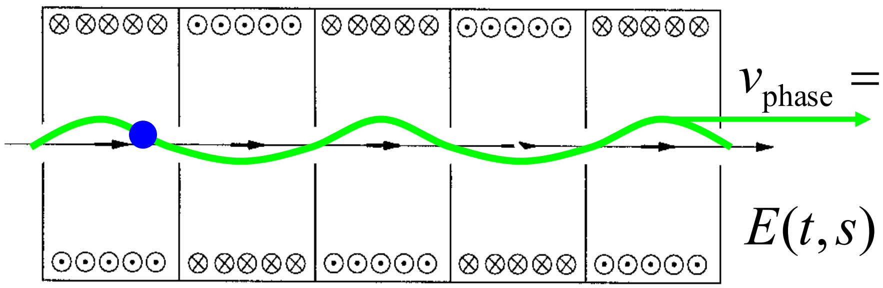
Accelerating cavities

- 1933: J.W. Beams uses resonant cavities for acceleration

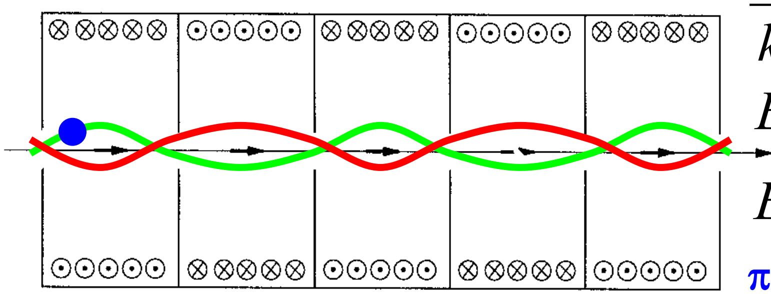
Traveling wave cavity:



$$v_{\text{phase}} = \frac{\omega}{k} = v_{\text{particle}}$$ Here v=c for electrons

$$E(t,s) \approx E_{\max} \sin(\omega t - ks)$$

Standing wave cavity:



$$\frac{\omega}{k} = v_{\text{particle}}$$

$$E(t,s) \approx E_{\max} \sin(\omega t)\sin(ks)$$

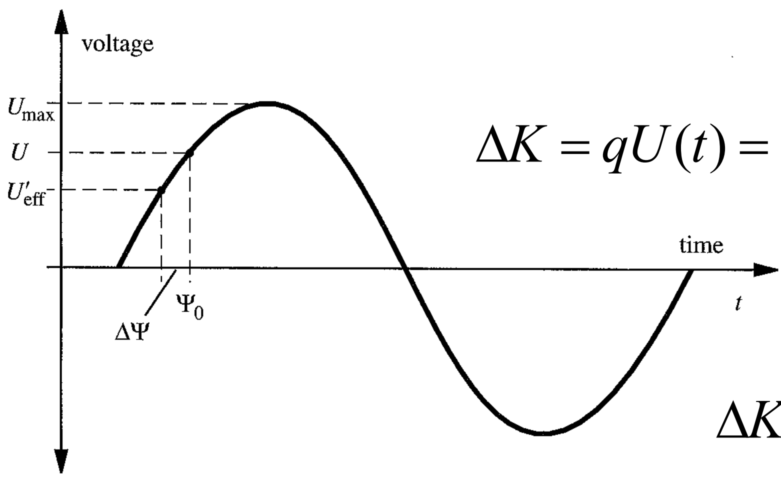
$$E(\tfrac{s}{v_{\text{particle}}}, s) \approx E_{\max} \sin^2(ks)$$

π or the 1/2βλ mode

Transit factor (for this example): $\langle E \rangle = \frac{1}{\lambda_{RF}} \int_0^{\lambda_{RF}} E(\tfrac{s}{v_{\text{particle}}}, s)\, ds = \frac{1}{2} E_{\max}$

Phase focusing

- 1945: Veksler (UDSSR) and McMillan (USA) realize the importance of phase focusing



$$\Delta K = qU(t) = qU_{max}\sin(\omega(t-t_0)+\psi_0)$$

Longitudinal position in the bunch:

$$\sigma = s - s_0 = -v_0(t-t_0)$$

$$\Delta K(\sigma) = qU_{max}\sin(-\tfrac{\omega}{v_0}(s-s_0)+\psi_0)$$

$\Delta K(0) > 0$ (Acceleration)

$\Delta K(\sigma) < \Delta K(0) \text{ for } \sigma > 0 \Rightarrow \frac{d}{d\sigma}\Delta K(\sigma) < 0$ (Phase focusing)

$$\left.\begin{aligned} qU(t) &> 0 \\ q\tfrac{d}{dt}U(t) &> 0 \end{aligned}\right\} \quad \psi_0 \in (0, \frac{\pi}{2})$$

Phase focusing is required in any RF accelerator.

Longitudinal phase space in a Linac

Other particles: $\frac{dE}{ds} = \hat{E}\cos\Phi$ Reference particle: $\frac{dE_0}{ds} = \hat{E}\cos\Phi_0$

$$\phi = \Phi - \Phi_0 = \omega(t - t_0)$$

$$\frac{d\delta}{ds} = \frac{\hat{E}}{E_0}\left(\cos(\Phi_0 + \phi) - \cos\Phi_0\right) \approx -\phi\frac{\hat{E}}{E_0}\sin\Phi_0$$

$$\frac{d\phi}{ds} = \omega\left(\frac{1}{v} - \frac{1}{v_0}\right) \approx \omega\left(\frac{1}{v_0 + \frac{dv}{d\delta}\big|_0 \delta} - \frac{1}{v_0}\right) \approx -\omega\frac{\frac{dv}{d\delta}\big|_0}{v_0^2}\delta = -\omega\frac{c^2}{v_0^3\gamma_0^2}\delta$$

$$\frac{dv}{d\delta} = E\frac{dv}{dE} = \gamma\frac{dv}{d\gamma} = c\gamma\frac{d\sqrt{1 - \frac{1}{\gamma^2}}}{d\gamma} = \frac{c}{\gamma^2\beta}$$

$$\frac{d^2\phi}{ds^2} \approx -\omega\frac{c^2}{v_0^3\gamma_0^2}\frac{d\delta}{ds} \approx \underline{\underline{\frac{\hat{E}}{E_0}\sin\Phi_0\omega\frac{c^2}{v_0^3\gamma_0^2}}}\phi$$

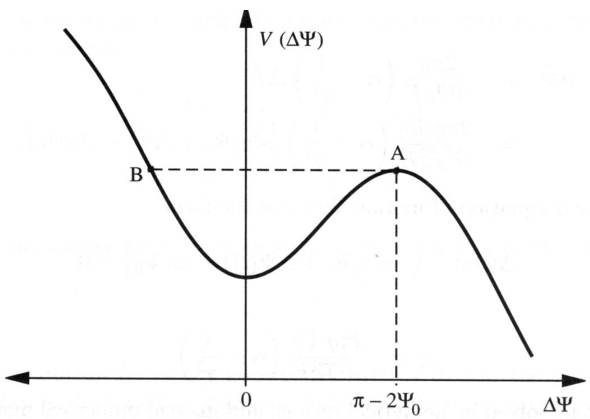
Stability for small phases when the factor on the right-hand side is negative.

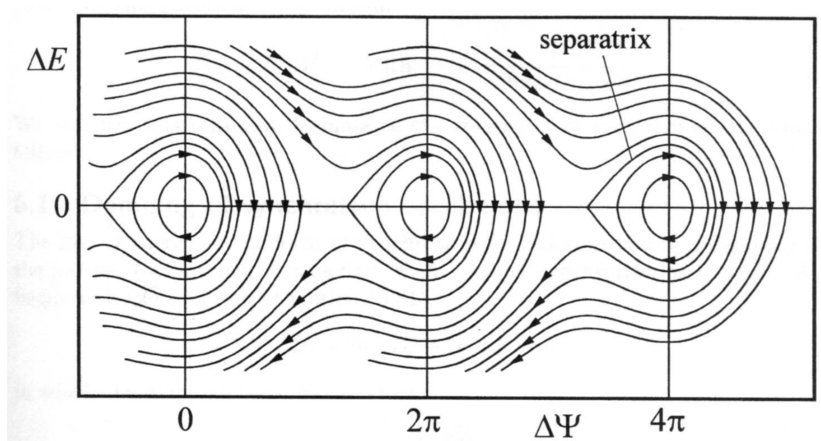
For not very small phases one cannot linearize.

$$\frac{d\delta}{ds}=\frac{\hat{E}}{E_0}\left(\cos(\Phi_0+\phi)-\cos\Phi_0\right) \qquad \frac{d\phi}{ds}\approx-\omega\frac{c^2}{v_0^3\gamma_0^2}\delta$$

$$H(\phi,\delta)=-\frac{q\overline{E_s}}{K_0}\left(\sin(\Phi_0+\phi)-\phi\cos\Phi_0\right)-\omega\frac{c^2}{v_0^3\gamma_0^2}\delta^2$$

$$\frac{d}{dt}\phi=\frac{\partial}{\partial\delta}H\ , \quad \frac{d}{dt}\delta=-\frac{\partial}{\partial\phi}H$$



