PROSPECTS FOR POLARIZED PROTONS AT HERA [a]

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This article explores the implications of obtaining polarized protons at high energy in HERA. Some likely modifications are explained and a possible fundamental limitation on the polarization attainable at very high energy is explained.

1 Introduction

HERA is the $e^{\pm}$-p colliding beam facility at DESY in Hamburg. The maximum $e^{\pm}$ energy is 30 GeV and the current proton energy is 820 GeV.

In May 1994 it was demonstrated that it is possible to obtain longitudinal electron polarization in HERA [1] and, at the time of writing, HERA regularly supplies positrons with 60 percent polarization to the HERMES experiment [2]. So it is now natural to consider if polarized protons could also be obtained. This would surely represent a significant enhancement of the physics potential of HERA–provided the luminosity could be maintained.

This article will explore some options for obtaining polarized protons at HERA and describe the modifications to the HERA complex likely to be necessary. An account of a possible fundamental limitation in the attainable polarization in high energy proton storage rings will also be given.

2 Three ways to obtain high energy polarized protons

In contrast to the electrons, which become polarized by the emission of synchrotron radiation [1] there is no analogous process for protons. Then the obvious way to obtain high energy polarized protons in HERA is to accelerate polarized protons provided by a source of polarized H^- ions. But as the reader will discover later in this article this would be a nontrivial undertaking. So two alternative concepts have been explored in which the polarization is obtained in the storage mode only after unpolarized protons have been accelerated from low energy. They involve spin transfer from low energy polarized electrons and self polarization using the Stern-Gerlach effect.

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The first alternative would involve providing an intense beam of spin polarized electrons travelling with the high energy ($\approx$ 820 GeV) proton beam over a distance of a few metres and at almost the same speed [b]. It has been shown by Horowitz and Meyer [3] that electron proton scattering would lead to a build up of proton polarization. However, the relative velocity of the electrons and protons is required to be low [a], so that the spin transfer rate would be impractically low although the spin transfer cross section is large. Since a scattering process would be involved, there would also be a proton spin diffusion process in competition with the polarizing process. The mathematical description of the combined polarization build up and depolarization could perhaps involve a generalised form of the Derbenev-Kondratenko formula [4].

The second alternative would involve exploiting the tiny Stern-Gerlach force acting on the spins in the inhomogeneous fields. The 'spin up' and 'spin down' parts of the ensemble would then be separated in transverse or longitudinal phase space [5,6,7,8]. Since there is at present no clear *practical* way to implement these ideas we do not go into more detail here although we do return briefly to this topic later. Instead we will concentrate on exploring the implications of the conventional route, namely acceleration of a polarized beam from low energy.

3 Acceleration from low energy

The acceleration chain for HERA consists of: $SOURCE \rightarrow RFQ(750KeV) \rightarrow LINAC\ III(50MeV) \rightarrow DESY\ III(7.5GeV) \rightarrow PETRA\ II(39GeV) \rightarrow HERA(820GeV)$. Now we discuss some of these systems. The connoisseur will recognize that we are drawing strongly on concepts already developed in the design studies for accelerating polarized protons to high energy at Fermilab and Brookhaven [9,10].

For polarized protons one needs a new H^- source. There are two basic types: the 'atomic beam' and 'optically pumped' sources. They achieve up to 80 percent polarization and in a few years it should be possible to reach currents of 5 mA [9]. However, to reach half of the usual linac output current of 10 mA it is likely that the existing RFQ will have to be replaced with a new version with much greater acceptance.

