LONGITUDINAL ELECTRON POLARIZATION AT HERA AND PROSPECTS FOR PROTON POLARIZATION AT VERY HIGH ENERGY. *

D.P. BARBER
Deutsches Elektronen Synchrotron, (DESY),
22603 Hamburg, Germany
E-mail: mpybar@dsyibm.desy.de

ABSTRACT

The HERA electron ring is the first and only high energy electron storage ring at which longitudinal spin polarization has been obtained. This paper describes the spin rotators, the optic, the first observation of longitudinal polarization and the current status. After this achievement the possibility of obtaining polarized protons in the HERA proton ring is under study. Preliminary ideas on this topic will be presented.

1. Longitudinal electron polarization

1.1. Introduction

HERA is the $e^{\pm}$-p colliding beam facility at DESY in Hamburg[1,2]. The maximum $e^{\pm}$ energy is 30 GeV and the current proton energy is 820 GeV. An integral part of the design of HERA has been the provision of longitudinally spin polarized electrons for the high energy physics experiments at the interaction points.

Stored electron beams can become vertically polarized due to the emission of spin flip synchrotron radiation–the Sokolov-Ternov (S-T) effect[3]. The maximum polarization achievable is 92.4% corresponding to a planar ring. To provide longitudinal polarization at an interaction point the naturally occuring vertical polarization in the arcs must be rotated into the longitudinal direction just before the interaction point (IP) and back to the vertical just after the IP using special magnet configurations called spin rotators.

Synchrotron radiation generates polarization but also causes depolarization[4]. This is especially the case in the presence of spin rotators. Furthermore the ratio: (depolarization rate/polarization rate) increases strongly with energy. However, the depolarizing effects can in principle be minimized by special choice of the optic called 'spin matching'. Nevertheless, owing to the difficulty of obtaining reliable numerical predictions of the polarization in the presence of rotators throughout the preparatory stage of the project and because of the initially very pessimistic predictions, it was by no means clear that longitudinal polarization could be obtained even after spin matching.

*Presented at the Adriatico Research Conference: "Trends in Collider Spin Physics", Trieste, 5–8 December 1995.

Since a detailed account of this work has now been published [1], this section takes the form of a short summary and an update. The reader seeking a more detailed account and sketches and plots should refer to reference 1.

1.2. The spin rotators

Spin precession in electric and magnetic fields is described by the T-BMT equation[5]:

$$\frac{d\vec{S}}{ds} = \vec{\Omega} \times \vec{S} \tag{1}$$

where $\vec{\Omega}$ depends on the fields, the velocity and the energy. In magnetic fields Eqn. (1) can be written as:

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc\gamma} \times ((1+a)\vec{B}_{||} + (1+a\gamma)\vec{B}_{\perp}) \quad , \quad a = \left(\frac{g-2}{2}\right) \tag{2}$$

where $\vec{B}_{||}$, $\vec{B}_{\perp}$ are the magnetic fields parallel and perpendicular to the trajectory, s is the longitudinal (azimuthal) coordinate and the other symbols have the usual meanings. The gyromagnetic anomaly, $\frac{g-2}{2}$, for electrons is about 0.00115. The full T-BMT equation contains terms depending on the electric field but these can be neglected if the electric field is almost parallel to the velocity as in a storage ring or linac.

At HERA we use spin rotators comprising strings of dipole magnets. These exploit the prediction of the Thomas-BMT equation that for motion transverse to the magnetic field, the spin precesses around the field at a rate which is $a\gamma$ faster than the rate of rotation of the orbit direction. We call $a\gamma$ the 'naive spin tune' since it is just the number of spin precessions around the vertical dipole fields of the ring per turn. We denote it by ν_0. Thus

$$\delta\theta_{spin} = \nu_0 \delta\theta_{orbit} \tag{3}$$

in an obvious notation. At HERA energies $a\gamma$ is between 60 and 80. So it can be arranged that small commuting deflections of the orbit can result in large noncommuting precessions of the polarization vector. For HERA, the Mini-Rotator design of Buon and Steffen [6] was adopted. In the Mini-Rotators the rotation is achieved by interleaving three horizontal bends with three vertical bends. The orbit bend angles are of the order of 10 mrads and the horizontal bends replace the bend angle of one arc cell. The whole rotator is inserted into the end of the arc with minimum disturbance to the rest of the machine. The total vertical bend of this rotator is zero and it is only 56 m long so that no quadrupoles need be included among the constituent dipoles. The vertical orbit excursion is about 20 cm. The required rotator dipole fields and hence the shape of the rotator depend on the energy chosen. By mounting