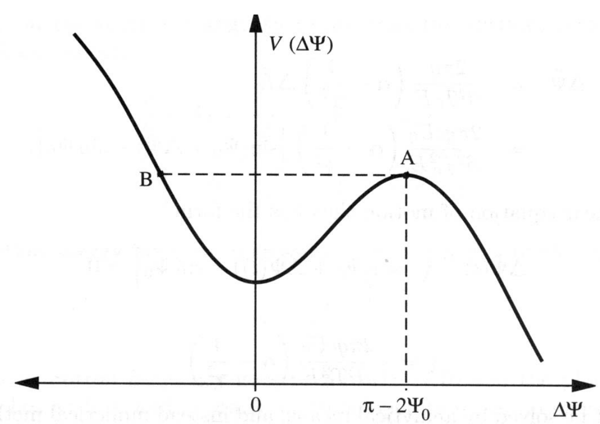


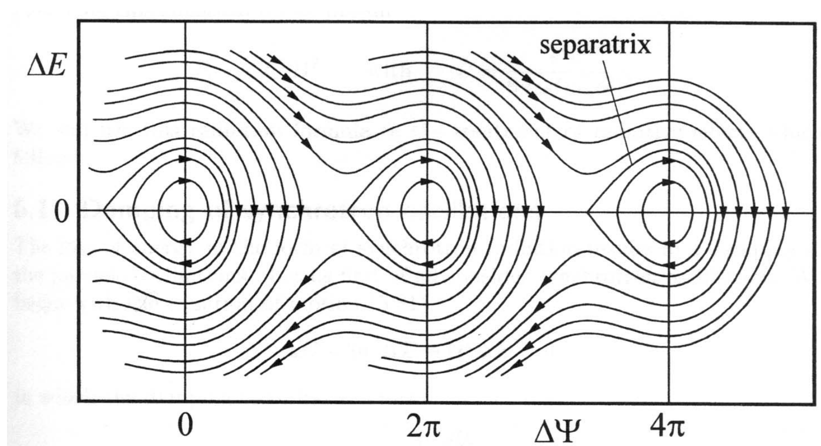
$$\frac{d\delta}{ds} = \frac{q\overline{E}_s}{K_0}\left(\cos(\Phi_0 + \phi) - \cos\Phi_0\right) \approx -\phi\,\frac{q\overline{E}_s}{K_0}\sin\Phi_0$$

$$\frac{d^2\phi}{ds^2} \approx -\omega\frac{c^2}{v_0^3\gamma_0^2}\frac{d\delta}{ds} = -\omega\frac{c^2}{v_0^3\gamma_0^2}\frac{q\overline{E}_s}{K_0}\left(\cos(\Phi_0 + \phi) - \cos\Phi_0\right)$$

$$= -\frac{d}{d\phi}\,\omega\frac{c^2}{v_0^3\gamma_0^2}\frac{q\overline{E}_s}{K_0}\left(\sin(\Phi_0 + \phi) - \phi\cos\Phi_0\right)$$

Effective potential





Georg.Hoffstaetter@Cor

Longitudinal phase space in rings

Reference particle: $\frac{dE_0}{ds} = \hat{E}\cos\Phi_0$

Other particles: $\frac{dE}{ds} = \hat{E}\cos\Phi$

$$\phi = \Phi - \Phi_0 = \omega(t - t_0)$$

$$\frac{d\delta}{ds} = \frac{\hat{E}}{E_0}\left(\cos(\Phi_0 + \phi) - \cos\Phi_0\right) \approx -\phi\frac{\hat{E}}{E_0}\sin\Phi_0$$

Momentum compaction $\alpha = \frac{p}{L}\frac{dL}{dp}$, closed orbit length change with momentum.

$$\frac{d\phi}{ds} = \omega(t - t_0) = \omega\left(\frac{L}{v} - \frac{L_0}{v_0}\right) = \omega\left(\frac{L_0}{v} + \alpha\frac{L_0}{p_0\, v_0}dp - \frac{L_0}{v_0}\right) = \frac{\omega L_0}{v_0}\left(\alpha\frac{dp}{p_0} - \frac{dv}{v_0}\right) = \frac{\omega L_0}{v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)\delta$$

Stability for small phases when the factor on the right-hand side is negative.

$$\frac{d^2\phi}{ds^2} = -\phi\frac{\omega L_0}{v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)sin\phi_0$$

Cornell University

Brookhaven National Laboratory

Transition energy and phase focusing

Stability for small phases when the factor on the right-hand side is negative.

$$\frac{d^2\phi}{ds^2} = -\phi \frac{\omega L_0}{v_0\, \beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right) sin\phi_0$$

For only natural ring focusing, $\rho \propto p \Rightarrow \alpha = 1$. For stronger focusing, $\alpha < 1$.

For small γ, $\alpha - \frac{1}{\gamma^2} < 0$ and $\phi_0 \epsilon [0, \pi]$ leads to a stable phase.

For large γ, $\alpha - \frac{1}{\gamma^2} < 0$ and $\phi_0 \epsilon [\pi, 2\pi]$ leads to a stable phase.

At the transition energy, one must jump the RF phase by π. This jump doers not have to happen very quickly, because at that energy, phases hardly change because of $\frac{d^2\phi}{ds^2} \approx 0$.

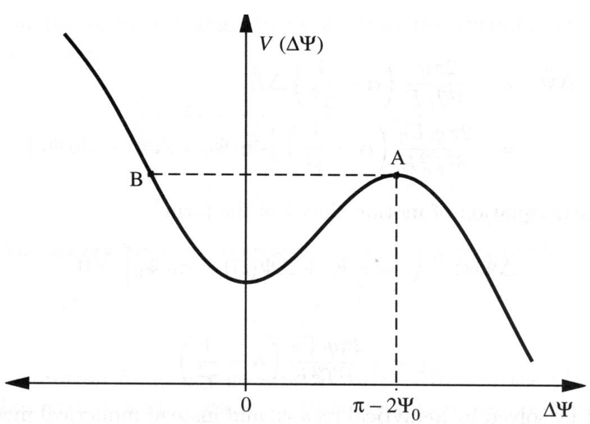
Accelerating longitudinal phase space

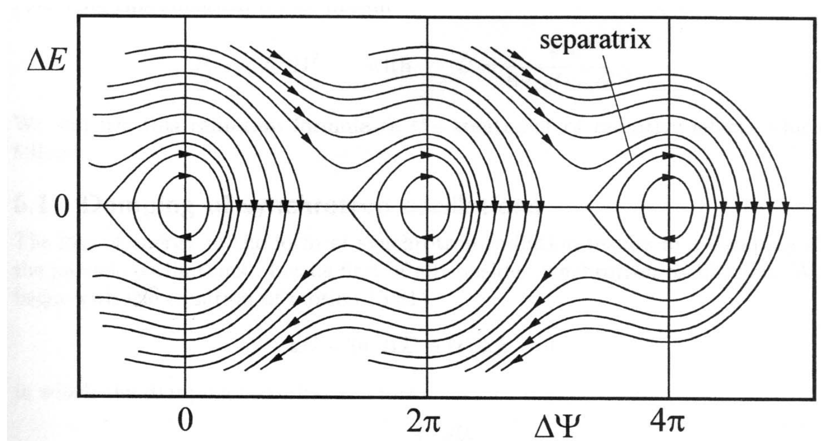
For not very small phases one cannot linearize.

$$\frac{d\delta}{ds} = \frac{\hat{E}}{E_0}\left(\cos(\Phi_0 + \phi) - \cos\Phi_0\right) \qquad \frac{d\phi}{ds} = -\frac{\omega L_0}{v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)\delta$$

$$H(\phi,\delta) = -\frac{qE}{E}\left(\sin(\Phi_0 + \phi) - \phi\cos(\Phi_0)\right) - \frac{\omega L_0}{2\,v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)\delta^2$$

$$\frac{d}{dt}\phi = \frac{\partial}{\partial\delta}H\,, \qquad \frac{d}{dt}\delta = -\frac{\partial}{\partial\phi}H$$





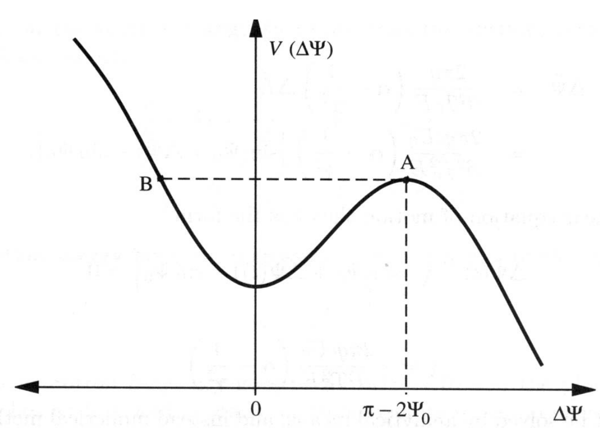
The effective accelerating potential

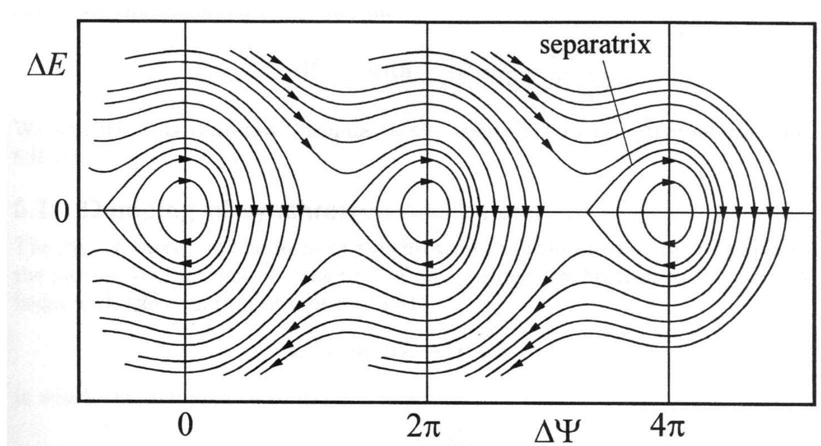
$$\frac{d\delta}{ds} = \frac{\hat{E}}{E_0}\left(\cos(\Phi_0 + \phi) - \cos\Phi_0\right) \qquad \frac{d\phi}{ds} = -\frac{\omega L_0}{v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)\delta$$

$$\frac{d^2\phi}{ds^2} = -\frac{\omega L_0}{v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{\hat{E}}{E_0}(\cos(\Phi_0 + \phi) - \cos(\Phi_0))$$

