FIRST-PRINCIPLE APPROACH FOR OPTIMIZATION OF CAVITY SHAPE FOR HIGH GRADIENT AND LOW LOSS *

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Abstract

Minimization of surface fields for a given accelerating rate is the subject of cavity optimization because high electric and magnetic fields lead to field emission or thermal breakdown, respectively. The ratio between peak electric and magnetic fields is a function of geometry and the desired ratio depends on application. For each application the optimal geometry may be different. The elliptic shape of the cavity have been found evolutionarily: starting from a pill-box with beam-pipes having rounded corners. No attempts up to now are known for a search of non-elliptical optimal shapes. Here we describe the search for a cavity shape that has the lowest surface fields, not restricting to the conventional elliptical cavity shapes.

INTRODUCTION

The commonly used superconducting cavity shape for high relativistic β values has been a result of evolution from a pillbox RF cavity with the beam tubes added and rounded walls – to decrease the peak electric field – to a shape consisting of elliptic arcs to prevent multipacting [1]. The relationship between the parameters of these arcs is a subject of a cavity optimization in search of the minimal magnetic surface field for a given peak electric surface field. The ratio between these peak fields depend on applications. All other figures of merit of a cavity also depend on the arc parameters but either they are defined by different requirements – aperture radius should be big enough to reduce the wakefields, or are connected with low magnetic fields: wall losses are minimal for a cavity optimized for lowest peak magnetic field [2].

So, in the present paper the attempt will be done to find the shape of a cavity with minimal peak magnetic field, or, more definitely, with minimal value of H_{pk}/E_{acc} for given values of E_{pk}/E_{acc}, wall slope angle α, and the aperture radius R_a. H_{pk} and E_{pk} are peak magnetic and electric fields, E_{acc} is the accelerating field.

As it was shown in [2] and [3], if these 3 parameters are given: E_{pk}/E_{acc}, α, and R_a, the minimal value of H_{pk}/E_{acc} can be found. If no limitations are used for the wall slope angle α, we will come to the reentrant shape having the minimal H_{pk}/E_{acc} from all possible for given E_{pk}/E_{acc} and R_a [4]. It is worthy to note that H_{pk}/E_{acc} is a monotone function of any of these 3 parameters: it decreases for smaller α and R_a, and for bigger E_{pk}/E_{acc}.

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ELLIPTIC GEOMETRY AND SURFACE FIELDS

All the optimizations for minimal H_{pk}/E_{acc} have usually been done under the assumption that the components of the shape are elliptic arcs connected with a straight segment, – we are talking about one only element of periodicity – one half-cell, Fig. 1.

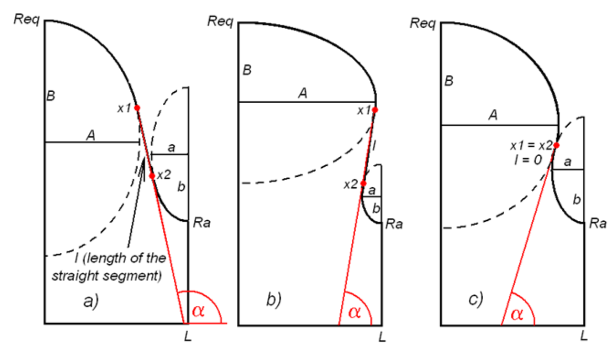

Figure 1: Geometry of the elliptic half-cell: non-reentrant – a), and reentrant – b) and c).

As an example, three known geometries of the central half-cells are shown in Fig. 2: TESLA [5] from DESY, the low-loss cavity [6] from JLab, and the reentrant cavity by Cornell [4].

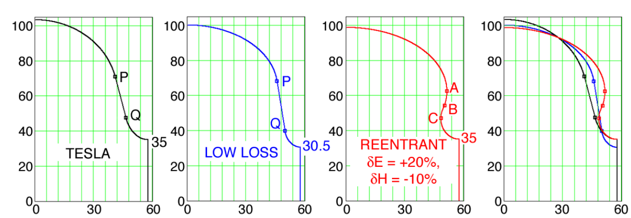

Figure 2: Comparison of three geometries of the elliptic half-cells.

