Possibilities for polarized protons at HERA *

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Abstract

This article explores the implications of obtaining polarized protons at high energy in HERA. A list of likely minimum modifications is given and a possible fundamental limitation on the polarization attainable at very high energy is explained.

1 Introduction

HERA is the $e^{\pm}$-p colliding beam facility at DESY in Hamburg. The maximum $e^{\pm}$ energy is 30 GeV and the current proton energy is 820 GeV. There are two experiments for observing $e^{\pm}$-p collisions, namely H1 and ZEUS at the north and south interaction points (IP) respectively. At the east straight section the HERMES experiment is installed [1] and the HERA-B experiment will be installed in the west area.

Electrons and positrons in storage rings can become spin polarized normal to the plane of the machine by the emission of synchrotron radiation [2]. However, the synchrotron radiation also causes stochastic disturbance to the orbit motion [3] and this can, owing to the necessary presence of quadrupole fields, lead to strong spin diffusion, i.e. depolarization, which limits the achievable polarization. This effect can, in principle, be particularly strong if there are parts of the ring where the polarization vector has been rotated into the horizontal plane by magnet systems called spin rotators.

Nevertheless, in May 1994 it was demonstrated that by means of spin rotators it is possible to obtain longitudinal polarization in HERA at the position of the HERMES experiment without significant loss in the value of the polarization. This was the first time that longitudinal polarization had been obtained in a high energy electron storage ring. Longitudinal polarization is now in routine use by HERMES and values up to 70 percent have been achieved [4].

Following this success it is natural to consider whether it would be possible to complement the polarized $e^{\pm}$ with polarized protons. This would surely represent a significant enhancement of the physics potential of HERA–provided the luminosity could be maintained.

However, owing to their high mass in comparison to electrons, protons at HERA-p emit almost no synchrotron radiation so that there is no natural polarization process analogous to the Sokolov-Ternov effect. Thus even if one has, by some means, stored polarized protons at high energy, one must be careful to minimize any depolarization effects. But on the positive side there is no spin diffusion due to synchrotron radiation.

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So the attainment of polarized protons in storage rings must proceed along lines very different from those for electrons.

This article will explore some options for HERA. No attempt will be made to present a final solution–too little work has been done so far for that to be possible. Instead, the minimum modifications to the HERA complex likely to be necessary to obtain polarized protons will be described and an account of a possible fundamental limitation in the attainable polarization in high energy proton storage rings will be given.

2 Three ways to obtain high energy polarized protons

The obvious way to obtain high energy polarized protons in HERA is to accelerate polarized protons provided by a source of polarized H^- ions. As the reader will discover later in this article this would be a nontrivial undertaking. So two alternative concepts have been explored in which the polarization is obtained in the storage mode only after unpolarized protons have been accelerated from low energy:

- Spin transfer from low energy polarized electrons.
- Self polarization using the Stern-Gerlach effect.

The first alternative would involve providing an intense beam of spin polarized electrons travelling with the high energy ($\approx$ 820 GeV) proton beam over a distance of a few metres and at almost the same speed [1]. It has been shown by Horowitz and Meyer [5] that electron proton scattering would lead to a build up of proton polarization. There would be the added advantage of a mechanism for maintaining the polarization. However, the relative velocity of the electrons and protons is required to be low [1], so that the spin transfer rate would be impractically low although the spin transfer cross section is large. Since a scattering process would be involved, there would also be a proton spin diffusion process in competition with the polarizing process in analogy with the effects of synchrotron radiation on electron beams. The mathematical description of the combined polarization build up and depolarization would be analogous to that for electrons and could even involve a generalised form of the Derbenev-Kondratenko [6] formula in order to get a coherent semiclassical description.

The second alternative has been discussed for many years and in an early version would involve running the machine on a spin-orbit resonance (see below) and exploiting the tiny Stern-Gerlach force acting on the spins in the spatially varying fields of the quadrupole magnets to generate a resonant increase in the transverse (betatron) amplitudes. The 'spin up' and 'spin down' parts of the ensemble would then be spatially separated [7,8,9]. A more recent version involves generating a separation in longitudinal phase space by use of strong longitudinal field gradients [10]. Since these Stern-Gerlach schemes are discussed by R. Rossmanith in this volume it is not necessary to go into more detail here although we do return briefly to this topic later. Instead we will concentrate on exploring the implications of the conventional route, namely acceleration of a polarized beam from low energy.