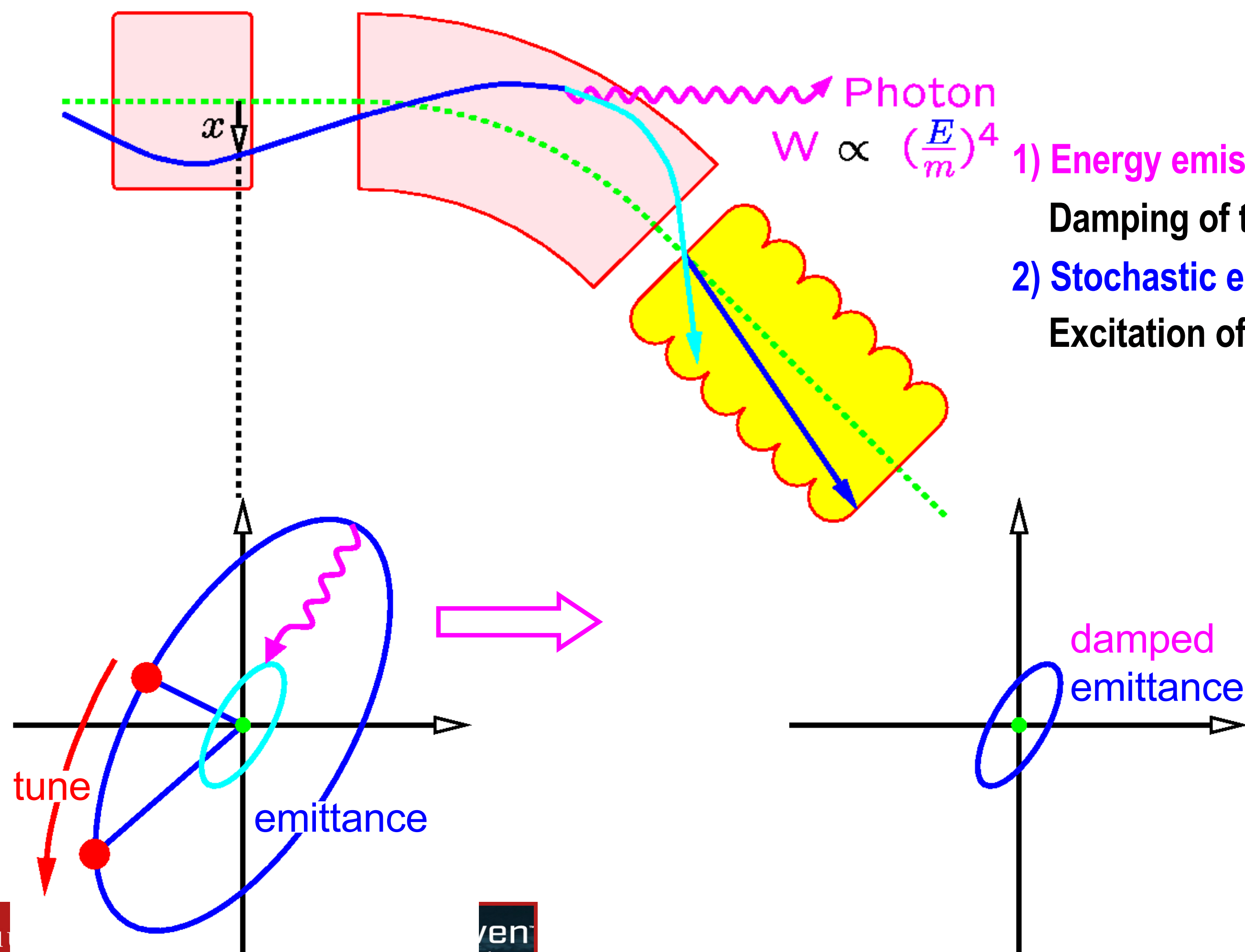
$$\frac{d^2\phi}{ds^2} = -\frac{\partial}{\partial\phi}\frac{\omega L_0}{v_0\,\beta_0^2}\left(\alpha - \frac{1}{\gamma^2}\right)\frac{\hat{E}}{E_0}(\sin(\Phi_0 + \phi) - \phi\cos(\Phi_0))$$

Effective potential



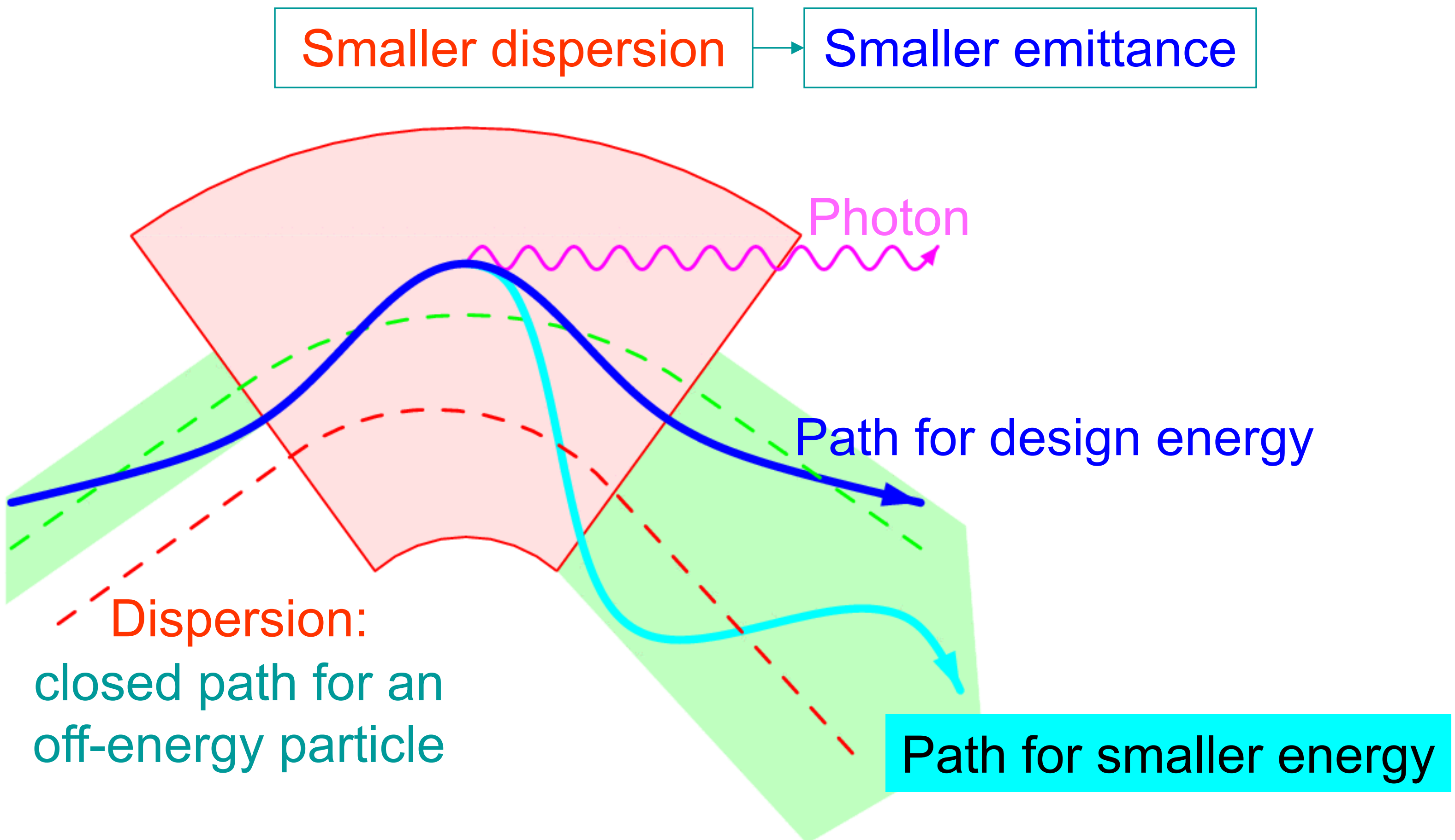


Radiative damping of the transverse emittance



1) **Energy emission:**
   Damping of the beam size
2) **Stochastic emission:**
   Excitation of the beam size

Radiative excitation of the transverse emittance



Radiative damping of the longitudinal emittance

1) Energy emission: Particles with larger energy radiate more, leading them closer to the average energy.

2) Stochastic emission: Random noise in the energy of emitted photons lead to an energy spread.

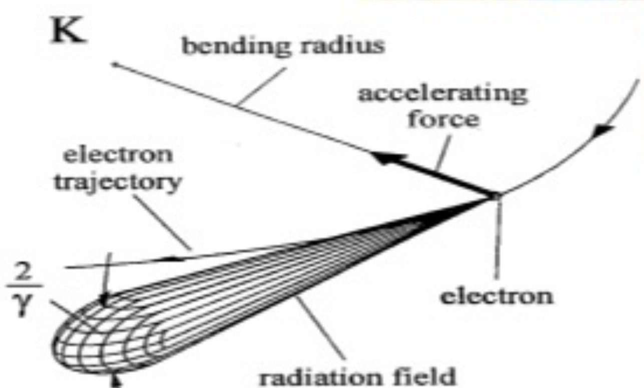
⇨

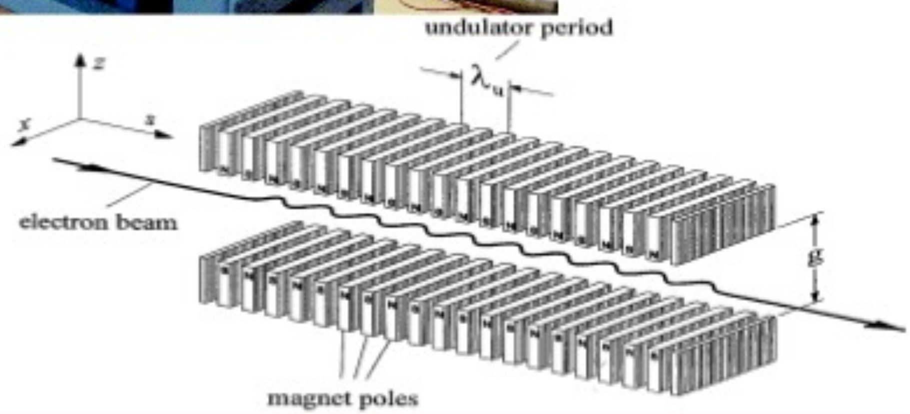
The equilibrium of the two effects leads to an equilibrium longitudinal emittance.