The surface electric and magnetic fields in these cavities are shown in Figs. 3 and 4. One can see that the electric field has irregularities in the points P and Q on the surface of non-reentrant cavities, – where the arcs conjugate with the segment of the straight line. The best shape (just aesthetically) should have a more regular dependence for these curves. In the case of the reentrant cavity, the maximum is not flat as it is on non-reentrant geometry. One can suppose that flattening of this maximum will decrease the peak electric field but the E_{acc} should not change significantly.

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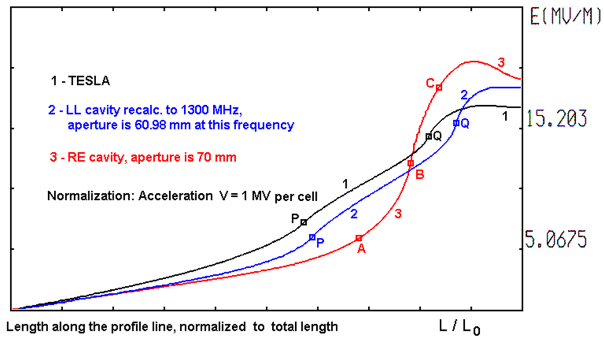

Figure 3: Surface electric field of the cavities from Fig. 2.

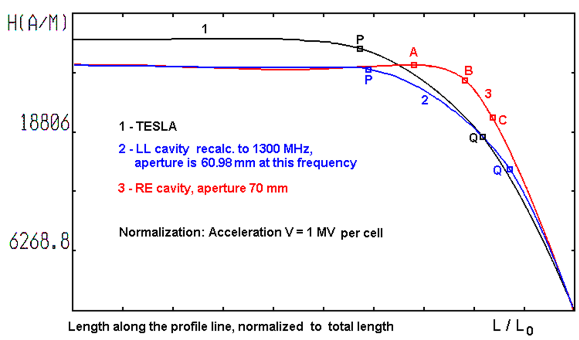

Figure 4: Surface magnetic field the cavities from Fig. 2.

Comparing the magnetic fields of the TESLA cell and the reentrant cell having the same aperture we can see that decrease of the maximal field is obtained through the lengthening of the maximal field region. Again, better flattening of the maximal values of this field could decrease the peak field though not much. The area under these curves is approximately the same in both cases. In the case of the Low Loss cavity the decrease of the maximal field is obtained due to smaller R_a as was noted above.

Let us remind that the record accelerating field obtained in the reentrant cavity [7] was achieved due to 10% lower magnetic peak field at the same E_{acc} compared to the TESLA geometry, though the value of E_{pk}/E_{acc} was 20% higher.

Because of a high cost of the superconducting cavities even a several percent further decrease of H_{pk}/E_{acc} is worth the candle, and we see that that this decrease is possible. On the other hand we have an interesting problem to find the shape with a given distribution of fields on its surface.

USAGE OF MAXWELL'S EQUATIONS

The elliptic shape is a lucky choice because fields have nearly flat maxima, are multipactor resistant, and technological, but it would be interesting to obtain the shape from the first principles.