Spin precession in magnetic fields is described by the T-BMT equation [11,12]:

$$\frac{d\vec{S}}{ds} = \vec{\Omega} \times \vec{S} \qquad (1)$$

[b] Corresponding to an electron energy of about 440 MeV for $\approx$ 820 GeV protons.

where $\vec{\Omega}$ depends on the electric and magnetic fields, the velocity and the energy. In magnetic fields eqn. 1 can be written as:

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc\gamma} \times ((1+a)\vec{B}_{\parallel} + (1+a\gamma)\vec{B}_{\perp}) \quad , \quad a = \left(\frac{g-2}{2}\right) \tag{2}$$

where $\vec{B}_{\parallel}$, $\vec{B}_{\perp}$ are the magnetic fields parallel and perpendicular to the trajectory, s is the longitudinal (azimuthal) coordinate and the other symbols have the usual meanings. The gyromagnetic anomaly, $\frac{g-2}{2}$, for protons is about 1.79. The full T-BMT equation contains terms depending on the electric field but these vanish if the electric field is parallel to the velocity as in a linac. The T-BMT equation shows that in one turn around the design orbit of a flat storage ring a nonvertical spin precesses $a\gamma$ times. We call this latter quantity the 'naive spin tune' and denote it by ν_0. It gives the natural spin precession frequency in the vertical dipole fields of the ring. At 820 GeV ν_0 is about 1550. If the polarization vector is vertical at the entrance to the RFQ it will be vertical on entering DESY. Up to that point the spins will have passed though the off-axis horizontal and vertical fields of the various quadrupoles in the linac and the transfer line only once. But acceleration in DESY, PETRA and HERA requires that the spin ensemble passes through the complex structure of quadrupole fields many millions of times. It is this point at which a detailed understanding of spin dynamics in storage rings becomes necessary.

If a circular accelerator only had vertical (dipole) fields, vertical spins would not be affected. But the spins 'see' radial fields in the quadrupoles or combined function magnets since the beam has a nonzero height and even in a nominally flat ring the periodic (closed) orbit is offset vertically owing to unavoidable magnet misalignments. According to the T-BMT equation these fields tend to tilt the spins away from the vertical. The disturbance is different for each spin since it depends on the position of the particle. For particles circulating for many turns the total disturbance can grow to become very big if there is coherence between the natural spin motion and the oscillatory motion in the beam characterized by the spin-orbit resonance condition:

$$\nu_{spin} = m + m_x \cdot Q_x + m_z \cdot Q_z + m_s \cdot Q_s \tag{3}$$

where the m's are integers and the Q's are respectively the horizontal, vertical and longitudinal tunes of the orbital oscillations. ν_{spin} is the real (i.e. not naive) spin tune. In the presence of misalignments, rotators and snakes (see below), the unit vector describing the periodic spin direction on the closed orbit, $\vec{n}_0(s)$, is not vertical so that the concept of spin tune must be generalised to be the effective number of spin precessions around $\vec{n}_0$ per turn. In general

$\nu_{spin} \neq \nu_0$ but in a nominally flat ring ν_{spin} will not deviate very far from $a\gamma$. $\vec{n}_0(s)$ also describes the direction of the average spin vector of a bunch i.e. of the polarization direction of a bunch.

Now we can see the difficulty: during acceleration, ν_{spin} passes through the resonance condition many thousands of times[c] and there is a good chance that the beam loses polarization at each resonance so that by full energy the polarization has vanished. The integer resonances ($|m_x| + |m_z| + |m_s| = 0$ in eqn. 3) are driven by the periodic fields along the closed orbit. The spin-synchrobeta resonances[d] ($|m_x| + |m_z| + |m_s| \neq 0$ in eqn. 3) are driven by the quasiperiodic fields seen by particles executing synchrobeta oscillations about the closed orbit. Even at fixed nominal beam energy, there are longitudinal (energy) oscillations due to the presence of the rf cavities installed to keep the beam bunched, and the consequent oscillations in the spin tune can, if they have large enough amplitude, also cause resonance crossing.

