# **BEAM-BEAM LIMIT WITH HOURGLASS EFFECT IN HERA**

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#### Abstract

After the design luminosity had been exceeded in the electron-proton collider HERA, the interaction regions have been rebuilt to boost the luminosity by roughly a factor 4 and are now being commissioned. While the specific luminosity is increased by a reduction of the beta functions at the interaction point and by a reduced horizontal electron emittance, the beam-beam tune shift of the e-beam will be increased and the vertical proton beta function at the interaction point will become comparable to the proton bunch length. To analyze these new beam dynamical conditions, the beam-beam force's effect on the tune spread and on high order resonances under the presence of the hourglass effect has been investigated, and accelerator studies have been performed in conditions for which the tune spread and the resonance strength are comparable to the luminosity upgrade parameters. In these experiments the nominal beam-beam parameter per experiment has been pushed to a record high of 0.5. Based on these tests, the future performance limits of the HERA e/p collider are discussed.

#### **1 LUMINOSITY UPGRADE**

With a luminosity of  $\mathcal{L} = 0.2 \times 10^{32} \text{ cm}^{-2} \text{s}^{-1}$  and an integrated luminosity of 67 pb<sup>-1</sup>, HERA has exceeded its design luminosity in its phase 1 run period which ended in September 2000. To increase the luminosity in a phase 2 period, an upgrade of the interaction region (IR) has been implemented. The North and South IRs around the colliding beam experiments ZEUS and H1 have been rebuilt [1, 2, 3] to move superconducting focusing magnets for electrons or positrons from 7 m to 2 m distance from the IP and to move the first focusing magnets for protons from 28 m to 11 m. The proton beam can now be strongly focused to  $\beta_x^* = 2.45$  m and  $\beta_y^* = 0.18$  m at the IP. The previous values have been  $\beta_x^* = 7$  m and  $\beta_y^* = 0.5$  m. The new nominal HERA parameters are:

	р	e
Energy–p/e (GeV)	920	27.5
Emit. hor/vert (nm)	5.1/5.1	20/3.4
$\beta^*$ at IP hor/vert (m)	2.45/0.18	0.63/0.26
Aperture hor/vert ( $\sigma$ )	12/12	20/20
p per bunch and e-cur.	$1.03 \cdot 10^{11}$	58 mA
Tune shift hor/vert $(10^{-3})$	1.6/0.4	34/51
Bunch Length (mm)	191	10.3

These parameters would lead to a luminosity of  $0.74 \cdot 10^{32} \text{ cm}^{-2} \text{s}^{-1}$  which includes a reduction by 0.92 due to the increase of the beta functions along the proton bunch during collision (the hourglass effect).

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# 2 INFLUENCE OF THE HOURGLASS EFFECT



Figure 1: The bunch size changes during the collision due to the hourglass effect.

Figure 1 shows how the proton beam size during an electron-proton collision can change when the bunch is longer than the beta function at the interaction point (IP). For short Gaussian bunches the luminosity is given by  $\mathcal{L} = f_b N_+ N_- / [2\pi \Sigma_x \Sigma_y]$  where the  $\Sigma$  are given by  $\sqrt{\sigma_1^2 + \sigma_2^2}$ . Therefore the hourglass effect diminishes the luminosity according to

$$\frac{\mathcal{L}^{\text{hg}}}{\mathcal{L}} = \iint_{\infty}^{\infty} \rho_1(s_1) \rho_2(s_2) \left. \frac{\Sigma_x(0) \Sigma_y(0)}{\Sigma_x(s) \Sigma_y(s)} \right|_{s=\frac{s_1+s_2}{2}} ds_1 ds_2 \,.$$
(1)

Here  $\rho_{1/2}$  are the longitudinal densities of the two beams. For Gaussian longitudinal densities, one integration can be performed leading to  $\frac{2}{\sqrt{2\pi\Sigma_s}}\exp(-\frac{2s+\xi}{2\Sigma_s^2})ds$  instead of  $\rho_1(s_1)\rho_2(s_2)ds_2ds_2$  where  $\xi$  is the distance between the bunches when one of them is at the IP. Well adjusted timing of the collisions leads to  $\xi = 0$ . If the hourglass effect is only important in one plain, also the second integral can be computed analytically [4]. The effect of this luminosity reduction on luminosity scans has been analyzed in [5]. The beam-beam parameter is given by  $\xi_{x,y} = C_{bb}\beta_{x,y}/[2\pi(\sigma_x^* + \sigma_y^*)\sigma_{xy}^*]$  with  $C_{bb} = \frac{q^*N^*r_c}{q\gamma}$ . The star refers to the opposing beam,  $r_c$  is the classical radius of the particle and  $N^*$  is the numbers of particle per bunch. The hourglass effect changes this tune shift,

$$\frac{\xi_{x/y}^{\rm hg}(s_1)}{\xi_{x/y}} = \int_{-\infty}^{\infty} \rho_2(s_2) \frac{\beta(s)\sigma_{x/y}^*(0)[\sigma_x^*(0) + \sigma_y^*(0)]}{\beta(0)\sigma_{x/y}^*(s)[\sigma_x^*(s) + \sigma_y^*(s)]} ds_2 ,$$
(2)