the dipoles on coupled, remotely controlled mechanical jacks and by having flexible joints between sections of vacuum pipe, it is possible to tune the rotator to give the correct spin transformation over the range 27-35 GeV and to obtain both helicities. The vertical bumps in a pair of rotators are antisymmetric with respect to the IP. The first two rotators were installed on either end of the East straight section at the beginning of 1994 so as to provide longitudinal polarization for the HERMES experiment[7] which uses a polarized internal gas target to study the deep inelastic spin structure of nucleons. At the chosen energy of 27.52 GeV, the S-T polarization level is reduced below 92.4% to about 89% with one pair of rotators by the presence of the vertical bend magnets. This energy corresponds to a fractional spin tune of 0.5 and is chosen to be maximally distanced from the resonances to be defined in the next subsection.

1.3. The optic

In addition to the S-T effect producing the polarization, there is depolarization, i.e. spin diffusion, due to the stochastic excitation of the electron orbits by emission of synchrotron radiation photons. The depolarization is especially strong if there is coherence between the natural spin motion and the oscillatory motion in the beam characterized by the resonance condition

$$\nu_0 = k + m_x Q_x + m_z Q_z + m_s Q_s \tag{4}$$

where k, m_x, m_z, m_s are integers and the $Q_{x,z,s}$ are orbital tunes (47.1,47.2 and 0.06 resp.).

In perfectly aligned planar rings the equilibrium spin axis is vertical, the beam would have almost zero vertical emittance and the depolarization would actually almost vanish.

But in real rings with their typical misalignments, the closed obit is distorted and the equilibrium spin axis defined along the closed orbit is tilted from the vertical and the spin diffusion can then take hold[1,4]. This depolarization has been successfully combatted by the application of harmonic spin-orbit corrections[1] using special families of closed bumps to return the equilibrium spin axis to the vertical.

However, the installation of spin rotators can result in additional strong depolarization even in an ideal ring. One reason is that between the rotators the spin axis is horizontal and can be considered to be maximally tilted. Another reason is that there is local vertical dispersion inside the rotators so that synchrotron radiation emission in the rotators excites vertical betatron motion. In principle these two consequences of the presence of rotators can, in HERA, be largely eliminated by a special choice of optics called 'strong spin matching'. This consists of arranging the quadrupole settings so that the relevant parts of the 2 x 6 matrix describing the coupling of the spin to the orbit in linear approximation, vanish in the section between the rotators

straddling the interaction point and in the remainder of the ring[1,8]. Since the rotators contain no quadrupoles, spin matching inside the rotators is not needed. A spin matched section of ring is said to be 'spin transparent'. Not only must the matrix elements be minimized but also the usual optical properties of the machine must be acceptable. At HERA, it has been possible to satisfy these requirements by giving every mirror symmetric pair of quadrupoles in the section between the rotators and in the first seven cells at the beginning of each arc its own power supply.

1.4. The first tests

In the years up to 1994 it had become routine to have obtain over 60% vertical polarization (measured using a Compton polarimeter in the West area)[1]. The first test of the rotators took place in May 1994 with electrons. After achieving about 65% vertical polarization with the vertical bends off and the rotators 'flat', by the usual application of the harmonic spin-orbit bumps, the rotators were activated and a beam was stored without additional orbit adjustment. Then without further ado the polarization as measured at the West area polarimeter rose according to the expected exponential law [1].

$$P(t) = P_\infty \left(1 - e^{t/\tau_{pol}}\right) . \tag{5}$$

to about 56%. Note that the *value* of the polarization is the same everywhere in the ring but the direction can be changed by rotators. Since a tilt of the polarization vector by more than a few tens of milliradians in the arcs would lead to very strong depolarization we conclude that we had achieved longitudinal polarization at the East IP.