Thankfully, the situation is not as bad as it might appear: in 1975 Derbenev and Kondratenko proposed an ingenious class of devices which are now called 'Siberian Snakes' [13]. They consist of magnet systems designed to rotate spins (and $\vec{n}_0$) by 180 degrees around an axis in the horizontal plane independently of the particle energy. With such devices it can be arranged that ν_{spin} is *independent* of the particle energy and that at least the first order resonance condition, ($|m_x| + |m_z| + |m_s| = 1$ in eqn.3) is never satisfied. Thus at one stroke depolarization during acceleration or due to energy oscillations is eliminated–at least if the effect of higher order resonances can be ignored. Typically one would arrange to set $\nu_{spin} = 1/2$. For detailed sketches of spin motion in the presence of snakes the reader is referred to ref. 14. From those sketches one can obtain an additional view of the resonance suppression mechanism. The spin perturbations accumulated during one turn tend to be cancelled on the next turn after the snake has flipped the spins by 180 degrees. This is very reminiscent of the spin echo effect used in spin-magnetic resonance investigations in materials.

From the T-BMT equation it is clear that at high energy, transverse magnetic fields are much more efficient than solenoid fields for rotating spins. For transverse fields the T-BMT equation takes the form

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc} \times (1/\gamma + a)\vec{B}_\perp \tag{4}$$

from which it is clear that for protons at high energy the spin precession rate is essentially independent of energy and proportional to the field. At 820

[c] ν_0 increases by one unit for every 523 MeV increase in energy.

[d] Often called 'intrinsic resonances'.

GeV with $\nu_0 \approx 1550$ the field needed to give a 1 mrad orbit deflection will cause about 90 degrees of spin precession. Equivalently, a simple rotation of 90 degrees requires a field integral of 2.74 Tesla metres. So at high energy, snakes should be built from strings of interleaved horizontal and vertical dipoles which generate a small local closed bump in the orbit but produce, via the noncommutation of large spin precessions, the required overall 180 degree spin rotation [e]. Since the snake fields are independent of energy, a key problem in snake design is to find solutions for which the orbit excursion at low energy is small, i.e. one or two centimetres. The ultimate in 'interleaving' is achieved using a helical dipole and a very elegant practical design has been proposed for RHIC [10,16]. That snake, which is superconducting and built out of four 2.4 metre modules is only 10 metres long and might also be suitable for the HERA ring–especially since the modules can be reconfigured to behave like spin rotators without excessive orbit excursion at low energy.

It is not yet clear how many snakes would be needed for suppression of depolarization during ramping to 820 GeV in HERA. For PETRA with its very different energy range, another snake design would be needed, probably consisting of discrete dipole magnets.

DESY poses a special problem. Down at a few GeV, dipole fields of the magnitude needed for snakes would cause unacceptable orbit distortions. On the other hand if longitudinal fields were to be used to produce 180 degree rotations around the beam direction, they would also have to be unacceptably large. A first step to a solution lies in the concept of the Partial Snake [10].

Recall that integer resonance depolarization is driven by periodic non-vertical fields along the closed orbit. It may be shown that the sensitivity of the spin motion to the orbital motion near the beam energy corresponding to an integer resonance depends on a certain component in a Fourier analysis of the field along the closed orbit [17,18]. If the strength of this harmonic (called the 'resonance strength') is large enough, and if one accelerates through the resonance sufficiently adiabatically, there is no decrease in the value of the polarization. Instead, all of the spins are inverted adiabatically just as when the excitation frequency is scanned through resonance during spin-magnetic resonance investigations in materials [f].

However, the resonance strengths vary widely with energy and since the ramp rate in an accelerator is more or less predetermined and since some integer resonances are not strong enough, not all resonances can be crossed adiabatically and polarization can be lost. This problem can be avoided by making tedious energy dependent orbit corrections to ensure that the resonance

[e] This is the principle of the HERA $e^{\pm}$ rotators [1,15].

