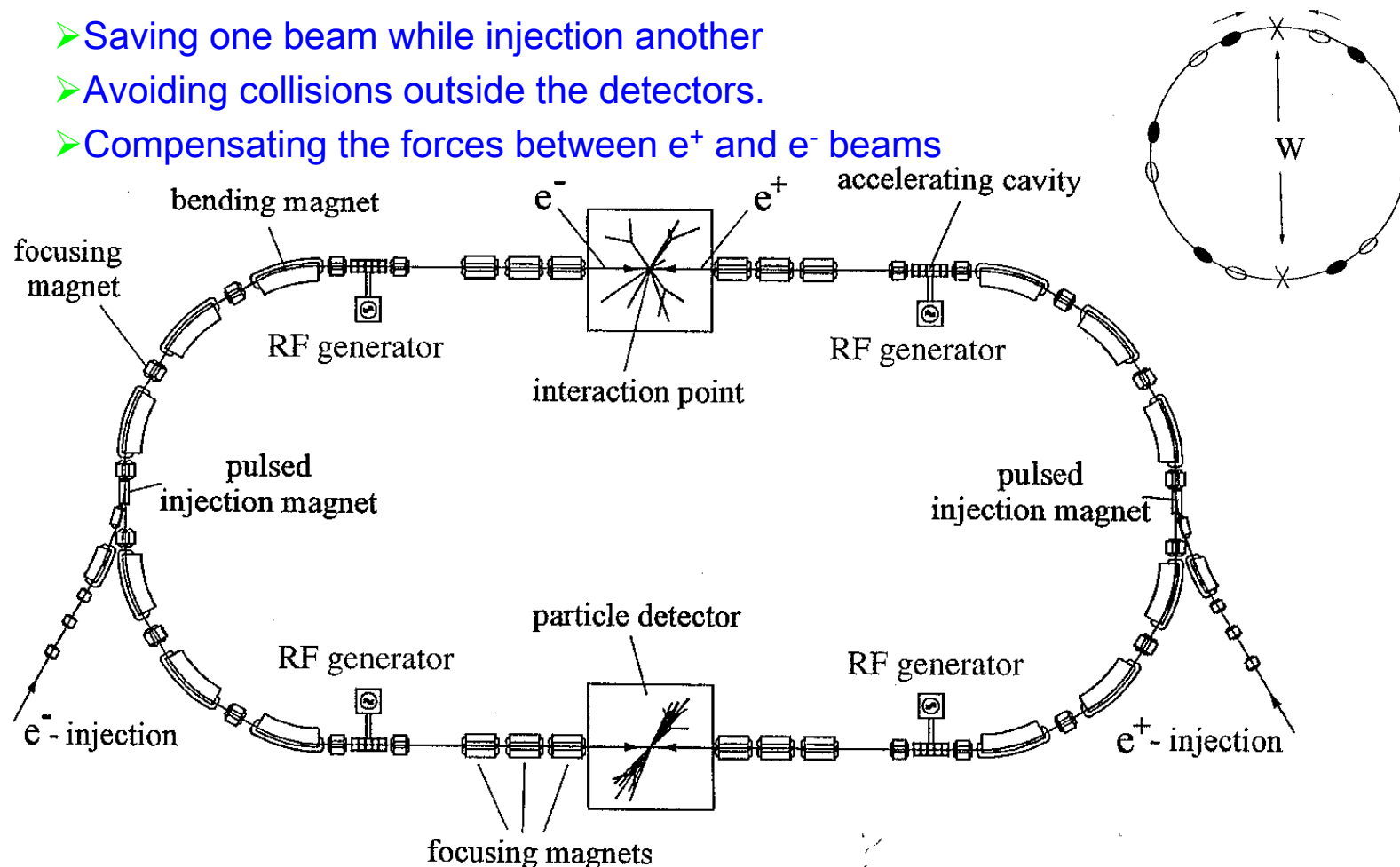
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Elements of a collider

- Saving one beam while injection another
- Avoiding collisions outside the detectors.
- Compensating the forces between e^+ and e^- beams



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Storage Rings

To avoid the loss of collision time during filling of a synchrotron, the beams in colliders must be stored for many millions of turns.