This was the first time in the history of high energy storage ring physics that longitudinal spin polarization had been achieved and demonstrated that, with suitable spin matching, it is indeed possible to rotate spins at the machine circulation frequency of 47.35 kHz and in a strongly stochastic environment. In the following days, after further machine optimization, a longitudinal polarization of about 65% was reached consistent with simulations from the Monte-Carlo spin tracking program SITROS[9].

1.5. The current status and future plans

In November 1994 about 65% longitudinal polarization was obtained with positrons and in 1995 we have been supplying the HERMES experiment regularly with longitudinal positron polarizations of over 40%. This is the first time that a physics experiment on a high energy electron storage ring has been supplied with longitudinal polarization. At the time of writing (December 1995) we have often been running with about 60% polarization. On one occasion in July 1995 the polarization remained at about 70% for 10 hours!

There is no clear indication of any degradation of the polarization due to the beam-beam effect from the $e^{\pm}$-p collisions at the H1 and ZEUS experiments with up to 40% of design luminosity (beam-beam electron tune shifts of about 0.02). This is also consistent with simulations by SITROS[9]. But SITROS simulations indicate that at full luminosity there could be beam-beam depolarization[10]. In that case extra spin matching will be needed and an algorithm has been prepared. It might also be necessary to increase the vertical emittance by introducing a vertical dispersion bump. The latter can also cause depolarization and the so called 'vertical harmonic dispersion matching' based on the generation of extra dispersion bumps[11] will be needed. Another major source of depolarization is of course the closed orbit distortion. A large reduction in the effect of the latter is expected with the implementation of the beam based calibration of the monitor positions[12]. SITROS simulations suggest that a longitudinal polarization of 80% could then be achieved in the absence of beam-beam effects.

2. Proton polarization in HERA at very high energy

2.1. Introduction

Now that longitudinal electron polarization is available it is natural to consider whether polarized protons could also be obtained. This would surely represent a significant enhancement of the physics potential of HERA–provided the luminosity could be maintained.

The remainder of this article will explore some options for obtaining polarized protons at HERA and describe the modifications to the HERA complex likely to be necessary. An account of a possible fundamental limitation in the attainable polarization in high energy proton storage rings will also be given.

2.2. Three ways to obtain high energy polarized protons

The obvious way to obtain high energy polarized protons in HERA is to accelerate polarized protons provided by a source of polarized H^- ions. As the reader will discover later in this article this would be a nontrivial undertaking. So two alternative concepts have been explored in which the polarization is obtained in the storage mode only after unpolarized protons have been accelerated from low energy. They involve spin transfer from low energy polarized electrons and self polarization using the Stern-Gerlach effect.

The first alternative would involve providing an intense beam of spin polarized electrons travelling with the high energy ($\approx$ 820 GeV) proton beam over a distance of a few metres and at almost the same speed [a]. It has been shown by Horowitz

[a]Corresponding to an electron energy of about 440 MeV for $\approx$ 820 GeV protons.

and Meyer[13] that electron proton scattering would lead to a build up of proton polarization. However, calculations for HERA show that the effect is too weak to be useful.

The second alternative would involve exploiting the tiny Stern-Gerlach force acting on the spins in the inhomogeneous fields[14,15,16,17]. Since this has been described by M. Conte at this meeting[18] we will concentrate on exploring the implications of the conventional route, namely acceleration of a polarized beam from low energy.

2.3. Acceleration from low energy

The acceleration chain for HERA consists of: $SOURCE \rightarrow RFQ(750KeV) \rightarrow LINAC\ III(50MeV) \rightarrow DESY\ III(7.5GeV) \rightarrow PETRA\ II(39GeV) \rightarrow HERA(820GeV)$. Now we discuss some of these systems. The expert will recognize that we are drawing strongly on concepts already developed in the design studies for accelerating polarized protons to high energy at Fermilab and Brookhaven[19,20].

For polarized protons one needs a new H^- source. There are two basic types: the 'atomic beam' and 'optically pumped' sources. They achieve up to 80 percent polarization and in a few years it should be possible to reach currents of 5 mA or more[19,21]. However, to reach half of the usual linac output current of 10 mA it is likely that the existing RFQ will have to be replaced with a new version with much greater acceptance.