Using the first Maxwell's equation

$$\oint_l H dl = \int_s \frac{\partial D}{\partial t} ds = \int_s \frac{\partial(\varepsilon_0 E)}{\partial t} ds$$

for the circular contours 1 and 2 (Fig. 5) with surface fields H_s and E_s, we have:

$$(H_s + \Delta H_s) \cdot 2\pi(R + \Delta R) - H_s \cdot 2\pi R =$$
$$H_s \cdot 2\pi \Delta R + \Delta H_s \cdot 2\pi R = -\omega\varepsilon_0 E_s 2\pi R \Delta L. \quad (1)$$

From (1) we have:

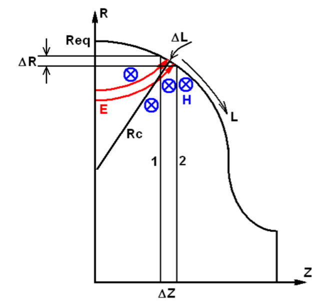

Figure 5: Geometry for the first Maxwell's equation.

$$\frac{\Delta R}{R} = -\frac{\omega\varepsilon_0 E_s}{H_s}\Delta L - \frac{\Delta H_s}{H_s}$$

and the expressions for the cavity shape:

$$R(L) = R_{eq} \cdot \exp\left[-\omega\varepsilon_0 \int_0^L \frac{e(s)}{h(s)} ds - \int_0^L \frac{1}{h(s)} \left(\frac{d}{ds} h(s)\right) ds\right]$$
$$= R_{eq} \cdot \frac{h(0)}{h(L)} \cdot \exp\left[-\omega\varepsilon_0 \int_0^L \frac{e(s)}{h(s)} ds\right], \quad (2)$$

$$Z(L) = \int_0^L \sqrt{1 - (dR(s)/ds)^2} ds. \quad (3)$$

Here, we presented E_s and H_s as functions of the coordinate along the profile line:

$$E_s = e(s), \quad H_s = h(s).$$

We can note that the fields in (2) appear in ratios only, this is because the optimal shape doesn't depend on the fields strength. So, we can introduce two new functions:

$$eh(s) = e(s)/h(s) \quad \text{and} \quad hh(s) = h(s)/h(0)$$

and change (2) to the following

$$R(L) = \frac{R_{eq}}{hh(L)} \cdot \exp\left[-\omega\varepsilon_0 \int_0^L eh(s) ds\right]. \quad (4)$$

From this we can obtain:

$$\frac{dR}{dL} = R(L)\left[-\omega\varepsilon_0\, eh(L) - \frac{1}{hh(L)} \cdot \frac{dhh}{dL}\right]. \quad (5)$$

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FIELDS ON THE SURFACE OF A PILLBOX CAVITY

Fields on the surface of a pill-box cavity (Fig. 6) can be presented in the form

$$e(L) = \begin{cases} 0, & L < a \\ E_0 \cdot J_0(kr), & a < L < a + R_{eq}, \end{cases}$$

$$h(L) = \begin{cases} (E_0/Z_0) \cdot J_1(\nu_{01}), & L < a \\ (E_0/Z_0) \cdot J_1(kr), & a < L < a + R_{eq}, \end{cases}$$

where $k = \omega/c$, $r = a + R_{eq} - L$, J_0 and J_1 are Bessel's functions of the first kind and ν_{01} is the first zero of $J_0(x)$.

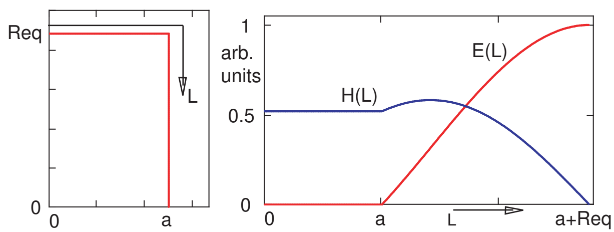

Figure 6: Fields on the surface of a pillbox cavity.

Substituting these fields into (2) and (3) we have

$$R(L) = R_{eq}, \quad Z(L) = L \quad \text{for} \quad 0 < L < a.$$

For $a < L < a + R_{eq}$ we will have

$$R(L) = R_{eq}\frac{J_1(kr)}{J_1(\nu_{01})} \cdot \exp\left[-\omega\varepsilon_0 Z_0 \int_{R_{eq}}^{r} \frac{J_0(ks)}{J_1(ks)} ds\right] = \\ = r = a + R_{eq} - L, \\ Z(L) = a.$$

So, the formulae (2) and (3) proved correct for the pill-box cavity.