[1] Corresponding to an electron energy of about 440 MeV for $\approx$ 820 GeV protons.

3 Acceleration from low energy

Before entering into details of spin dynamics in accelerators and storage rings and explaining the operating principles of some special devices we will simply list some required modifications to the $SOURCE \to RFQ \to LINAC\ III \to DESY\ III \to PETRA\ II \to HERA$ chain. The modifications are underlined.

- The source: <u>atomic beam or optically pumped source of polarized H^- ions.</u>
- Radio frequency quadrupole (RFQ) to 750 KeV: <u>bigger acceptance.</u>
- Proton linac to 50 MeV.
- Transfer line to DESY III.
- DESY III: stripping foil injection, $50MeV \to 7.5GeV$: <u>partial snake and pulsed quads.</u>
- Transfer line DESY III to PETRA II: <u>snakelike spin direction tuner</u>.
- PETRA II: $7.5GeV \to 39GeV$: <u>two Siberian Snakes.</u>
- Transfer line PETRA II to HERA: <u>snakelike spin direction tuner</u>.
- HERA: $39GeV \to 820GeV$: <u>Siberian Snakes and spin rotators.</u>
. <u>Rotators might be integrated into the vertical bends.</u>
- <u>Polarimeters.</u>

Now we discuss these entries one by one. The connoisseur will recognize that we are drawing strongly on concepts already developed in the design studies for accelerating polarized protons to high energy at Fermilab and Brookhaven. See the articles by A. Krisch and T. Roser in this volume [11,12].

The acceleration chain for HERA-p begins with the H^- ion source. The ions are first accelerated in the RFQ up to an energy at which they can be captured and accelerated in the linac. Then after the linac, at an energy of 50 MeV, they are injected though a stripping foil into DESY. The stripping foil removes the electrons from the ions and allows protons to be accumulated in the phase space already occupied by circulating protons.

For polarized protons one needs a new H^- source. There are two basic types: the 'atomic beam' and 'optically pumped' sources. They achieve up to 80 percent polarization and in a few years it should be possible to reach currents of 5 mA [11]. However, to reach half of the usual linac output current of 10 mA it is likely that the existing RFQ will have to be replaced with a new version with much greater acceptance.

Spin precession in magnetic fields is described by the Thomas-Bargmann-Michel-Telegdi (T-BMT) equation [13,14]:

$$\frac{d\vec{S}}{ds} = \vec{\Omega} \times \vec{S} \tag{1}$$

where $\vec{\Omega}$ depends on the electric and magnetic fields, the velocity and the energy. In magnetic fields eqn. 1 can be written as:

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc\gamma} \times ((1+a)\vec{B}_{\|} + (1+a\gamma)\vec{B}_{\perp}) \ , \quad a = \left(\frac{g-2}{2}\right) \tag{2}$$

where $\vec{B}_{||}$, $\vec{B}_{\perp}$ are the magnetic fields parallel and perpendicular to the trajectory, s is the longitudinal (azimuthal) coordinate and the other symbols have the usual meanings. The gyromagnetic anomaly, $\frac{g-2}{2}$, for protons is about 1.79. The full T-BMT equation contains terms depending on the electric field but these vanish if the electric field is parallel to the velocity as in a linac. The T-BMT equation shows that in one turn around the design orbit of a flat storage ring a nonvertical spin precesses $a\gamma$ times. We call this latter quantity the 'naive spin tune' and denote it by ν_0. It gives the natural spin precession frequency in the vertical dipole fields of the ring. At 820 GeV ν_0 is about 1550. If the polarization vector is vertical at the entrance to the RFQ it will be vertical on entering DESY. Up to that point the spins will have passed though the off-axis horizontal and vertical fields of the various quadrupoles in the linac and the transfer line only once. But acceleration in DESY, PETRA and HERA requires that the spin ensemble passes through the complex structure of quadrupole fields many millions of times. It is this point at which a detailed understanding of spin dynamics in storage rings becomes necessary.