The gyromagnetic anomaly for protons is about 1.79. Thus at 820 GeV ν_0 is about 1550. If the polarization vector is vertical at the entrance to the RFQ it will be vertical on entering DESY. Up to that point the spins will have passed though the off-axis horizontal and vertical fields of the various quadrupoles in the linac and the transfer line only once. But acceleration in DESY, PETRA and HERA requires that the spin ensemble passes through the complex structure of quadrupole fields many millions of times. This is the point at which a detailed understanding of spin dynamics in storage rings becomes necessary.

If a circular accelerator only had vertical (dipole) fields, vertical spins would not be affected. But the spins 'see' radial fields in the quadrupoles or combined function magnets since the beam has a nonzero height and even in a nominally flat ring the periodic (closed) orbit is offset vertically owing to unavoidable magnet misalignments. According to the T-BMT equation these fields tend to tilt the spins away from the vertical. The disturbance is different for each spin since it depends on the position of the particle. For particles circulating for many turns the total disturbance can grow to become very big close to spin -orbit resonances. In the presence of misalignments, rotators and snakes (see below), the unit vector describing the periodic spin direction on the closed orbit, $\vec{n}_0(s)$, is not vertical so that the concept of spin tune must be generalised to be the effective number of spin precessions around $\vec{n}_0$ per turn. We denote this by ν_{spin}. In general $\nu_{spin} \neq \nu_0$ but in a nominally flat ring ν_{spin} will not

deviate very far from $a\gamma$. $\vec{n}_0(s)$ also describes the direction of the average spin vector of a bunch i.e. of the polarization direction of a bunch.

Now we can see the difficulty: during acceleration, ν_{spin} passes through the resonance condition many thousands of times [b] and there is a good chance that the beam loses polarization at each resonance so that by full energy the polarization has vanished. The integer resonances ($|m_x| + |m_z| + |m_s| = 0$ in Eqn. (4)) are driven by the periodic fields along the closed orbit. The spin-synchrobeta resonances [c] ($|m_x| + |m_z| + |m_s| \neq 0$ in Eqn. (4)) are driven by the quasiperiodic fields seen by particles executing synchrobeta oscillations about the closed orbit. Even at fixed nominal beam energy, there are longitudinal (energy) oscillations due to the presence of the rf cavities installed to keep the beam bunched, and the consequent oscillations in the spin tune can, if they have large enough amplitude, also cause resonance crossing.

Thankfully, the situation is not as bad as it might appear: in 1975 Derbenev and Kondratenko proposed an ingenious class of devices which are now called 'Siberian Snakes' [22]. They consist of magnet systems designed to rotate spins (and $\vec{n}_0$) by 180 degrees around an axis in the horizontal plane independently of the particle energy. With such devices it can be arranged that ν_{spin} is almost *independent* of the particle energy and that at least the first order resonance condition, ($|m_x| + |m_z| + |m_s| = 1$ in Eqn. (4)) is never satisfied. Thus at one stroke depolarization during acceleration or due to energy oscillations is eliminated–at least if the effect of higher order resonances can be ignored. Typically one would arrange to set $\nu_{spin} = 1/2$. For detailed sketches of spin motion in the presence of snakes the reader is referred to reference 23. From those sketches one can obtain an additional view of the resonance suppression mechanism. The spin perturbations accumulated during one turn tend to be cancelled on the next turn after the snake has flipped the spins by 180 degrees. This is very reminiscent of the spin echo effect used in spin-magnetic resonance investigations in materials.

From Eqn. (2) it is clear that at high energy, transverse magnetic fields are much more efficient than solenoid fields for rotating spins. This is why dipole rotators were chosen for the electrons. For transverse fields the T-BMT equation takes the form

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc} \times (1/\gamma + a)\vec{B}_\perp \tag{6}$$

Since for protons a is about 1.79, at high energy the spin precession rate is essentially independent of energy and proportional to the field. At 820 GeV with $\nu_0 \approx 1550$ the field needed to give a 1 mrad orbit deflection will cause about 90 degrees of spin precession. Equivalently, a simple rotation of 90 degrees requires a field integral of 2.74 Tesla metres. So at high energy, snakes should be built from strings of interleaved

[b] ν_0 increases by one unit for every 523 MeV increase in energy.
[c] Often called 'intrinsic resonances'.

horizontal and vertical dipoles which generate a small local closed bump in the orbit but produce, via the noncommutation of large spin precessions, the required overall 180 degree spin rotation. Since the snake fields are independent of energy, a key problem in snake design is to find solutions for which the orbit excursion at low energy is small, i.e. one or two centimetres. The ultimate in 'interleaving' is achieved using a helical dipole and a very elegant practical design has been proposed for RHIC[20,24]. That snake, which is superconducting and built out of four 2.4 metre modules is only 10 metres long and might also be suitable for the HERA ring–especially since the modules can be reconfigured to behave like spin rotators without excessive orbit excursion at low energy.