where again  $s = \frac{s_1+s_2}{2}$ . The tune shift has an increasing and a decreasing component. The beta function increases with distance from the IP whereas the inverse beam size of the opposing beam decreases with this distance. When averaging the tune shift over the interaction time, it can therefore be larger or smaller than the tune shift without hourglass effect. Furthermore the tune shift depends on the particles longitudinal position in a bunch and therefore oscillates relatively slowly with the synchrotron frequency giving rise to synchrotron sidebands. Also the beam-beam potential changes due to the hourglass effect,

$$U^{hg}(Jx, J_y, \phi_x, \phi_y, s_1) = \frac{C_{bb}}{2\pi} \int_{-\infty}^{\infty} \rho_2 U^{\sigma_x^*(s), \sigma_y^*(s)}(x, y) ds_2 ,$$
(3)

with  $x = \sqrt{2J_x\beta_x(s)}\sin(\phi_x + \delta\phi_x(s))$ ,  $y = \sqrt{2J_y\beta_y(s)}\sin(\phi_y + \delta\phi_y(s))$ ,  $U^{\sigma_x^*,\sigma_y^*}(x,y)$  is the conventional beam-beam potential and  $\delta\phi(s) = \int_0^s [\frac{1}{\beta_x(s)} - \frac{2\pi Q_x}{L}]ds$  takes into account that the betatron phase changes during the interaction [6]. For Gaussian beams one can derive the Fourier expansion of this potential as a series of products of Bessel functions [7, 8]. The influence of the hourglass effect is then computed by integrating  $s_2$  over these Bessel functions. The 0th Fourier coefficient of this expansion leads to the amplitude dependent tune shift and the higher order components lead to the resonance strength.

These resonance strengths depend on the longitudinal position  $s_1$  of the particles in the bunch. Here we do not compute the strength of synchrotron sideband resonances, but we compute the maximum of the resonances along the bunch since this describes the maximum transverse resonance strength that a particle suffers for an appreciable amount of time. The formulas for the resonance strength contain high powers of the beta functions and some of the resonances can therefore strongly increase due to the hourglass effect, especially for particles in the head or tail of the proton bunch.

In the calculations which will be mentioned below, all integrals were first simplified by assuming Gaussian longitudinal bunch profiles and by expanding in Bessel functions, then numerical integration was performed without any approximation on the betatron phase variation over the bunch length or about the bunch shape for each electron or proton bunch in HERA [9].

### **3** ACCELERATOR STUDIES

Before displaying results of these computations for HERA's phase 2 we will show performance experience at extreme operation conditions. We will then compare these conditions with phase 2 parameters. The new low emittance optics of the leptons with a betatron phase advance of  $72^{\circ}$  per FODO cell and an rf frequency shift of 300 Hz has been implemented and tested [10, 11]. An emittance value of 22 nm and the corresponding increase in specific luminosity was verified. The normalized dynamic aperture was measured and polarization of was obtained. Consequently the design value of the specific luminosity has been reached rather early in the commissioning process of phase 2.

In a beam-beam study, the protons were collided with lepton bunches of different intensities inducing 180% of



Figure 2: Top: Measured emittances of the lepton beam as a function of the vertical beta function. Bottom: Measured and computed specific luminosity.

the proton beam-beam tune shift values expected for the upgraded optics [12]. While there was an enhancement of a factor 2–4 in tail population  $(x > 5\sigma_{px})$ , the core of the proton beam and therefore the specific luminosity remained unaffected during 10 h of collisions. We thus conclude that there should be a sufficient margin in the proton beam-beam tune shifts.

The vertical beam-beam tune shift of the lepton beam will increase by 50% compared to the year 2000 operation. Therefore, the tune shift limit has been explored by increasing the lepton  $\beta_y^*$  [10, c]. Values of  $\beta_{ex}^* = 2.5$  m (corresponding to the expected horizontal tune shift for 140 mA of protons) and  $\beta_{ey}^*(\mathbf{m}) \in \{1, 1.5, 2, 3, 4\}$  have been implemented and collided with a 90 mA proton beam. The beam sizes were not matched. We observed a monotonically increasing blow-up of the vertical lepton emittance with increasing  $\beta_y^*$  by factors up to 8 as shown in figure 2 (top). At the value of  $\beta_{ey}^* = 1.5$  m which corresponds to a beam-beam tune shift of 0.069, the blow-up factor was 1.5. Figure 2 (bottom) shows the measured specific luminosity and the luminosity computed from the measured beam emittances. These measurements justify that no emittance blow-up should be expected for the nominal values of the upgraded lattice. The lifetime and operation conditions were good even with the tune shift of 0.5. This is probably due to a depopulation of the bunch center, which is constant with the fact that the luminosity at large  $\beta_{ey}^*$ was smaller than computed from the measured emittances. Additionally the shift in the coherent oscillation frequency for  $\beta_{ex}^* = 2.5$ m and  $\beta_{ey}^* = 4$ m was measured to be only  $\Delta \nu_x = 0.005 \text{ and } \Delta \nu_y = 0.071.$ 