The damping time is the time it takes to radiate off the energy of the beam, while it is kept at constant energy with RF cavities. It is usually a few 100 revolutions.

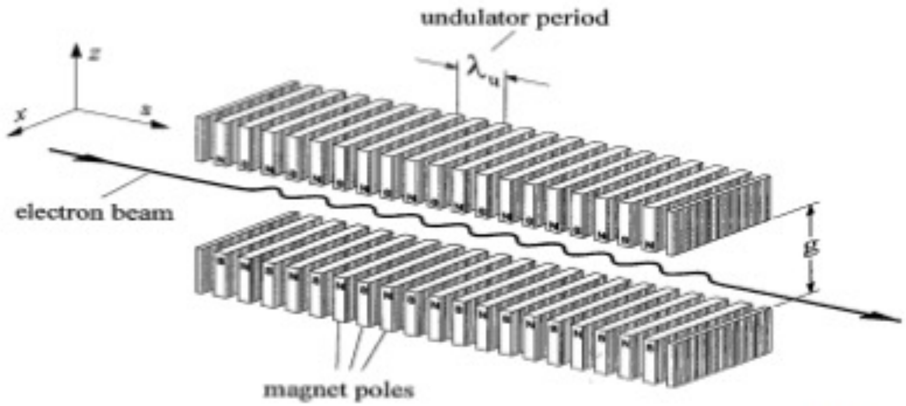
During this damping time the beam forgets its history, particle coordinates are reshuffled within the beam.

Radiative production by electrons

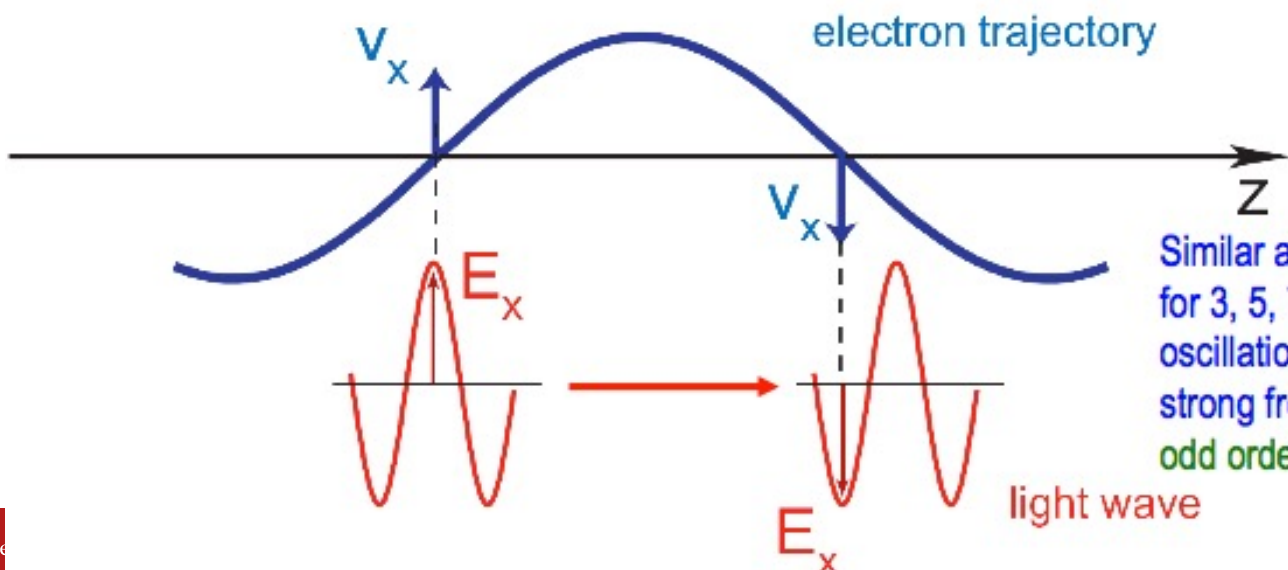




Radiative production in undulators

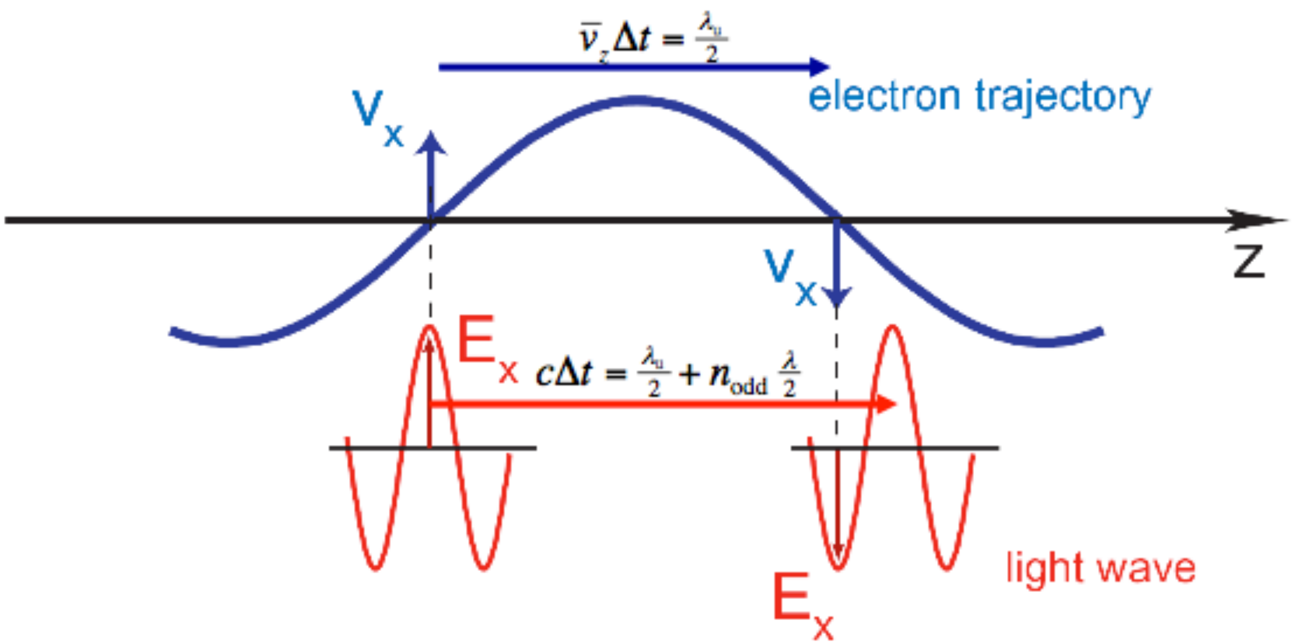


When the light wave passes the electron beam by half a wavelength per half beam oscillation, the radiation from each beam oscillation adds.



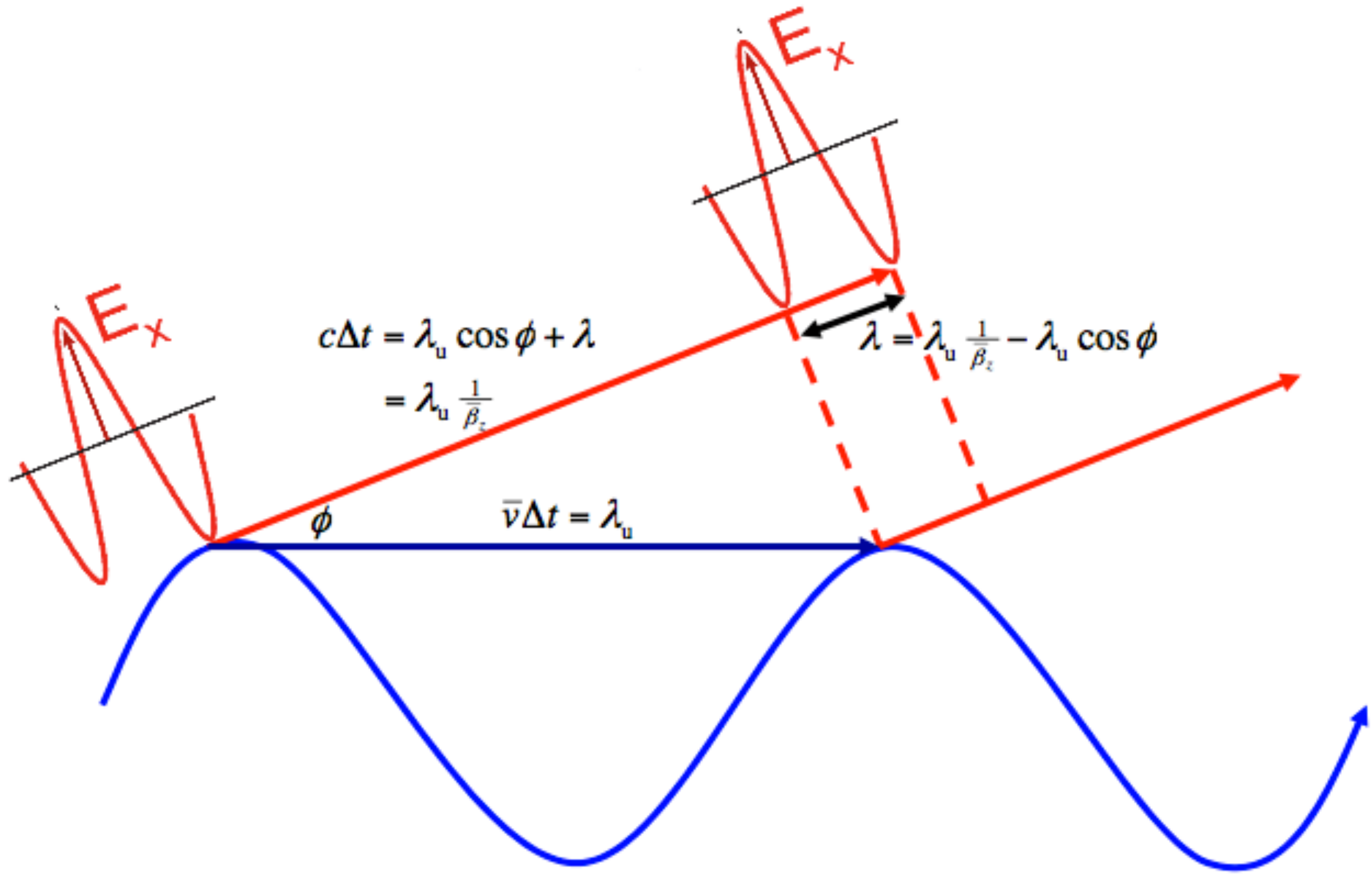
Similar adding can happen for 3, 5, 7, etc. half beam oscillations. For every strong frequency there are odd order harmonics.