It is not yet clear how many snakes would be needed for suppression of depolarization during ramping to 820 GeV in HERA. For PETRA with its very different energy range, another snake design would be needed, probably consisting of discrete dipole magnets.

DESY poses a special problem. Down at a few GeV, dipole fields of the magnitude needed for snakes would cause unacceptable orbit distortions. On the other hand if longitudinal fields were to be used to produce 180 degree rotations around the beam direction, they would also have to be unacceptably large. A first step to a solution lies in the concept of the Partial Snake[20].

Recall that integer resonance depolarization is driven by periodic non-vertical fields along the closed orbit. It may be shown that the sensitivity of the spin motion to the orbital motion near the beam energy corresponding to an integer resonance depends on a certain component in a Fourier analysis of the field along the closed orbit[25,26]. If the strength of this harmonic (called the 'resonance strength') is large enough, and if one accelerates through the resonance sufficiently adiabatically, there is no decrease in the value of the polarization. Instead, all of the spins are inverted adiabatically just as when the excitation frequency is scanned through resonance during spin-magnetic resonance investigations in materials [d]

However, the resonance strengths vary widely with energy and since the ramp rate in an accelerator is more or less predetermined and since some integer resonances are not strong enough, not all resonances can be crossed adiabatically and polarization can be lost. This problem can be avoided by making tedious energy dependent orbit corrections to ensure that the resonance strengths are either zero or very big. A better way is to use a Partial Snake. This is a device which creates a spin perturbation which is small but which nevertheless generates a resonance strength dominating that due to the orbit distortion at all integer resonances without significantly influencing the orbit motion. The partial snake could, for example, be a solenoid which is ramped with the energy so as to give a constant spin rotation very much smaller than 180 degrees and which therefore does not need an impractically large field. For DESY a rotation angle of just a few degrees (nevertheless representing a very high precession rate)

[d] Resonance strengths for spin-synchrobeta resonances can be defined too.

would be sufficient to ensure that the usual ramp rate of about 7 GeV in 1.6 seconds would be adiabatic. Integer resonances in DESY would then be crossed by deliberate, controlled and repeated full spin flip. This approach has been spectacularly successful at the AGS[20,27]. A full snake can be thought of as the extreme case of a partial snake: in a full snake, any practical ramp rate is adiabatic.

Traversal of the most important spin-synchrobeta resonances in DESY, namely the first order Q_z resonances, would probably have to be carried out by so called 'tune jumping' using special pulsed quadrupoles to cause very fast changes in Q_z. Here one tries to achieve the exact opposite of adiabaticity: one maintains the polarization by pushing the spins through resonance before they have a chance to react. The alternative of making energy dependent changes to the optics to reduce the strengths of the Q_z resonances to zero would be impractical [e]. Spin tune jumping[28] should also be investigated as well as the deliberate driving of spin flip by exciting coherent vertical betatron motion.

The transfer lines between DESY and PETRA and between PETRA and HERA are not planar and the vertically bending dipoles rotate the spins away from the vertical. Consequently, snakelike systems of spin direction tuners would be needed to match the $\vec{n}_0$ of the lower energy machine to that of the higher energy machine.

This completes the outline of the modifications needed for acceleration from low energy.

2.4. The equilibrium polarization distribution

Clearly, before embarking on the kinds of upgrades described above one should simulate the whole or at least a part of the acceleration process on a computer by integrating the equations of combined spin-orbit motion with a tracking program for an ensemble of particles. The simulation of determinstic non-dissipative orbital motion must preserve the volume of phase space (i.e. be symplectic) and the spin transformations should be pure rotations.