If a circular accelerator only had vertical (dipole) fields, vertical spins would not be affected. But the spins 'see' radial fields in the quadrupoles or combined function magnets since the beam has a nonzero height and even in a nominally flat ring the periodic (closed) orbit is offset vertically owing to unavoidable magnet misalignments. According to the T-BMT equation these fields tend to tilt the spins away from the vertical. The disturbance is different for each spin since it depends on the position of the particle. For particles circulating for many turns the total disturbance can grow to become very big if there is coherence between the natural spin motion and the oscillatory motion in the beam characterized by the spin-orbit resonance condition:

$$\nu_{spin} = m + m_x \cdot Q_x + m_z \cdot Q_z + m_s \cdot Q_s \qquad (3)$$

where the m's are integers and the Q's are respectively the horizontal, vertical and longitudinal tunes of the orbital oscillations. ν_{spin} is the real (i.e. not naive) spin tune. In the presence of misalignments, rotators and snakes (see below), the unit vector describing the periodic spin direction on the closed orbit, $\vec{n}_0(s)$, is not vertical [2] so that the concept of spin tune must be generalised to be the effective number of spin precessions around $\vec{n}_0$ per turn. In general $\nu_{spin} \neq \nu_0$ but in a nominally flat ring ν_{spin} will not deviate very far from $a\gamma$. $\vec{n}_0(s)$ also describes the direction of the average spin vector of a bunch i.e. of the polarization direction of a bunch.

Now we can see the difficulty: during acceleration, ν_{spin} passes through the resonance condition many thousands of times [3] and there is a good chance that the beam loses polarization at each resonance so that by full energy the polarization has vanished. The integer resonances ($|m_x| + |m_z| + |m_s| = 0$ in eqn. 3) are driven by the periodic fields along the closed orbit. The spin-synchrobeta resonances [4] ($|m_x| + |m_z| + |m_s| \neq 0$ in eqn. 3) are driven by the quasiperiodic fields seen by particles executing synchrobeta oscillations about the closed orbit. Even at fixed nominal beam energy, there are longitudinal (energy) oscillations due to the presence of the rf cavities installed to keep the beam bunched, and the consequent oscillations in the spin tune can, if they have large enough amplitude, also cause resonance crossing.

Thankfully, the situation is not as bad as it might appear: in 1975 Derbenev and Kondratenko proposed an ingenious class of devices which are now called 'Siberian Snakes' [15]. They consist of magnet systems designed to rotate spins (and $\vec{n}_0$) by 180 degrees around an

[2] Indeed for the HERA *electron* ring $\vec{n}_0$ is made *longitudinal* at the east IP by means of rotators.
[3] ν_0 increases by one unit for every 523 MeV increase in energy.
[4] Often called 'intrinsic resonances'.

axis in the horizontal plane independently of the particle energy. With such devices it can be arranged that ν_{spin} is *independent* of the particle energy and that at least the first order resonance condition, ($|m_x|+|m_z|+|m_s|=1$ in eqn.3) is never satisfied. Thus at one stroke depolarization during acceleration or due to energy oscillations is eliminated–at least if the effect of higher order resonances can be ignored. Typically one would arrange to set $\nu_{spin}=1/2$. For detailed sketches of spin motion in the presence of snakes the reader is referred to [16]. From those sketches one can obtain an additional view of the resonance suppression mechanism. The spin perturbations accumulated during one turn tend to be cancelled on the next turn after the snake has flipped the spins by 180 degrees. This is very reminiscent of the spin echo effect used in spin-magnetic resonance investigations in materials.

From the T-BMT equation it is clear that at high energy, transverse magnetic fields are much more efficient than solenoid fields for rotating spins. For transverse fields the T-BMT equation takes the form

$$\frac{d\vec{S}}{ds}=\frac{e\vec{S}}{mc}\times(1/\gamma+a)\vec{B}_{\perp} \tag{4}$$

from which it is clear that for protons at high energy the spin precession rate is essentially independent of energy and proportional to the field. At 820 GeV with $\nu_0\approx 1550$ the field needed to give a 1 mrad orbit deflection will cause about 90 degrees of spin precession. Equivalently, a simple rotation of 90 degrees requires a field integral of 2.74 Tesla metres. So at high energy, snakes should be built from strings of interleaved horizontal and vertical dipoles which generate a small local closed bump in the orbit but produce, via the noncommutation of large spin precessions, the required overall 180 degree spin rotation [5]. Since the snake fields are independent of energy, a key problem in snake design is to find solutions for which the orbit excursion at low energy is small, i.e. one or two centimetres. The ultimate in 'interleaving' is achieved using a helical dipole and a very elegant practical design has been proposed for RHIC [12,17]. That snake, which is superconducting and built out of four 2.4 metre modules is only 10 metres long and might also be suitable for the HERA ring–especially since the modules can be reconfigured to behave like spin rotators without excessive orbit excursion at low energy. Spin motion in the HERA proton ring is complicated by the presence of interleaved horizontal and vertical bends on either side of the north, south and east IP's. These bends serve to bring the proton beam in line with the electron beam and cause $\vec{n}_0(s)$ to be nonvertical at almost all s. If this were to lead to significant depolarization, even in the presence of snakes, energy dependent spin rotators might have to be incorporated in among these bends to ensure that $\vec{n}_0(s)$ were vertical in the arcs during acceleration. Rotators are needed in any case to obtain the correct spin orientation at the IP's.