Coherent addition of radiation



$$\frac{c}{\bar{v}_z}\frac{\lambda_u}{2} = \frac{\lambda_u}{2} + n_{odd}\frac{\lambda}{2} \quad \Rightarrow \quad \boxed{\lambda = \frac{1}{n_{odd}}\lambda_u\left(\frac{1}{\beta_z} - 1\right)}$$

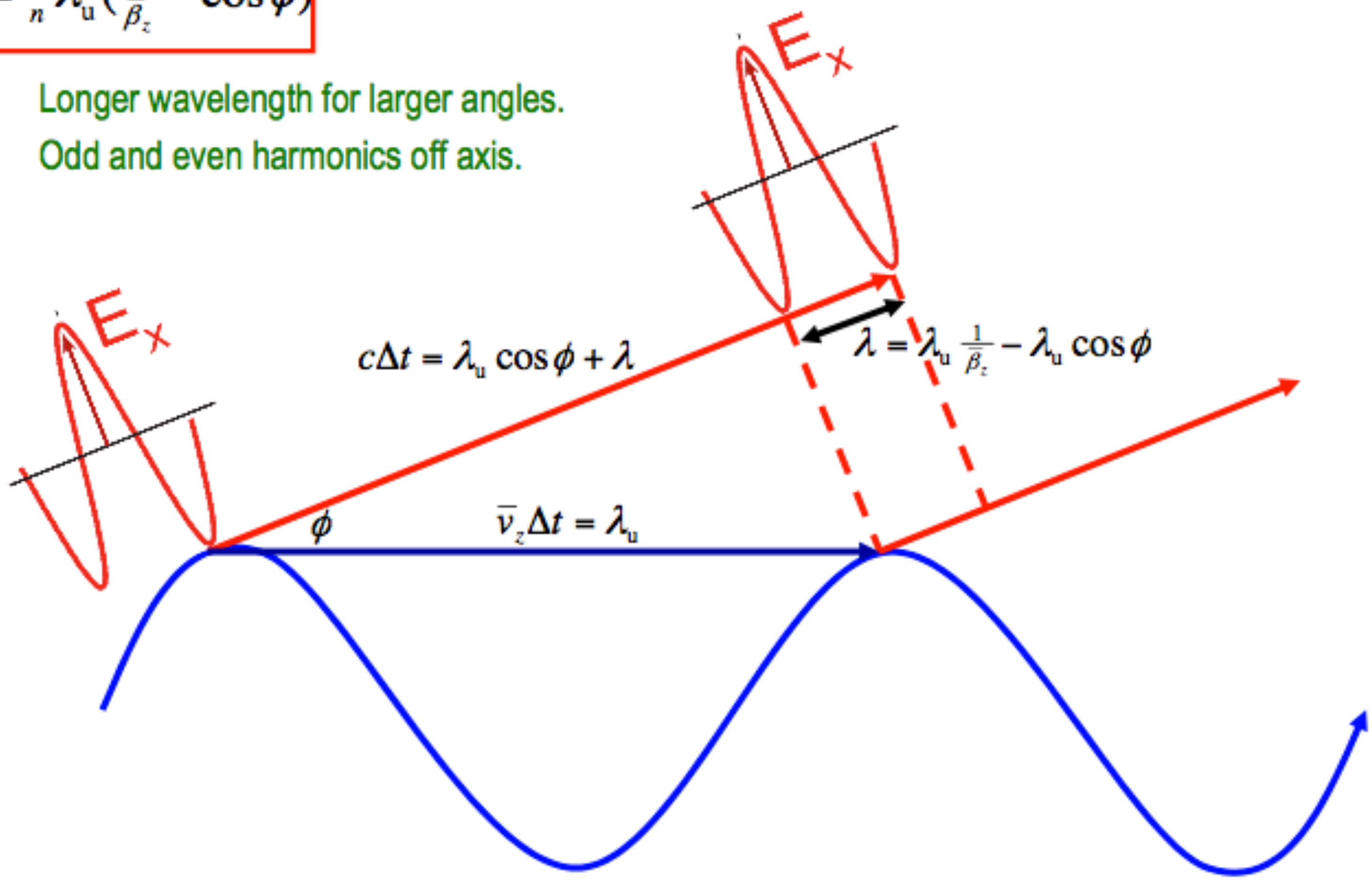
Coherent addition at angles



Radiation production at angles

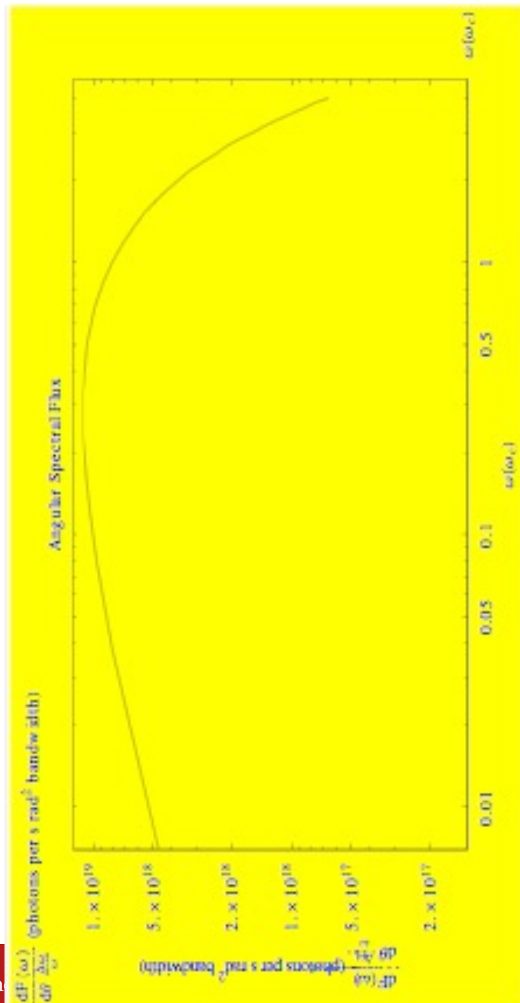
$$\lambda = \frac{1}{n}\lambda_u\left(\frac{1}{\beta_z} - \cos\phi\right)$$

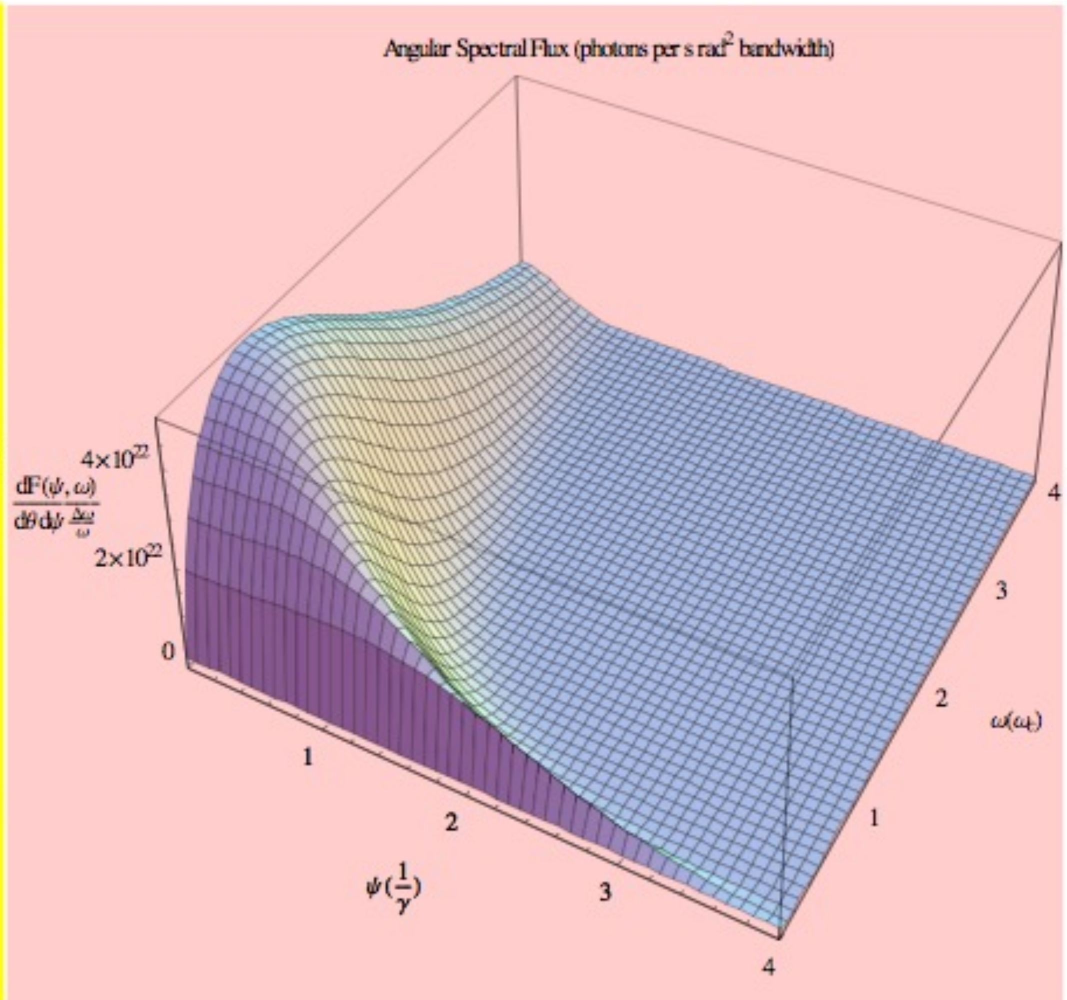
1) Longer wavelength for larger angles.
2) Odd and even harmonics off axis.