We have seen that at the very high proton energy of HERA the spin motion is extremely sensitive to disturbances. So we should try to estimate the lifetime of the polarization at very high energy in the presence of beam-beam interaction, various sources of noise and of intrabeam scattering. This is the current strategy DESY and we use concepts from the theory of electron polarization.

We begin by making the approximation that the proton beam phase space can be described in terms of a continuous density, $\omega(\vec{u};s)$, which is a scalar function of the vector $\vec{u} = (x, p_x, z, p_z, \delta t, \delta E)$ of canonical orbital coordinates and the azimuth s. If the beam is stable, i.e. if it's at equilibrium, ω is periodic in s. In the absence of dissipation, ω is constant along a phase space trajectory and obeys a relation of the

[e] This is called 'vertical harmonic spin matching' [6].

Liouville type:

$$\frac{\partial w}{\partial s} \;=\; \{H_{orb}, w\} \tag{7}$$

where H_{orb} is the orbital Hamiltonian. We now assign a polarization $\vec{P}(\vec{u};s)$ to each point. $\vec{P}$ is the average of the unit spin vector directions at $(\vec{u};s)$. Since the T-BMT equation is linear in the spin and since in this picture the spins at $(\vec{u};s)$ all see the same $\vec{\Omega}(\vec{u};s)$, $\vec{P}(\vec{u};s)$ obeys the T-BMT equation

$$\frac{d\vec{P}}{ds} = \vec{\Omega}(\vec{u};s) \times \vec{P}\ . \tag{8}$$

Because Eqn. (8) describes precession, $|\vec{P}(\vec{u};s)|$ is constant along a phase space trajectory. To make closer contact with the synchrobeta motion, we can rewrite Eqn. (8) as[29]:

$$\frac{\partial \vec{P}}{\partial s} \;=\; \{H_{orb}, \vec{P}\} + \vec{\Omega}(\vec{u};s) \times \vec{P} \tag{9}$$

which is analogous to Eqn. (7).

At equilibrium, $\vec{P}(\vec{u};s)$ not only obeys the T-BMT equation, but it is also periodic in s. One also expects on physical grounds that it is a smooth function of $\vec{u}$. If the orbital motion can be expressed in terms of action-angle variables $((I_k, \Psi_k), k = 1 \rightarrow 3)$ [f] we see that $\vec{P}(\vec{u};s)$ is also periodic in the Ψ_k. We write the equilibrium $\vec{P}$ as $\vec{P}_{eq}$ and we denote the unit vector along $\vec{P}_{eq}(\vec{u};s)$ by $\vec{n}(\vec{u};s)$

Obviously $\vec{n}$ also obeys Eqn. (8). Furthermore, it may be shown that $\vec{n}$ is unique (up to a sign) except at the resonances of Eqn. (4)[31] [g].

So if we are sufficiently away from strong resonances and if we can fish out the T-BMT solution at each point in phase space which satisfies the required periodicity conditions, we know the $(\vec{u};s)$ dependence of the polarization direction across phase space at equilibrium! If we know $\omega(\vec{u};s)$, we know the statistical weight to apply to the polarization at each point. Clearly, this is an immensely useful result: if we imagine that each point in phase space is fully polarized ($\vec{P}_{eq}(\vec{u};s) = \vec{n}$) we can immediately obtain the upper limit on the average polarization of the beam at azimuth s by taking the ensemble average $< \vec{n} >$ at s.

A knowledge of $\vec{P}_{eq}(\vec{u};s)$ and $\omega(\vec{u};s)$ might also be necessary for evaluating $e^{\pm}$-p collision data in future experiments!

The vector $\vec{n}$ is a key object in the analytical theory of equilibrium electron polarization and was first introduced by Derbenev and Kondratenko from the viewpoint of finding action-angle variables for the combined spin-orbit Hamiltonian[15,30,31].

[f] Linear (i.e. integrable) orbital motion can be described in terms of three generalised harmonic oscillators.

[g] This leads to nontrivial questions regarding the existance of solutions and the value of approximations[32].

At HERA electron beam energies $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ is very small and it is often sufficient to calculate $e^{\pm}$ polarization in an approximation in which $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ is a linear function of $\vec{u}$ using the program SLIM originally created by A.W. Chao[33].