Challenges:

- Required vacuum of pressure below 10^{-7} Pa = 10^{-9} mbar, 3 orders of magnitude below that of other accelerators.
- Fields must be stable for a long time, often for hours.
- Field errors must be small, since their effect can add up over millions of turns.
- Even though a storage ring does not accelerate, it needs acceleration sections for phase focusing and to compensate energy loss due to the emission of radiation.



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Further developments of Colliders

- 1981: Rubbia and van der Meer use stochastic cooling of anti-protons and discover W^+ , W^- and Z vector bosons of the weak interaction
- 1987: Start of the superconducting TEVATRON at FNAL
- 1989: Start of the 27km long LEP electron positron collider
- 1990: Start of the first asymmetric collider, electron (27.5GeV) proton (920GeV) in HERA at DESY
- 1998: Start of asymmetric two ring electron positron colliders KEK-B / PEP-II
- Today: 27km, 6.8 TeV proton collider LHC; Higgs discovery in 2012



NP 1984
Carlo Rubbia
Italy 1934 -



NP 1984
Simon van der Meer
Netherlands 1925 -



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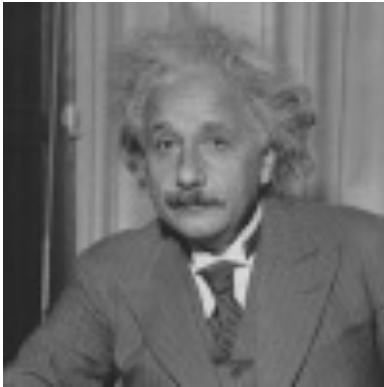
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Special Relativity

$$E = mc^2$$



Albert Einstein, 1879-1955
Nobel Prize, 1921
Time Magazine Man of the Century

Four-Vectors:

Quantities that transform according to the Lorentz transformation when viewed from a different inertial frame.

Examples:

$$X^\mu \in \{ct, x, y, z\}$$

$$P^\mu \in \{\frac{1}{c}E, p_x, p_y, p_z\}$$

$$\Phi^\mu \in \{\frac{1}{c}\phi, A_x, A_y, A_z\}$$

$$J^\mu \in \{c\rho, j_x, j_y, j_z\}$$

$$K^\mu \in \{\frac{1}{c}\omega, k_x, k_y, k_z\}$$

$$X^\mu \in \{ct, x, y, z\} \Rightarrow X^\mu X_\mu = (ct)^2 - \vec{x}^2 = \text{const.}$$

$$P^\mu \in \{\frac{1}{c}E, p_x, p_y, p_z\} \Rightarrow P^\mu P_\mu = \left(\frac{E}{c}\right)^2 - \vec{p}^2 = (m_0c)^2 = \text{const.}$$



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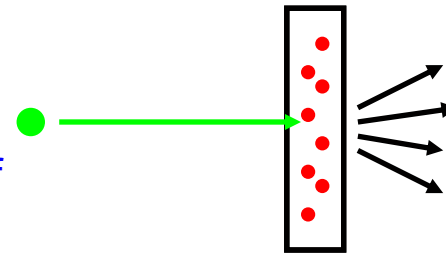
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Available Energy in Collisions

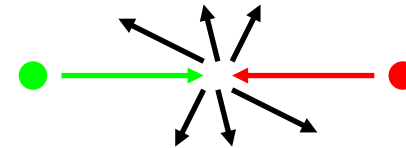
$$\begin{aligned}
 \frac{1}{c^2} E_{\text{cm}}^2 &= (P_1^\mu + P_2^\mu)_{\text{cm}} (P_{1\mu} + P_{2\mu})_{\text{cm}} \\
 &= (P_1^\mu + P_2^\mu)(P_{1\mu} + P_{2\mu}) \\
 &= \frac{1}{c^2} (E_1 + E_2)^2 - (p_{z1} - p_{z2})^2 \\
 &= 2\left(\frac{E_1 E_2}{c^2} + p_{z1} p_{z2}\right) + (m_{01} c)^2 + (m_{02} c)^2
 \end{aligned}$$

Operation of synchrotrons: fixed target experiments
where some energy is in the motion of the center of mass of the scattering products



$$E_1 \gg m_{01} c^2, m_{02} c^2; p_{z2} = 0; E_2 = m_{02} c^2 \Rightarrow E_{\text{cm}} = \sqrt{2 E_1 m_{02} c^2}$$