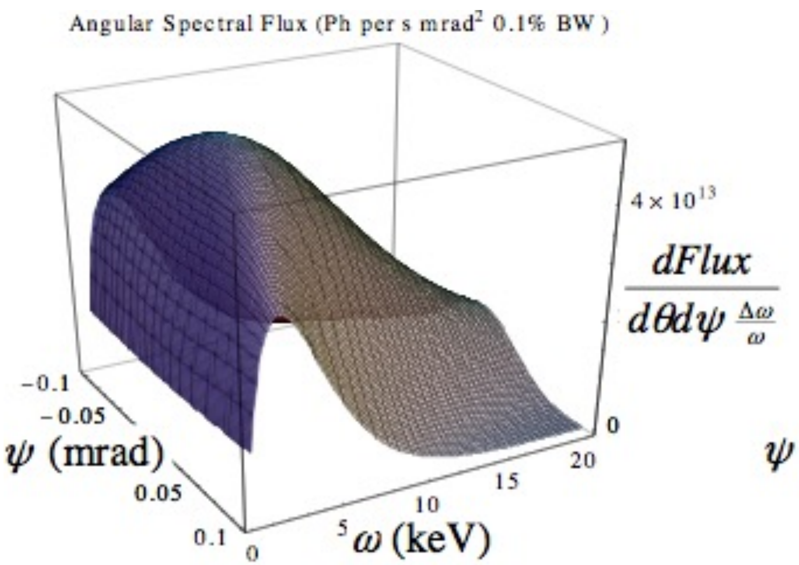
Lasing at the JLAB FEL

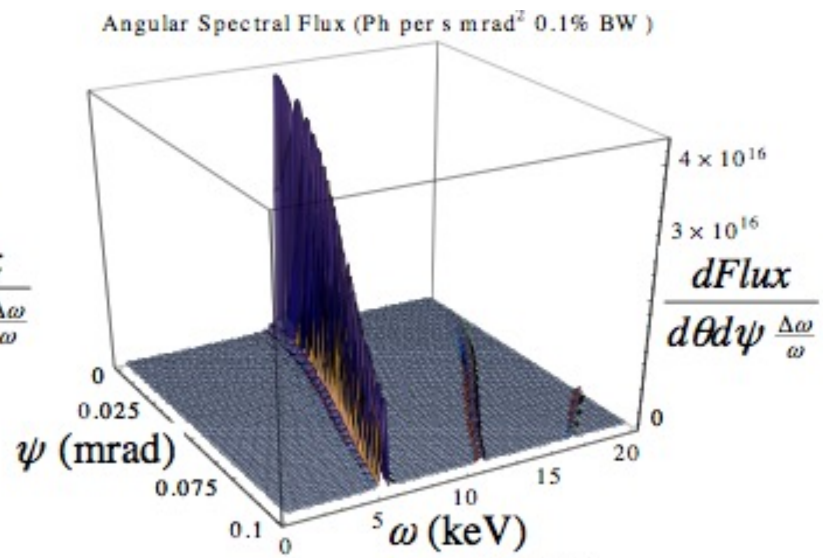
Radiative from bending magnets

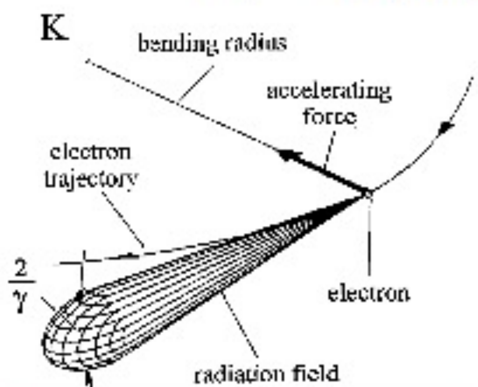


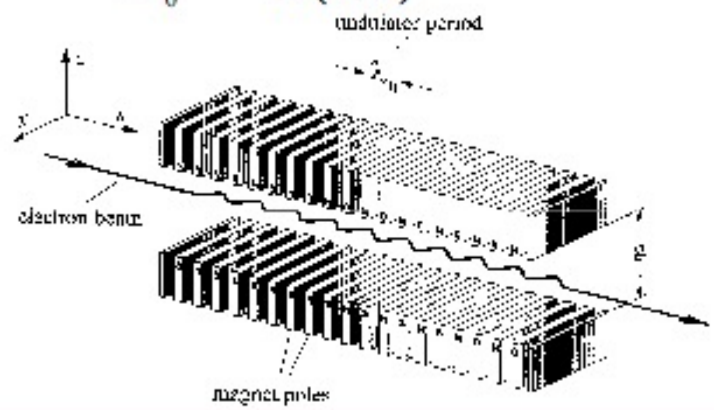


Photon flux from bends and undulators

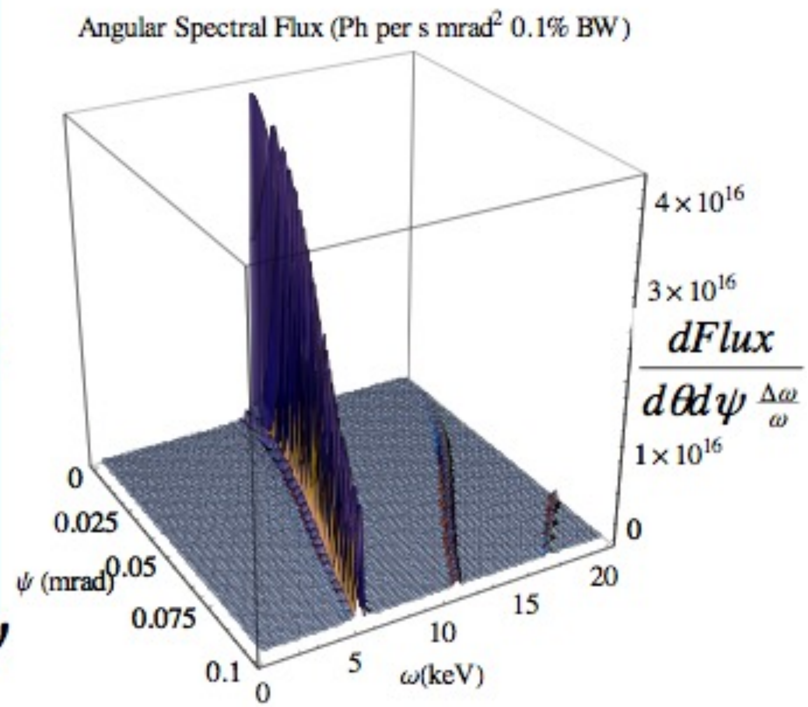








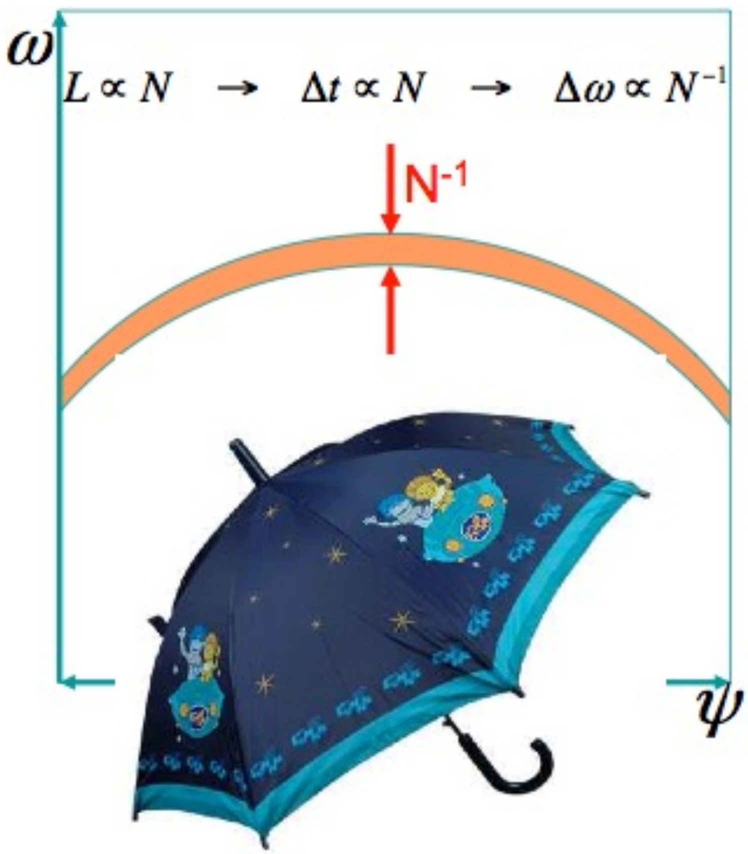
The umbrella of N-pole undulator radiation