SLIM can also be applied to HERA-p and with the vertical bends switched off (for simplicity in interpreting the results), with perfect alignment (so that $\vec{n}_0(s)$ is vertical [h]) and with a realistic average invariant vertical emittance of 4π mm.mrad., it is found that the average angle between $\vec{n}(\vec{u};s)$ and $\vec{n}_0(s)$, δ^{ave}_{n,n_0}, obtained by averaging over the phase space is 1 radian or more at 820 GeV! As expected it is even larger near resonances. Related calculations using very different but equivalent algorithms by G. Hoffstaetter[34] and by V. Balandin and N. Golubeva[26], confirm this finding. However, since δ^{ave}_{n,n_0} is large, the linear approximation is no longer valid and it is clear in retrospect that SLIM results for HERA-p at high energy must be treated with caution. Nevertheless the message that δ^{ave}_{n,n_0} is large remains valid. This implies a potentially strong limitation on the polarization of the whole bunch. For example with a δ^{ave}_{n,n_0} of 60 degrees, the upper limit on the polarization of the whole bunch is $\approx \cos(60^0) = 0.5$.

The largeness of δ^{ave}_{n,n_0} is also reflected in some exotic behaviour observed when tracking spins at fixed energy which are initially set parallel to $\vec{n}_0$ [26].

In view of the obvious need to be certain about δ^{ave}_{n,n_0}, it is now planned to implement a new version of the SODOM algorithm[35] and SPRINT, a new non-perturbative algorithm [36] developed just before this article was submitted. SPRINT and SODOM will still require careful evaluation and checking, especially when studying the effects of the very slow synchrotron motion ($Q_s \approx 0.001$).

SPRINT, SODOM and SLIM allow $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ and δ^{ave}_{n,n_0} to be extracted from one turn spin-orbit maps. For very low emittances results from all three should coincide.

It is obvious that δ^{ave}_{n,n_0} decreases as both the invariant emittance and the beam energy decrease. But reliable estimates of the potential gains will only be available when SODOM or SPRINT are running. Since δ^{ave}_{n,n_0} increases with the strength of nearby resonances, it might be possible to reduce δ^{ave}_{n,n_0} at the interaction points by strong[1,8], or harmonic spin matching[6] –providing the optics is sufficiently flexible.

Although the orbital motion is not exactly integrable we believe that the investigation of $\vec{n}$ provides a powerful way to understand the polarization properties of a beam. Indeed, since the beam blow-up due to beam-beam effect and power supply ripple is due to deterministic external electromagnetic forces and is adiabatic, and if we can neglect direct spin flip effects, knowledge of $\vec{n}(\vec{u};s)$ enables us to estimate the effect on the polarization: we speculate that the main result of beam blow-up will be an adiabatic increase of δ^{ave}_{n,n_0} resulting directly from an increase in the beam size and will lead to a decrease in the average polarization of the beam.

[h] Then in this (first order) calculation only vertical betatron motion contributes, not synchrotron motion[8].

How would the polarization distribution at high energy arrive at equilibrium? The key to understanding this is to note that $\vec{n}(\vec{u};s)\cdot\vec{S}$ is an adiabatic invariant. At low energy δ^{ave}_{n,n_0} is small and so is the angle between $\vec{n}_0$ and $\vec{S}$ for each particle. Thus at low energy the average angle $\delta^{ave}_{n,S}$, between $\vec{n}(\vec{u};s)$ and $\vec{S}$ is also small and if mishaps are avoided during the adiabatic acceleration it is still small at high energy. Then the polarization at high energy will be almost parallel to $\vec{n}$.

Note that the equilibrium function $\vec{P}_{eq}(\vec{u};s)$ is by definition completely independent of the method used to polarize the beam. In particular, if δ^{ave}_{n,n_0} is very large, it could be that $\vec{P}_{eq}(\vec{u};s)$ at the edge of phase space is perpendicular to $\vec{n}_0$. This could have important implications for methods for polarization based either on the transverse or on the longitudinal Stern-Gerlach effect[18].

Polarimetry based on scattering protons out of the tails of the beam could also be difficult.