[f] Resonance strengths for spin-synchrobeta resonances can be defined too.

strengths are either zero or very big. A better way is to use a Partial Snake. This is a device which creates a spin perturbation which is small but which nevertheless generates a resonance strength dominating that due to the orbit distortion at all integer resonances without significantly influencing the orbit motion. The partial snake could, for example, be a solenoid which is ramped with the energy so as to give a constant spin rotation very much smaller than 180 degrees and which therefore does not need an impractically large field. For DESY a rotation angle of just a few degrees (nevertheless representing a very high precession rate) would be sufficient to ensure that the usual ramp rate of about 7 GeV in 2.5 seconds would be adiabatic. Integer resonances in DESY would then be crossed by deliberate, controlled and repeated full spin flip. This approach has been spectacularly successful at the AGS [10,19]. A full snake can be thought of as the extreme case of a partial snake: in a full snake, any practical ramp rate is adiabatic.

Traversal of the most important spin-synchrobeta resonances in DESY, namely the first order Q_z resonances, would probably have to be carried out by so called 'tune jumping' using special pulsed quadrupoles to cause very fast changes in Q_z. Here one tries to achieve the exact opposite of adiabaticity: one maintains the polarization by pushing the spins through resonance before they have a chance to react. The alternative of making energy dependent changes to the optics to reduce the strengths of the Q_z resonances to zero would be impractical[g]. Spin tune jumping as suggested in ref. 20 should also be investigated as well as the deliberate driving of spin flip by exciting coherent vertical betatron motion.

The transfer lines between DESY and PETRA and between PETRA and HERA are not planar and the vertically bending dipoles rotate the spins away from the vertical. Consequently, snakelike systems of spin direction tuners would be needed to match the $\vec{n}_0$ of the lower energy machine to that of the higher energy machine.

This completes the outline of the modifications needed for acceleration of polarized protons from low energy.

So far we have dealt only with protons but there is some interest in the particle physics community in obtaining high energy deuterons. Since polarized deuteron sources exist we should perhaps also consider how to accelerate these particles. For deuterons, $\frac{g-2}{2}$ is only about –0.14 and the deuteron mass is about twice that of the proton. So there are far fewer spin-orbit resonances to cross. But equivalently, the influence of dipole fields on deuteron spins is very small and the design of spin rotators would be far from simple. Nevertheless the possiblity should be kept in mind.

[g] This is called 'vertical harmonic spin matching'[15].

4 The equilibrium polarization distribution

Clearly, before embarking on the kinds of upgrades described above one should simulate the whole or at least a part of the acceleration process on a computer by integrating the equations of combined spin-orbit motion with a tracking program for an ensemble of particles. The simulation of determinstic non-dissipative orbital motion must preserve the volume of phase space (i.e. be symplectic) and the spin transformations should be pure rotations.

We have seen that at the very high proton energy of HERA the spin motion is extremely sensitive to disturbances. So we should try to estimate the lifetime of the polarization at very high energy in the presence of beam-beam interaction, various sources of noise and of intrabeam scattering. This is the current strategy DESY and we use concepts from the theory of electron polarization.

We begin by making the approximation that the proton beam phase space can be described in terms of a continuous density, $\omega(\vec{u};s)$, which is a scalar function of the vector $\vec{u} = (x, p_x, z, p_z, \delta t, \delta E)$ of canonical orbital coordinates and the azimuth s. If the beam is stable, i.e. if it's at equilibrium, ω is periodic in s. In the absence of dissipation, ω is constant along a phase space trajectory and obeys a relation of the Liouville type:

$$\frac{\partial w}{\partial s} \;=\; \{H_{orb}, w\} \tag{5}$$

where H_{orb} is the orbital Hamiltonian. We now assign a polarization $\vec{P}(\vec{u};s)$ to each point. $\vec{P}$ is the average of the unit spin vector directions at $(\vec{u};s)$ [h]. Since the T-BMT equation is linear in the spin and since in this picture the spins at $(\vec{u};s)$ all see the same $\vec{\Omega}(\vec{u};s)$, $\vec{P}(\vec{u};s)$ obeys the T-BMT equation

