Matching of Siberian Snakes

9 November 2002
AGS Polarization Workshop

\[ \nu = \frac{1}{2} \]

snake

snake
Driven spin perturbation on a trajectory

Integer values of spin-tune $n \pm \text{tune } n_y$ lead to coherent disturbances of spin motion.

Remedy:
Siberian Snakes avoid resonances by making the spin-tune $n = \frac{1}{2}$ independent of energy.

$$\phi_{\vec{S}} \propto \phi_{\vec{p}} \propto y = y_0 \sin(\psi_0 + nQ_y)$$

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Equation of motion for spin fields

\[
\begin{align*}
\frac{d}{d\theta} \vec{S} &= -\vec{\Omega}(\vec{z}, \theta) \times \vec{S} \\
\frac{d}{d\theta} \vec{f} &= \partial_\theta \vec{f} + [\vec{v}(\vec{z}, \theta) \cdot \partial_{\vec{z}}] \vec{f} = -\vec{\Omega}(\vec{z}, \theta) \times \vec{f}
\end{align*}
\]
A) Maximum polarization: \[ P_{\text{lim}} = \left< \vec{n}(\vec{z}) \right>_{\text{Phase space}} \]

For a large divergence, the average polarization is small, even if the local polarization is 100%.

B) \( \vec{n}(\vec{z}) \cdot \vec{S} \) is an adiabatic invariance!

C) \( \vec{n}(\vec{z}) \) Defines an amplitude dependent spin tune!
Low energies: First order theories agree
Medium energies: **resonances still isolated**
High energies: Resonances are no longer isolated. Isolated resonance model becomes invalid.
Polarized Deuterons

- Resonances are 25 times weaker and 25 times rarer for D than for p
- Transverse polarization could be achieved without Siberian Snakes
- Transverse RF dipoles could be used to rotate and stabilize longitudinal polarization
Siberian Snakes rotate spins at each energy $\frac{1}{2}$ times.

Freedom: direction of the rotation axis in the horizontal.
CO spin motion with 2N Siberian Snake

\[ A = \prod_{j=1}^{2N} ie^{-i\frac{\psi_j}{2}\sigma_3} (\sigma_1 \cos \alpha_j + \sigma_2 \sin \alpha_j) \]

\[ = i^N e^{-i\frac{\psi_{2N}}{2}\sigma_3} \prod_{j=1}^{N} (\sigma_1 \cos \alpha_{2j} + \sigma_2 \sin \alpha_{2j})(\sigma_1 \cos \alpha_{2j-1} + \sigma_2 \sin \alpha_{2j-1}) \]

\[ = i^N e^{-i\frac{\Delta \psi}{2}\sigma_3} \prod_{j=1}^{N} [\cos (\alpha_{2j} - \alpha_{2j-1}) - i \sin (\alpha_{2j} - \alpha_{2j-1})\sigma_3] \]

\[ A = i^N e^{-i\frac{\Delta \psi + 2\Delta \alpha}{2}\sigma_3} \]

\[ \Delta \psi = 0 \quad \text{, to make} \quad n_0 \text{ independent of energy} \]

\[ \Delta \alpha = \frac{\pi}{2} \quad \text{, to make} \quad n_0 = 0.5 \]

\[ \nu_0 = \frac{\Delta \psi + 2\Delta \alpha}{2\pi} \]

\[ \vec{n}_0 = \vec{e}_y \]
Some structure of the 1st order resonances remains after Siberian Snakes have been installed.
The spin tune deviates from $\frac{1}{2}$ for particles which oscillate around the design trajectory with amplitude $J_y$. 

\[ \nu = 1 - \nu_y \]
\[ \nu = 3 - 8\nu_y \]
\[ \nu = 5 - 15\nu_y \]
\[ \nu = 16\nu_y - 4 \]
\[ \nu = 2\nu_y \]
High Order Resonance Strength

The higher order Froissart-Stora formula

- Resonances up to 19th order can be observed
- Resonance strength can be determined from tune jump.

Tracked depolarization as expected

\[ P_{lim} = \langle \vec{n}(\vec{z}) \rangle \]

Spin tune

Computations performed in SPRINT, Hoffstaetter and Vogt, DESY/00

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1st Order: 4 harmonics of the spin perturbation in each section.

With 4 snakes only **2** can be compensated.

With 8 snakes all **4** can be compensated.
$P_{lim}$ after Snake Matching

8 matched snakes

8 snakes standard scheme
Spin Tune after Snake Matching

\[ n = Q_y^2 - \frac{1}{2} \]

4 snakes in standard scheme

8 matched snakes
Matching the betatron phases

\[ \Delta \Psi_{24} = 2(\varphi_W - \varphi_N) \]

\[ \Delta \Psi_{13} = 2(\varphi_N - \varphi_E) \]

A proper choice of betatron phase advances allows snake matching with 4 snakes.
Spin Tune after Snake Matching

\[ \nu = 2Q_y \]

\[ \nu = 1 - 2Q_y \]

4 snakes in standard scheme

4 matched snakes
Snake matching allows to have significantly larger beams.
TESLA with Röntgen FEL

- Superconducting Electron Linac
- Detector and Experimental Area
  - Wiggler for the Positron Source
  - Cryogenic Halls
- Superconducting Positron Linac
- Tunnel
- Damping Ring
- Röntgen FEL
- HERA