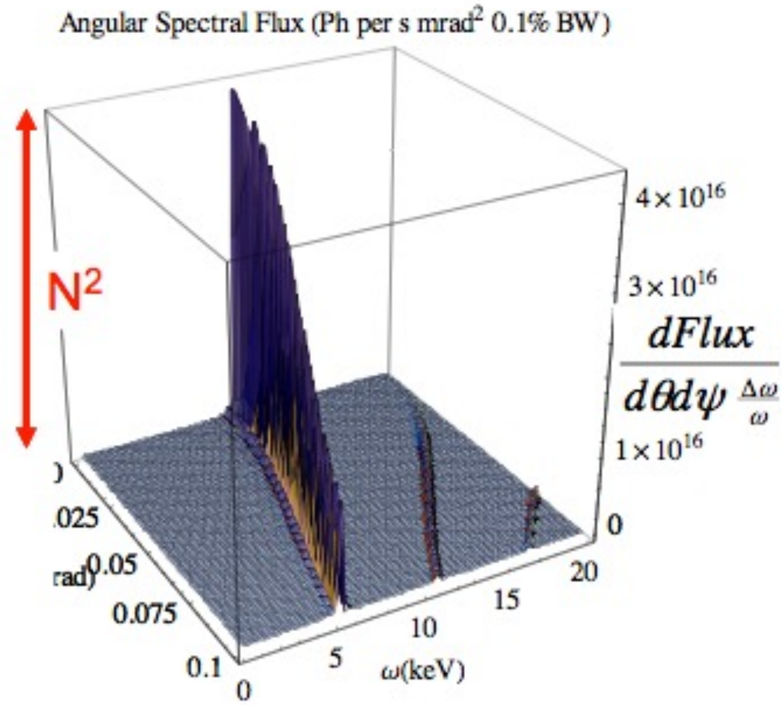


$$F_{total} \propto N$$

Flux from N poles is N times the flux from one pole

The umbrella of N-pole undulator radiation

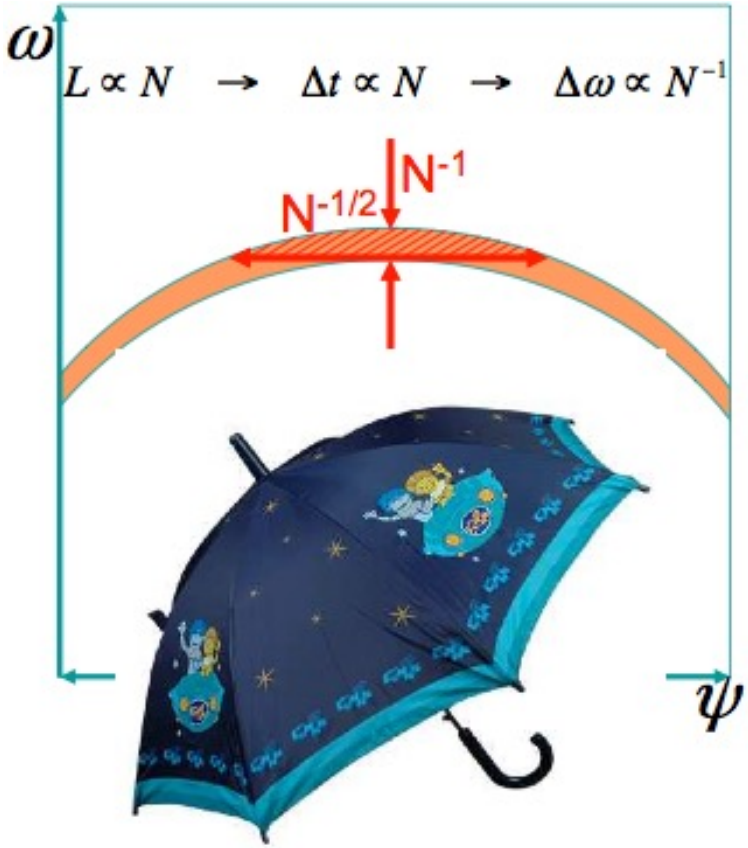


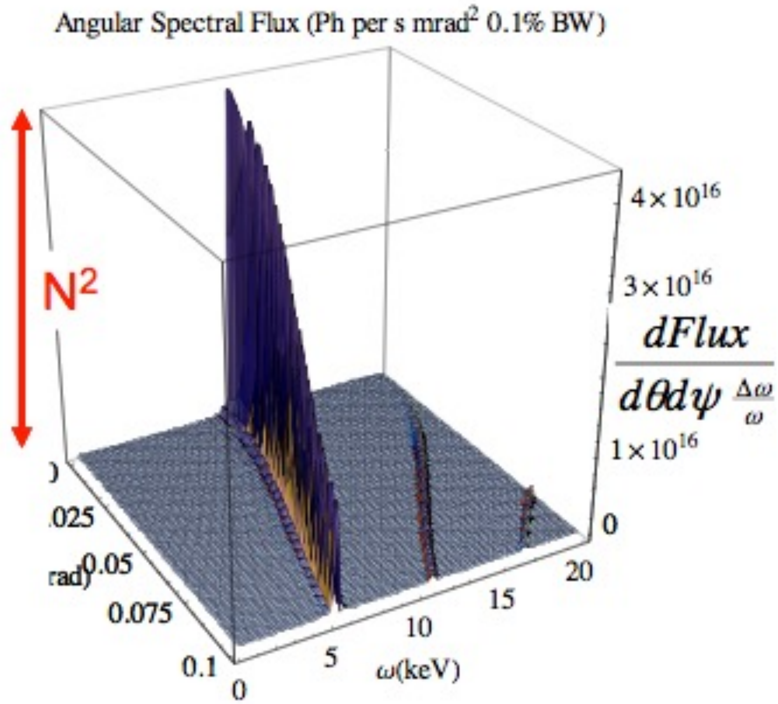


$$F_{total} \propto N$$

Flux from N poles is N times the flux from one pole

The umbrella of N-pole undulator radiation



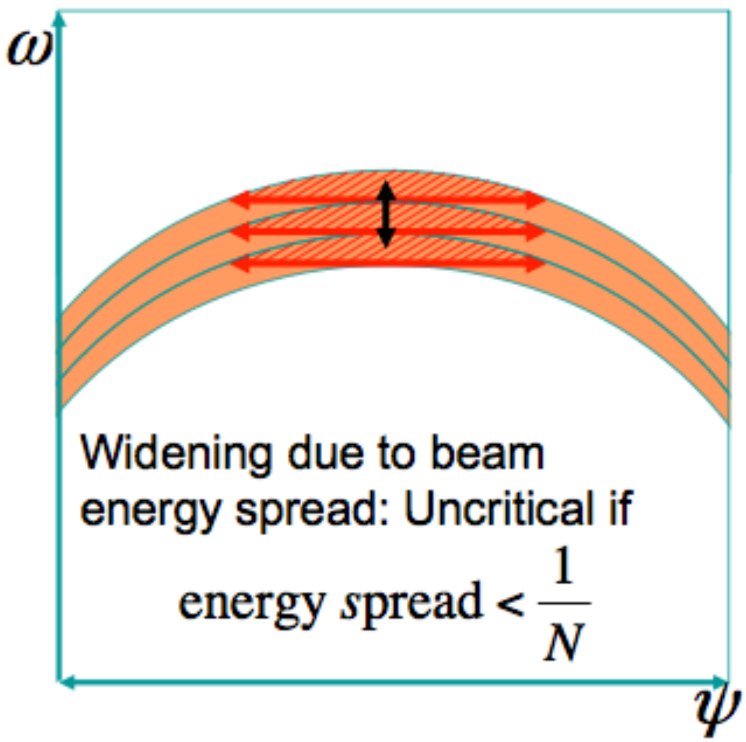


$F_{total} \propto N$

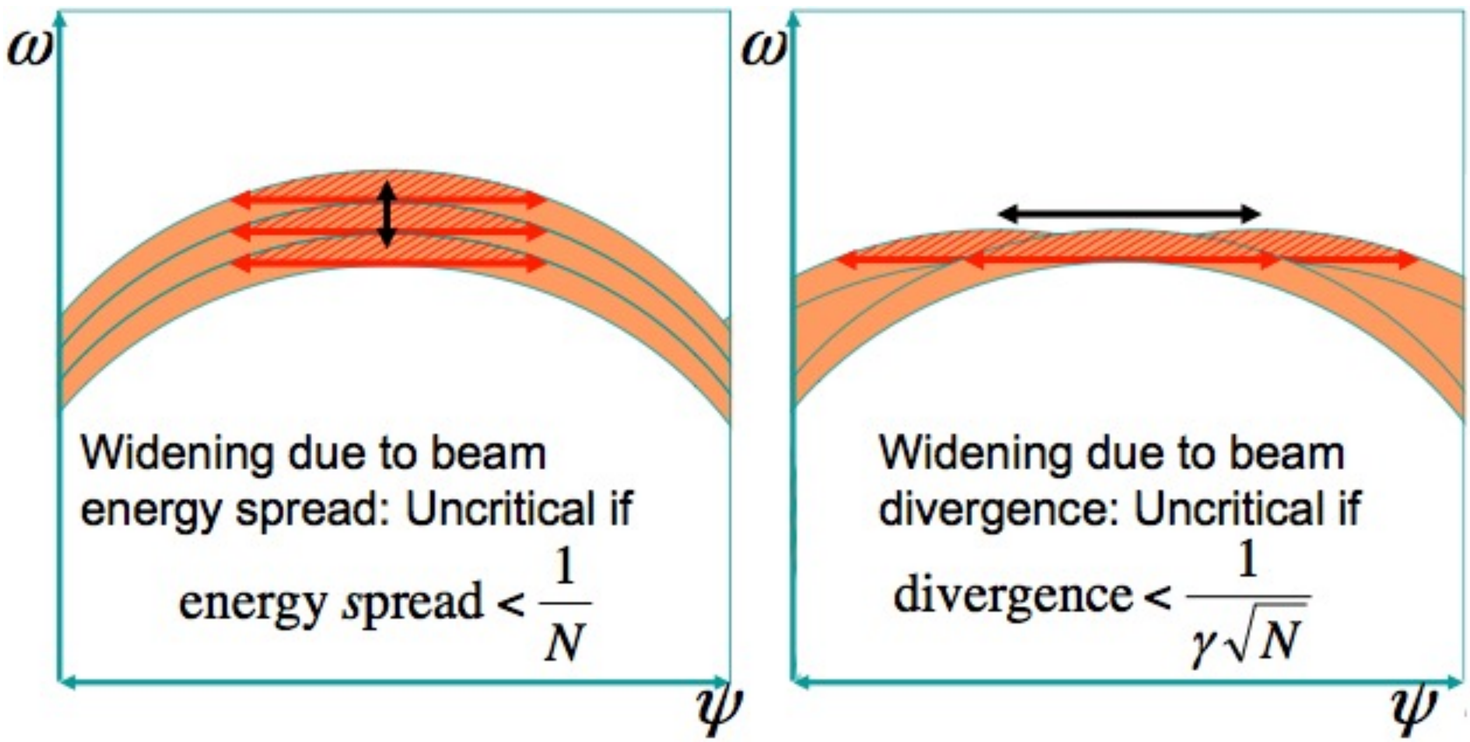
The power in the central cone is in dependent of N

Flux from N poles is N times the flux from one pole

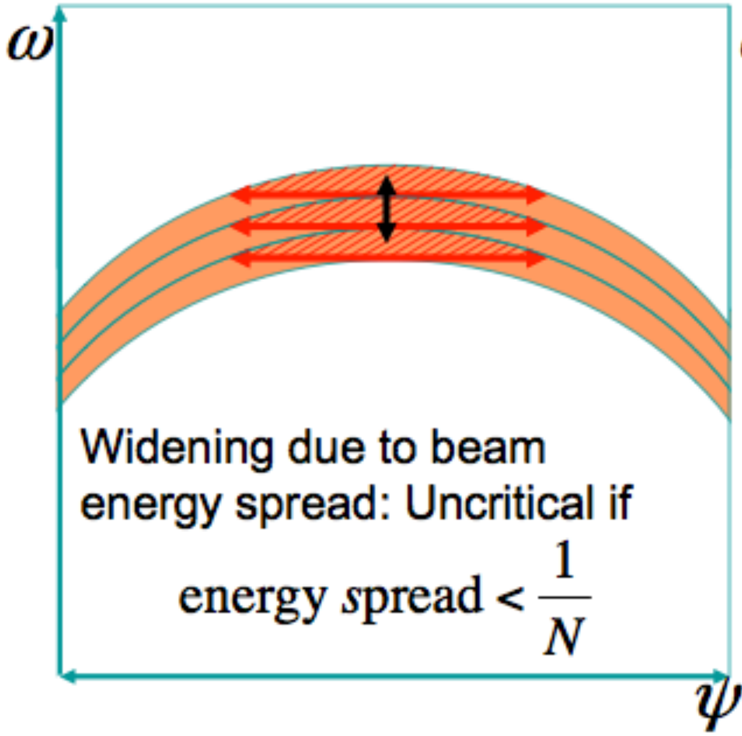
Brightness reduction by beam properties