Operation of colliders:
the detector is in the center of mass system



$$E_1 \gg m_{01} c^2; E_2 \gg m_{02} c^2 \Rightarrow E_{\text{cm}} = 2\sqrt{E_1 E_2}$$



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Example: Fixed-target production of p-bar

- 1954: Operation of Bevatron, first proton synchrotron for 6.2 GeV, production of the anti-proton by Chamberlain and Segre

$$p + p \mapsto p + p + p + \bar{p}$$

$$\frac{1}{c^2} E_{\text{cm}}^2 = 2 \left(\frac{E_1 E_2}{c^2} + p_{z1} p_{z2} \right) + (m_{01} c)^2 + (m_{02} c)^2$$

$$(4m_{p0}c)^2 < \frac{E_{\text{cm}}^2}{c^2} = 2 E_1 m_{p0} + (m_{p0}c)^2 + (m_{p0}c)^2$$

$$7m_{p0}c^2 < E_1$$

$$\underline{K_1 = E_1 - m_0 c^2} > \underline{6m_{p0}c^2 = 5.628 \text{ GeV}}$$



NP 1959

Emilio Gino Segre

Italy 1905 – USA 1989



NP 1959

Owen Chamberlain

USA 1920 - 2006



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Example: production of c / c-bar states

- 1974: Observation of $c - \bar{c}$ resonances (J/Ψ) at $E_{cm} = 3095\text{MeV}$ at the e^+/e^- collider SPEAR

$$\frac{1}{c^2} E_{cm}^2 = 2\left(\frac{E_1 E_2}{c^2} + p_{z1} p_{z2}\right) + (m_{01}c)^2 + (m_{02}c)^2$$

Resonance in c/c-bar creation when $E_{cm} = 2m_{c0}c^2$ $E_1 = E_2 \Rightarrow E_{cm}^2 = 4E^2$

Energy per beam: $K = E - m_0c = \underline{1547\text{MeV}}$

Beam energy needed for an equivalent fixed target experiment: $\frac{E_{cm}^2}{c^2} = 2[Em + (mc)^2]$



NP 1976
Burton Richter
USA 1931 -

$$K = E - m_{0e}c^2 = \frac{E_{cm}^2}{2m_{0e}c^2} - 2m_{0e}c^2 = \underline{9.4\text{TeV}}$$



NP 1976
Samuel CC Ting
USA 1936 -



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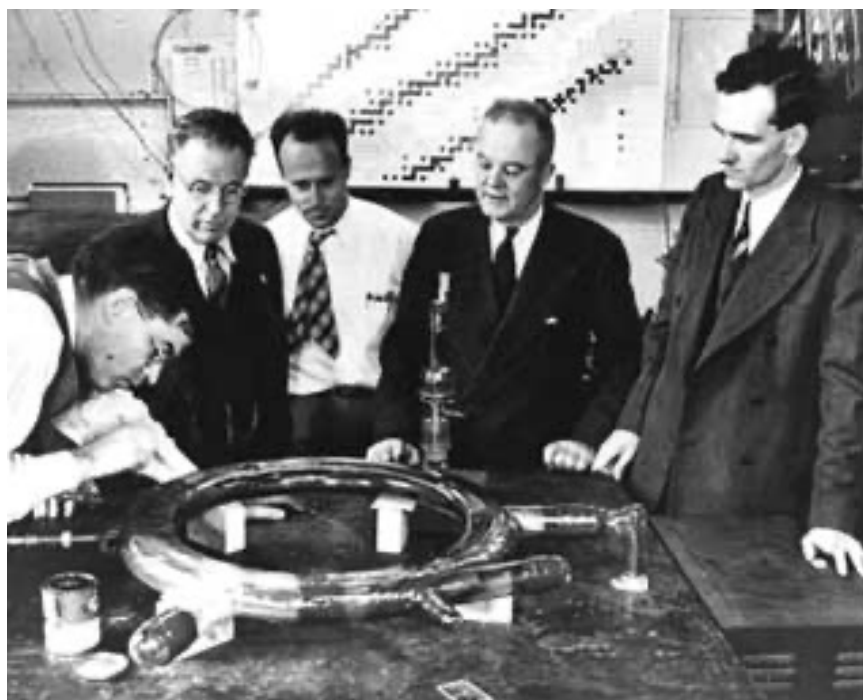
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Rings for Synchrotron Radiation