$$\frac{d\vec{P}}{ds} = \vec{\Omega}(\vec{u};s) \times \vec{P} \;. \tag{6}$$

Because eqn. 6 describes precession, $|\vec{P}(\vec{u};s)|$ is constant along a phase space trajectory. To make closer contact with the synchrobeta motion, we can rewrite eqn. 6 as [21]:

$$\frac{\partial \vec{P}}{\partial s} \;=\; \{H_{orb}, \vec{P}\} + \vec{\Omega}(\vec{u};s) \times \vec{P} \tag{7}$$

which is analogous to eqn. 5.

At equilibrium, $\vec{P}(\vec{u};s)$ not only obeys the T-BMT equation, but it is also periodic in s. If the orbital motion can be expressed in terms of action-angle

[h] This is equivalent to associating a spin density matrix with each point in phase space.

variables $((I_k, \Psi_k), k = 1 \rightarrow 3)$ [i], we see that $\vec{P}(\vec{u};s)$ is also periodic in the Ψ_k. We write the equilibrium $\vec{P}$ as $\vec{P}_{eq}$ and we denote the unit vector along $\vec{P}_{eq}(\vec{u};s)$ by $\vec{n}(\vec{u};s)$ [j].

Obviously $\vec{n}$ also obeys eqn. 6. Furthermore, it may be shown that $\vec{n}$ is unique (up to a sign) except at the resonances of eqn. 3 [22 k].

So if we are sufficiently away from strong resonances and if we can fish out the T-BMT solution at each point in phase space which satisfies the required periodicity conditions, we know the $(\vec{u};s)$ dependence of the polarization direction across phase space at equilibrium! If we know $\omega(\vec{u};s)$, we know the statistical weight to apply to the polarization at each point. Clearly, this is an immensely useful result: if we imagine that each point in phase space is fully polarized ($\vec{P}_{eq}(\vec{u};s) = \vec{n}$) we can immediately obtain the upper limit on the average polarization of the beam at azimuth s by taking the ensemble average $< \vec{n} >$ at s.

A knowledge of $\vec{P}_{eq}(\vec{u};s)$ and $\omega(\vec{u};s)$ might also be necessary for evaluating $e^{\pm}$-p collision data in future experiments!

The vector $\vec{n}$ is a key object in the analytical theory of equilibrium electron polarization and was first introduced by Derbenev and Kondratenko from the viewpoint of finding action-angle variables for the combined spin-orbit Hamiltonian [4,6,22].

At HERA electron beam energies $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ is very small and it is often sufficient to calculate $e^{\pm}$ polarization in an approximation in which $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ is a linear function of $\vec{u}$ using the program SLIM originally created by A.W. Chao [24].

SLIM can also be applied to HERA-p and with the vertical bends switched off (for simplicity in interpreting the results), with perfect alignment (so that $\vec{n}_0(s)$ is vertical [l]) and with a realistic average invariant vertical emittance of 4π mm.mrad., it is found that the average angle between $\vec{n}(\vec{u};s)$ and $\vec{n}_0(s)$, δ^{ave}_{n,n_0}, obtained by averaging over the phase space is 1 radian or more at 820 GeV! As expected it is even larger near resonances. Related calculations using very different but equivalent algorithms by G. Hoffstätter [26] and by V. Balandin and N. Golubeva [18], confirm this finding. However, since δ^{ave}_{n,n_0} is large, the linear approximation is no longer valid and it is clear in retrospect that SLIM results for HERA-p at high energy must be treated with caution. Nevertheless

[i] Linear (i.e. integrable) orbital motion can be described in terms of three generalised harmonic oscillators.

[j] With respect to $\vec{n}(\vec{u};s)$ the spin density matrix at $(\vec{u};s)$ is diagonal.

[k] This leads to nontrivial questions regarding the existance of solutions and the value of approximations [23].