### **4 MAXIMAL HOURGLASS EFFECT**

The upgraded lattice provides a physical aperture of  $12\sigma$  for the protons, while it has been  $10\sigma$  before the upgrade. The  $\beta$ -functions of the protons at the IP could thus be some 30% smaller than the nominal values yielding  $\beta_{px}^* = 170$  cm,  $\beta_{py}^* = 12.5$  cm. Thus, the new lattice permits proton  $\beta^*$ -functions as small or even smaller than the bunch length of the protons. The proton bunch length is ultimately limited by intra-beam scattering and grows slowly during the storage time. Currently it's RMS value is about 15 cm at the beginning of a store and reaches 28 cm after 10 h storage. At this limit not only the hourglass effect on tune shift and resonance strength can become disturbing. Additionally, the horizontal beam size of the proton beam becomes smaller than that of the electrons, and that enhances nonlinear effects on the electron beam.

For HERA the beam–beam tune shift for particles in the center of the bunch and at the  $5\sigma$  head or tail of the bunch are shown with the initial parameters for which HERA is being comissioned. These are the nominal parameters with currents reduced to  $0.73 \cdot 10^{11}$  protons per bunch and 50 mA of positrons. These shifts can be compared to those for the ultimate parameters with the smallest possible  $\beta^*$  and small proton emittances and to those for the extreme conditions of the performed accelerator studies. In the following table the tune shifts computed without consideration of the hourglass effect are indicated by an index 0.

	$\Delta \nu_{x0}$	$\Delta  u_x^{5\sigma}$	$\Delta \nu_{y0}$	$\Delta  u_y^{5\sigma}$
initial p	0.0016	0.00081	0.00044	0.00011
ultimate p	0.0022	0.00060	0.00059	0.00147
studies p	0.0022	0.0017	0.00061	0.00080
initial e	0.024	0.024	0.045	0.044
ultimate e	0.034	0.036	0.069	0.070
studies e	0.041	0.041	0.085	0.083

We also list the resonance strength computed without hourglass effect as well as the maximum resonance strength for particles between  $\pm 5\sigma_s$  along the bunch. These numbers can be compared with the resonance strength which were encountered in the extreme conditions of the performed beam-beam studies. The strength (given in units of  $10^{-8}$ ) for initial parameters, for ultimate parameters, and for the accelerator studies are indicated by the superscripts i, u, and s in the table below. Other beam-beam resonances of reasonable order are not close to the working points. We have nearly reached most of the resonance strength that would occur with ultimate beta functions already in our accelerator studies. Those resonances that are stronger for ultimate parameters than in the studies would not be stronger without the bunch-length effect. Reducing the bunch length would therefore not only reduce the hourglass reduction of the luminosity, it would also reduce the resonance strengths to a level for which a stable operation of the proton beam under luminosity conditions was already demonstrated.

	$\delta_0^i$	$\delta^i_{max}$	$\delta^u_{max}$	$\delta^s_{max}$
$\delta(4Q_{ex} + 2Q_{ey})$		20000	31000	45000
$\delta(2Q_{ex} + 8Q_{ey})$		1200	1800	3600
$\delta(10Q_{px})$	175	175	230	220
$\delta(8Q_{px} + 2Q_{py})$	73	73	97	99
$\delta(6Q_{px} + 4Q_{py})$	43	55	65	60
$\delta(4Q_{px}+6Q_{py})$	24	55	60	44
$\delta(2Q_{px} + 8Q_{py})$	14	65	68	32
$\delta(10\dot{Q}_{py})$	22	251	300	68
$\delta(14Q_{px})$	3.1	3.1	4.1	4.1
$\delta(12Q_{px} + 2Q_{py})$	2.9	2.9	3.8	3.7
$\delta(10Q_{px} + 4Q_{py})$	4.4	4.4	5.8	5.8
$\delta(8Q_{px}+6Q_{py})$	5.9	5.5	5.8	5.3
$\delta(6Q_{px} + 8Q_{py})$	2.6	7.0	6.0	4.3
$\delta(4Q_{px}+10Q_{py})$	1.2	8.8	8.0	3.2
$\delta(2Q_{px} + 12Q_{py})$	0.37	8.5	6.6	1.4
$\delta(14Q_{py})$	0.33	22	22	1.7

### **5** CONCLUSIONS

Computations of the beam-beam tune shift, the amplitude-dependent tune shift, and high order resonance strength have been performed including their enhancement due to the hourglass effect. All relevant integrals were performed numerically without approximations. The computed values for extreme focusing in HERA's pase 2 were compared to conditions which had been found usable in previous accelerator physics experiments and most of the dynamical parameters turn out to be more relaxed in phase 2 operation than in these experiments, even when the beta functions are pushed to their limits. Beam dynamics problems due to the hourglass effect should therefore not avoid reaching the ultimately focused beta functions in HERA.

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