- 1947: First detection of synchrotron light at General Electrics.
- 1952: First accurate measurement of synchrotron radiation power by Dale Corson with the Cornell 300MeV synchrotron.
- 1968: TANTALOS (U of Wisconsin), first dedicated storage ring for synchrotron radiation



Dale Corson
Cornell's 8th president
USA 1914 –



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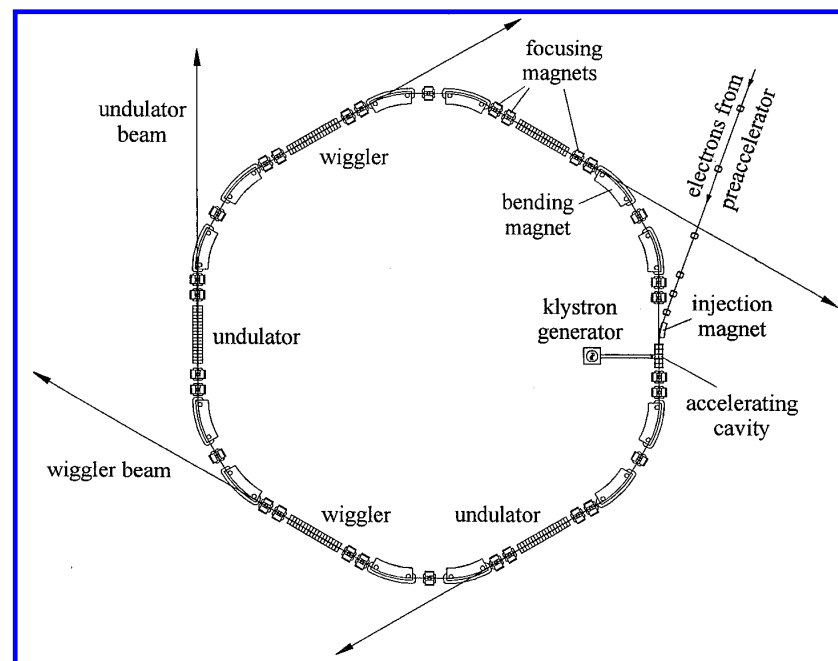
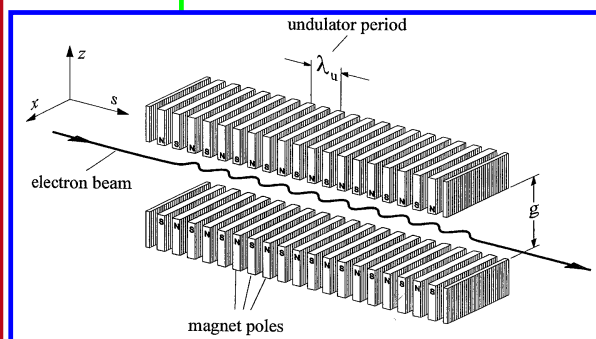
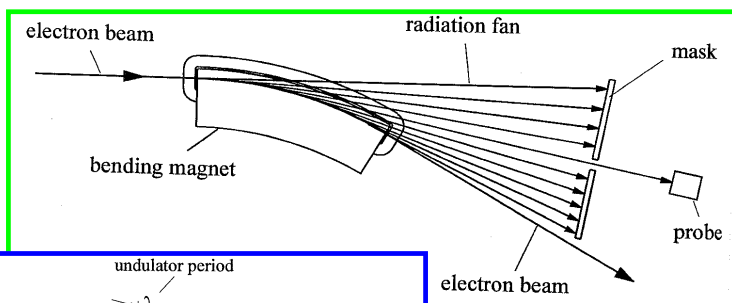
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4 Generations of Light Sources

- 1st Generation (1970s): Many HEP rings are parasitically used for X-ray production
- 2nd Generation (1980s): Many dedicated X-ray sources (light sources)
- 3rd Generation (1990s): Several rings with dedicated radiation devices (wigglers and undulators)
- Today (4th Generation): Construction of Free Electron Lasers (FELs) driven by LINACs



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