[l] Then in this (first order) calculation only vertical betatron motion contributes, not synchrotron motion [25].

the message that δ^{ave}_{n,n_0} is large remains valid. This implies a potentially strong limitation on the polarization of the whole bunch. For example with a δ^{ave}_{n,n_0} of 60 degrees, the upper limit on the polarization of the whole bunch is $\approx \cos(60^0) = 0.5$.

The largeness of δ^{ave}_{n,n_0} is also reflected in some exotic behaviour observed when tracking spins at fixed energy which are initially set parallel to $\vec{n}_0$ [18].

In view of the obvious need to be certain about δ^{ave}_{n,n_0}, it is now planned to implement a new version of the SODOM algorithm[27]. The SODOM algorithm will still require careful evaluation and checking, especially when studying the effects of the very slow synchrotron motion ($Q_s \approx 0.001$).

Both SODOM and SLIM allow $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ and δ^{ave}_{n,n_0} to be extracted from one turn spin-orbit maps. For very low emittances SLIM and SODOM results should coincide.

It is obvious that δ^{ave}_{n,n_0} decreases as both the invariant emittance and the beam energy decrease. But reliable estimates of the potential gains will only be available when SODOM or an equivalent program is running. Since δ^{ave}_{n,n_0} increases with the strength of nearby resonances, it might be possible to reduce δ^{ave}_{n,n_0} at the interaction points by strong [1,25] or harmonic spin matching[15] –providing the optics is sufficiently flexible.

Although the orbital motion is not exactly integrable we believe that the investigation of $\vec{n}$ provides a powerful way to understand the polarization properties of a beam. Indeed, since the beam blow-up due to beam-beam effect and power supply ripple is due to deterministic external electromagnetic forces and is adiabatic, and if we can neglect direct spin flip effects, knowledge of $\vec{n}(\vec{u};s)$ enables us to estimate the effect on the polarization: we speculate that the main result of beam blow-up will be an adiabatic increase of δ^{ave}_{n,n_0} resulting directly from an increase in the beam size and will lead to a decrease in the average polarization of the beam.

How would the polarization distribution at high energy arrive at equilibrium? The key to understanding this is to note that $\vec{n}(\vec{u};s)\cdot\vec{S}$ is an adiabatic invariant. At low energy δ^{ave}_{n,n_0} is small and so is the angle between $\vec{n}_0$ and $\vec{S}$ for each particle. Thus at low energy the average angle $\delta^{ave}_{n,S}$, between $\vec{n}(\vec{u};s)$ and $\vec{S}$ is also small and if mishaps are avoided during the adiabatic acceleration it is still small at high energy. Then the polarization at high energy will be almost parallel to $\vec{n}$.

Note that the equilibrium function $\vec{P}_{eq}(\vec{u};s)$ is by definition completely independent of the method used to polarize the beam. In particular, if δ^{ave}_{n,n_0} is very large, it could be that $\vec{P}_{eq}(\vec{u};s)$ at the edge of phase space is perpendicular to $\vec{n}_0$. This could have important implications for methods for polarization

based either on the transverse or on the longitudinal Stern-Gerlach effect.

Polarimetry based on scattering protons out of the tails of the beam could also be difficult.

5 Summary and Conclusion

The HERA $e^{\pm}$-p colliding beam facility is unique: it provides a high energy, high quality and highly polarized electron (positron) beam for collisions with 820 GeV protons. One of the next steps should be to try to polarize the proton beam.

As has been pointed out above, a large amount of work has already been invested in thinking of ways to accelerate polarized protons up to high energy and most of those ways could be adapted for HERA.

But calculations based on concepts adapted from the theory of radiative electron polarization suggest that for proton beam energies of the order of 1 TeV and above, there could be a fundamental limitation on the polarization (averaged over the beam) attainable.

For HERA this might mean that both the beam energy and the emittance must be reduced.

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