3. Conclusion and outlook

At HERA a major milestone in the science of radiative spin polarization in storage rings has been achieved. In 1995 we are supplying longitudinal polarization to an experiment on a regular basis. Two more pairs of spin rotators are in manufacture and will be installed in the '96-'97 shutdown to provide longitudinal polarization for H1 and ZEUS. The next step should be to get proton polarization.

4. Acknowledgements

I would like to thank members of the HERA machine group and in particular members of the HERA Electron Polarization Group as well as V. Balandin, Ya. Derbenev, E. Gianfelice-Wendt, N. Golubeva, K. Heinemann, G. Hoffstätter, G.Ripken and M. Vogt for their contributions to the material presented here. I would also like to thank A. Krisch, A. Luccio, S. Nurushev, V. Ptitsin, T. Roser and Yu. Shatunov for useful discussions.

5. References

1. D.P. Barber et al., Phys.Letts., **B343** (1995) 436.
2. R. Brinkmann, 1995 Particle Accelerator Conf., Dallas, USA., (1995).
3. A.A. Sokolov and I.M. Ternov, Sov.Phys.Doklady, **8** (1964) 1203.
4. D.P. Barber, Proc. 10th International Symposium on High Energy Spin Physics, Nagoya, Japan, 1992. Universal Academy Press (1993) 83.
5. L. Thomas, Philos.Mag., **3** (1927) 1.
   V. Bargmann, L. Michel and V.L. Telegdi, Phys.Rev.Lett., **2** (1959) 435.
6. J. Buon and K. Steffen, Nucl.Inst.Meth., **A245** (1986) 248.

7. W-D. Nowak, in these proceedings.
8. D.P. Barber et al., Part.Accel., **17** (1985) 243.
9. M. Boege, Thesis, DESY Report 94-87 (1994).
10. M. Boege and T. Limberg, 1995 Particle Accelerator Conf., Dallas, USA., (1995).
11. E. Gianfelice-Wendt, Private communication.
12. M. Boege and R. Brinkmann, Proc. 4th European Particle Accelerator Conference, London, U.K. 1994, World Scientific (1994) 938.
13. C. Horowitz and H.O. Meyer, Phys.Rev.Lett., **72** (1994) 981.
14. T.O. Niinikoski and R. Rossmanith, Nucl.Inst.Meth., **A255** (1987) 460.
15. D.P. Barber, K. Heinemann and G. Ripken, Z.f.Physik., **C64** (1994) 143.
16. Ya.S. Derbenev,University of Michigan - Ann Arbor, preprint, UM-HE 90-32 (1990).
17. M. Conte, A. Penzo and M. Pusterla, Nuovo Cimento, **108A** (1995) 127.
18. M. Conte, in these proceedings.
19. A. Krisch, in these proceedings and references therein.
20. T. Roser, 1995 Particle Accelerator Conf., Dallas, USA., (1995).
21. A. Zelenski, private communication 1996.
22. Ya.S. Derbenev and A. Kondratenko, Sov.Phys.Doklady, **20** (1976) 562 and later papers.
23. B. Montague, Physics Reports, **113** (1984).
24. V.I. Ptitsin and Yu.M. Shatunov, Proc. 3rd Workshop on Siberian Snakes and Spin Rotators, Brookhaven 1994, BNL 52453 (1995).
25. E.D. Courant and R.D. Ruth, Brookhaven report BNL 51270 (1980).
26. V. Balandin, N. Golubeva and D.P. Barber, DESY Report M-96-04 (1996).
27. H. Huang et al., Phys.Rev.Lett., **73** (1994) 2982.
28. Yu.N. Filatov et al., Proc. 5th Workshop on High Energy Spin Physics, Protvino, Russia 1993, IHEP Protvino (1994) 299.
29. K. Heinemann, private communication 1995 and thesis in preparation.
30. Ya.S. Derbenev and A. Kondratenko, Sov.Phys.JETP., **37** (1973) 968.
31. K. Yokoya, DESY Report 86-57 (1986).
32. V. Balandin, private communication 1995.
33. A.W. Chao, Nucl.Inst.Meth., **180** (1981) 29.
34. G. Hoffstaetter, private communication 1995.
35. K. Yokoya, KEK Report 92-6 (1992).
36. K. Heinemann and G. Hoffstaetter, DESY report 96-78. Submitted to Phys.Rev.E.