It is not yet clear how many snakes would be needed for suppression of depolarization during ramping to 820 GeV in HERA. For PETRA with its very different energy range, another snake design would be needed, probably consisting of discrete dipole magnets.

DESY poses a special problem. Down at a few GeV, dipole fields of the magnitude needed for snakes would cause unacceptable orbit distortions. On the other hand if longitudinal fields were to be used to produce 180 degree rotations around the beam direction, they would also have to be unacceptably large. A first step to a solution lies in the concept of the Partial Snake [12].

Recall that integer resonance depolarization is driven by periodic non-vertical fields along the closed orbit. It may be shown that the sensitivity of the spin motion to the orbital motion near the beam energy corresponding to an integer resonance depends on a certain component

[5] This is the principle of the HERA $e^{\pm}$ rotators [4].

in a Fourier analysis of the field along the closed orbit [18,19]. If the strength of this harmonic (called the 'resonance strength') is large enough, and if one accelerates through the resonance sufficiently adiabatically, there is no decrease in the value of the polarization. Instead, all of the spins are inverted adiabatically just as when the excitation frequency is scanned through resonance during spin-magnetic resonance investigations in materials [6].

However, the resonance strengths vary widely with energy and since the ramp rate in an accelerator is more or less predetermined and since some integer resonances are not strong enough, not all resonances can be crossed adiabatically and polarization can be lost. This problem can be avoided by making tedious energy dependent orbit corrections to ensure that the resonance strengths are either zero or very big. A better way is to use a Partial Snake. This is a device which creates a spin perturbation which is small but which nevertheless generates a resonance strength dominating that due to the orbit distortion at all integer resonances without significantly influencing the orbit motion. The partial snake could, for example, be a solenoid which is ramped with the energy so as to give a constant spin rotation very much smaller than 180 degrees and which therefore does not need an impractically large field. For DESY a rotation angle of just a few degrees (nevertheless representing a very high precession rate) would be sufficient to ensure that the usual ramp rate of about 7 GeV in 2.5 seconds would be adiabatic. Integer resonances in DESY would then be crossed by deliberate, controlled and repeated full spin flip. This approach has been spectacularly successful at the AGS. Details can be found in the talk by T. Roser in this volume [12]. A full snake can be thought of as the extreme case of a partial snake: in a full snake, any practical ramp rate is adiabatic.

Traversal of the most important spin-synchrobeta resonances in DESY, namely the first order Q_z resonances, would probably have to be carried out by so called 'tune jumping' using special pulsed quadrupoles to cause very fast changes in Q_z. Here one tries to achieve the exact opposite of adiabaticity: one maintains the polarization by pushing the spins through resonance before they have a chance to react. The alternative of making energy dependent changes to the optics to reduce the strengths of the Q_z resonances to zero would be impractical [7]. Spin tune jumping as suggested in [20] should also be investigated as well as the deliberate driving of spin flip by exciting coherent vertical betatron motion.

The transfer lines between DESY and PETRA and between PETRA and HERA are not planar and the vertically bending dipoles rotate the spins away from the vertical. Consequently, snakelike systems of spin direction tuners would be needed to match the $\vec{n}_0$ of the lower energy machine to that of the higher energy machine.

The need for polarimeters at each stage is obvious. The details of polarimeter design are beyond the scope of this article but material can be found in [11]. At very high energy $\vec{n}_0$ is very sensitive to misalignments since ν_0 is very large (≈ 1550). Therefore it would be useful if the polarimeter could give information on the direction of $\vec{n}_0$.

One also needs precise control of the final beam energy so that high order resonances which have not been suppressed by the snakes can be avoided.

We have now completed the outline explanation and justification of the modifications proposed at the beginning of this section.