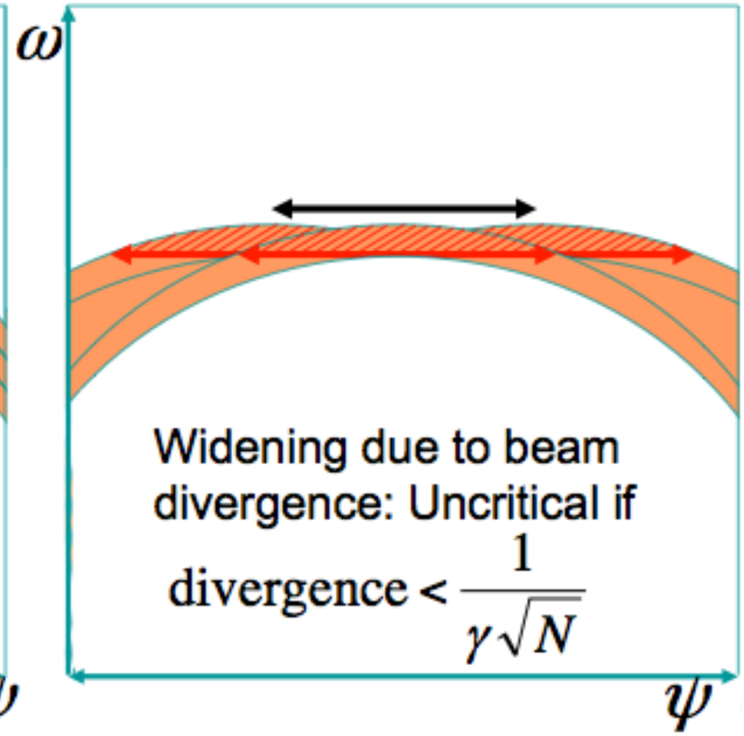


Brightness reduction by beam properties



Brightness reduction by beam properties

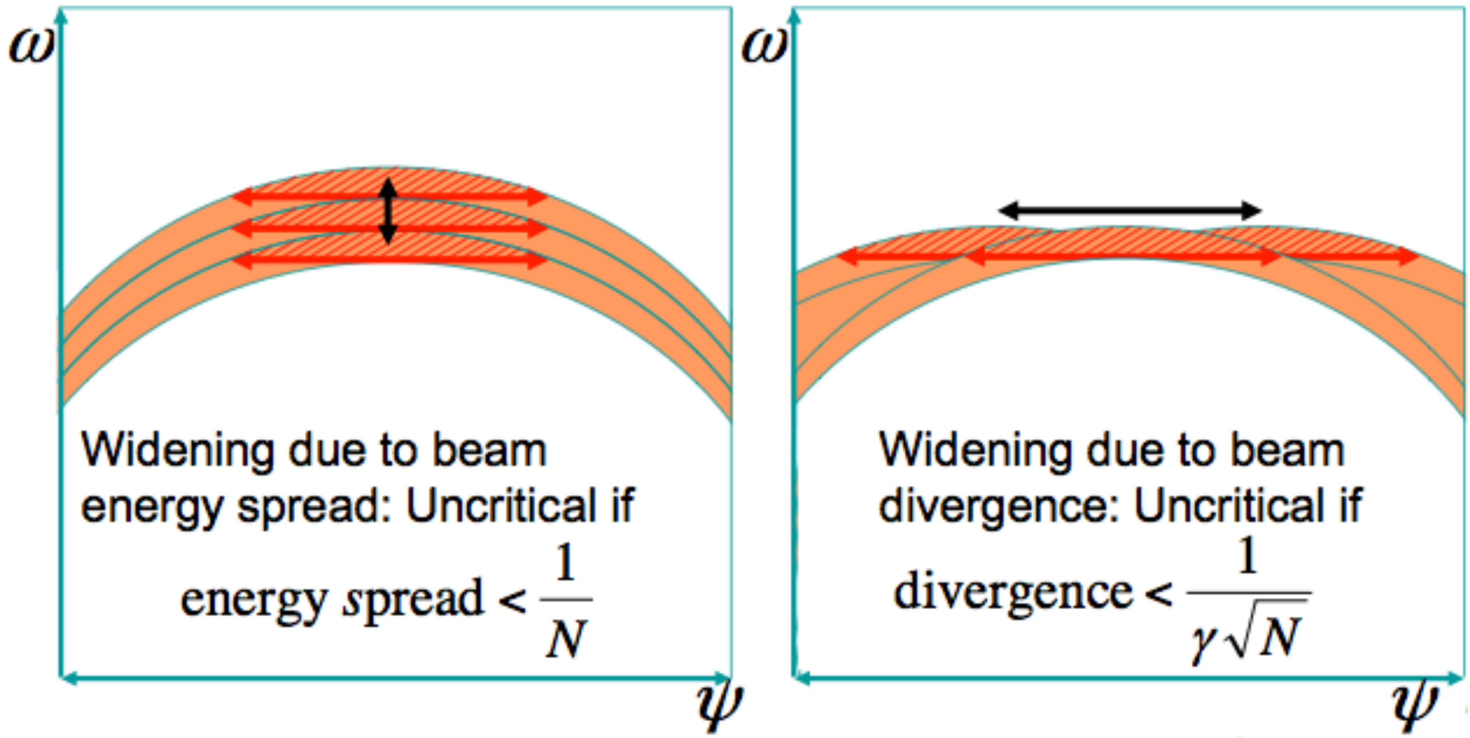




Field from a single electron cannot be distinguished from field from a spot with:

$$\text{spot size} < \frac{\lambda}{\text{divergence}}$$

Brightness reduction by beam properties



Field from a single electron cannot be distinguished from field from a spot with:

$$\text{spot size} < \frac{\lambda}{\text{divergence}}$$

To take advantage of many undulator poles, the electron beam needs to have little energy spread, little divergence, and small beam size.