Analogously, the formulae were checked for a spherical cavity which also has an analytic solution for fields.

IN SEARCH OF THE IDEAL SHAPE

So, any cavity shape, $R(L)$ and $Z(L)$, produces surface fields, $e(L)$ and $h(L)$, and this shape can be restored from these fields using (3) and (4). Unfortunately, the inverse problem is not so simple: arbitrarily prescribed surface fields can not lead to a shape at all, these two pairs of functions should be self-consistent. Some additional requirements should be defined, e. g. maximal wall slope angle, E_{pk}/E_{acc}, etc. Because of the exponential dependence of $R(L)$ on the fields even small deviations of these fields change the shape dramatically.

A good approximation for $hh(L)$ can be

$$hh(s) = 1 - \exp[-\alpha(L_{max} - L) - \beta(L_{max} - L)^2]. \quad (6)$$

This approximation is very flat in the region where we want to correct the magnetic field of the elliptic cavity and coincides with this field when L is close to L_{max}.

A possible approximation for $eh(L)$ is

$$eh(L) = \frac{P \cdot L}{L_{max}^2 - L^2}. \quad (7)$$

Here, $P = 2/\omega\varepsilon_0$. For small $\Delta = L_{max} - L$ we have

$$eh(L) = \frac{P}{2\Delta} - \frac{P}{4L_{max}},$$

and, substituting (7) for $dR/dL = 0$ in (5) we obtain:

$$\frac{1}{2L_{max}} = \frac{\alpha}{2} - \frac{\beta}{\alpha}, \quad \text{or} \quad \beta = \frac{\alpha^2}{2} - \frac{\alpha}{2L_{max}}. \quad (8)$$

The integral in (4) can be taken for $eh(L)$ from (7):

$$\int_0^L eh(s)ds = \frac{P}{2} \ln \frac{L_{max}^2}{L_{max}^2 - L^2}. \quad (9)$$

Substituting (9) into (4) for $L = L_{max}$ we have:

$$R(L_{max}) = R_a = \frac{2R_{eq}}{\alpha L_{max}}, \quad \text{or} \quad \alpha = \frac{2R_{eq}}{R_a L_{max}}. \quad (10)$$

These values of $\alpha = 0.0562$ and $\beta = 0.00130$ from (10) and (8) are close but not exactly the same as obtained for the optimized cavity with the slope angle of 90°: 0.0549 and 0.00173, respectively.

Comparison of the approximation (7) with the ratio E/H of the optimized elliptic cavities shows that their difference is defined by the wall slope angle and some correction functions should be added to (7) for each chosen angle.

CONCLUSIONS

The first attempt to find an ideal cavity shape from first principles is undertaken. A formula connecting the cavity shape and field distribution along the surface is found. Further efforts are needed to find self-consistent solutions of the problem.

REFERENCES

[1] Hasan Padamsee et al. *RF Superconductivity for Accelerators.* John Wiley & Sons, 1998.

[2] V. Shemelin. Phys. Rev. Special topics - Accelerators and Beams **12**, 114701 (2009).

[3] V. Shemelin. LEPP Report SRF070614–02, Cornell University, 2007.

[4] V. Shemelin, H. Padamsee, R.L. Geng. Nuclear Instruments and Methods in Physics Research, A. Vol. **496**, issue 1, pp. 1-7, 2003.

[5] B. Aune et al., Phys. Rev. ST Accel. Beams **3**, 092001 (2000).

[6] J. Sekutowicz, P. Kneisel, G. Ciovati, H. Wang. JLab–TN–02–023, June 2002.

[7] R. L. Geng, H. Padamsee, A. Seaman, V. Shemelin. PAC 2005, May 16 - 20, Knoxville, Tennessee 2005.

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