So far we have dealt only with protons but there is some interest in the particle physics community in obtaining high energy deuterons. Since polarized deuteron sources exist we should perhaps also consider how to accelerate these particles. For deuterons, $\frac{g-2}{2}$ is only about –0.14 and the deuteron mass is about twice that of the proton. So there are far fewer spin-orbit

[6] Resonance strengths for spin-synchrobeta resonances can be defined too.

[7] This is called 'vertical harmonic spin matching' [27].

resonances to cross. But equivalently, the influence of dipole fields on deuteron spins is very small and the design of spin rotators would be far from simple. Nevertheless the possiblity should be kept in mind.

4 The equilibrium polarization distribution

Clearly, before embarking on the kinds of upgrades described above one should simulate the whole or at least a part of the acceleration process on a computer by integrating the equations of combined spin-orbit motion with a tracking program for an ensemble of particles. In order to economise on computer time it is usually necessary to make some reasonable and carefully controlled approximations. Nevertheless, the simulation of determinstic non-dissipative orbital motion must preserve the volume of phase space (i.e. be symplectic) and the spin transformations should be pure rotations.

We have seen that at the very high proton energy of HERA the spin motion is extremely sensitive to disturbances. It is then perhaps inadvisable to assume that experience gained at low energy is applicable at very high energy. Indeed, for the very high final energy of HERA-p we should instead be confronting another matter first, namely of how to understand the term 'polarization' in a very high energy stored beam and of deciding whether this 'polarization' is useful. Then we should try to estimate the lifetime of the polarization at very high energy in the presence of beam-beam interaction, various sources of noise and of intrabeam scattering. This is the current strategy at DESY and to this end we are drawing on concepts which have long been common currency at DESY in our calculations of electron polarization.

We begin by making the approximation that the proton beam phase space can be described in terms of a continuous density, $\omega(\vec{u};s)$, which is a scalar function of the vector $\vec{u} = (x, p_x, z, p_z, \delta t, \delta E)$ of canonical orbital coordinates and the azimuth s. If the beam is stable, i.e. if it's at equilibrium, ω is periodic in s and we write it as ω_{eq}. In the absence of dissipation, ω is constant along a phase space trajectory and obeys a relation of the Liouville type:

$$\frac{\partial w}{\partial s} = \{H_{orb}, w\} \tag{5}$$

where H_{orb} is the orbital Hamiltonian. Having assigned a phase space density to each point in phase space we now assign a polarization $\vec{P}(\vec{u};s)$ to each point [8]. $\vec{P}$ is the average of the unit spin vector directions at $(\vec{u};s)$. Since the T-BMT equation is linear in the spin and since in this picture the spins at $(\vec{u};s)$ all see the same $\vec{\Omega}(\vec{u};s)$, $\vec{P}(\vec{u};s)$ obeys the T-BMT equation

$$\frac{d\vec{P}}{ds} = \vec{\Omega}(\vec{u};s) \times \vec{P} \; . \tag{6}$$

Because eqn. 6 describes precession, $|\vec{P}(\vec{u};s)|$ is constant along a phase space trajectory. To make closer contact with the synchrobeta motion, we can rewrite eqn. 6 as [21]:

$$\frac{\partial \vec{P}}{\partial s} = \{H_{orb}, \vec{P}\} + \vec{\Omega}(\vec{u};s) \times \vec{P} \tag{7}$$

which is analogous to eqn. 5.

[8]This is equivalent to associating a spin density matrix with each point in phase space.

At equilibrium, $\vec{P}(\vec{u};s)$ not only obeys the T-BMT equation, but it is also periodic in s. If the orbital motion can be expressed in terms of action-angle variables $((I_k,\Psi_k), k=1\rightarrow 3)$ [9], we see that $\vec{P}(\vec{u};s)$ is also periodic in the Ψ_k. We write the equilibrium $\vec{P}$ as $\vec{P}_{eq}$ and we denote the unit vector along $\vec{P}_{eq}(\vec{u};s)$ by $\vec{n}(\vec{u};s)$ [10].

Obviously $\vec{n}$ also obeys eqn. 6. Furthermore, it may be shown that $\vec{n}$ is unique (up to a sign) except at the resonances of eqn. 3 [22] [11].

So if we are sufficiently away from strong resonances and if we can fish out the T-BMT solution at each point in phase space which satisfies the required periodicity conditions, we know the $(\vec{u};s)$ dependence of the polarization direction across phase space at equilibrium! If we know $\omega(\vec{u};s)$, we know the statistical weight to apply to the polarization at each point. Clearly, this is an immensely useful result: if we imagine that each point in phase space is fully polarized ($\vec{P}_{eq}(\vec{u};s)=\vec{n}$) we can immediately obtain the upper limit on the average polarization of the beam at azimuth s by taking the ensemble average $<\vec{n}>$ at s.

A knowledge of $\vec{P}_{eq}(\vec{u};s)$ and $\omega_{eq}(\vec{u};s)$ might also be necessary for evaluating $e^{\pm}$-p collision data in future experiments!

The vector $\vec{n}$ is a key object in the analytical theory of equilibrium electron polarization and was first introduced by Derbenev and Kondratenko from the viewpoint of finding action-angle variables for the combined spin-orbit Hamiltonian [6,8,22].

Although, the particle motion in electron beams is not fully deterministic owing to synchrotron radiation, the direction of the electron polarization, $\vec{P}^{e}_{eq}(\vec{u};s)$, at equilibrium is still essentially parallel to the $\vec{n}(\vec{u};s)$ defined above for protons [21]. In the HERA electron ring at the running energy of about 27.5 GeV and away from resonances $\vec{n}(\vec{u};s)$ deviates from $\vec{n}_0(s)$ by just a few milliradians. However, the *value* of $\vec{P}^{e}_{eq}(\vec{u};s)$ at equilibrium depends on the balance between the polarization and the depolarization, both due to synchrotron radiation [12] and is independent of the history of the beam. In a badly optimized machine, some resonances can, as a function of beam energy be hundreds of MeV wide. In contrast, for protons $\vec{P}_{eq}(\vec{u};s)$ does depend on the history of the beam and in particular on the relative mixture of spin states at the source and on how successfully resonances have been dealt with during acceleration. Thus although electron and proton spins both obey the T-BMT equation, the origins of depolarization are somewhat different.

Since, for electrons in HERA, $\vec{n}(\vec{u};s)-\vec{n}_0(s)$ is very small it is often sufficient to calculate $e^{\pm}$ polarization in an approximation in which $\vec{n}(\vec{u};s)-\vec{n}_0(s)$ is a linear function of $\vec{u}$ using the program SLIM originally created by A.W. Chao [24]. SLIM is one of the basic tools for calculating $e^{\pm}$ polarization at HERA.

SLIM can also be applied to HERA-p and with the vertical bends switched off (for simplicity in interpreting the results), with perfect alignment (so that $\vec{n}_0(s)$ is vertical [13]) and with a realistic average invariant vertical emittance of 4π mm.mrad., it is found that the average angle between $\vec{n}(\vec{u};s)$ and $\vec{n}_0(s)$, δ^{ave}_{n,n_0}, obtained by averaging over the phase space is 1 radian or more at 820 GeV! As expected it is even larger near resonances. Related calculations using very different but equivalent algorithms by G. Hoffstätter [25] and by V. Balandin and N. Golubeva [19], confirm this finding. However, since δ^{ave}_{n,n_0} is large, the linear approximation is no longer valid and it is clear in retrospect that SLIM results for HERA-p at high energy

[9]Linear (i.e. integrable) orbital motion can be described in terms of three generalised harmonic oscillators.

[10]With respect to $\vec{n}(\vec{u};s)$ the spin density matrix at $(\vec{u};s)$ is diagonal.

[11]This leads to nontrivial questions regarding the existance of solutions and the value of approximations [23].

[12]The depolarization rate is proportional to the ensemble and azimuthal average of $(\frac{\partial\vec{n}}{\partial(\delta E)})^2$ [6].

[13]Then in this (first order) calculation only vertical betatron motion contributes, not synchrotron motion [3].

must be treated with caution. Nevertheless the message that δ^{ave}_{n,n_0} is large remains valid. This implies a potentially strong limitation on the polarization of the whole bunch. For example with a δ^{ave}_{n,n_0} of 60 degrees, the upper limit on the polarization of the whole bunch is $\approx \cos(60^0) = 0.5$.

The largeness of δ^{ave}_{n,n_0} is also reflected in some exotic behaviour observed when tracking spins at fixed energy which are initially set parallel to $\vec{n}_0$ [19].

In view of the obvious need to be certain about δ^{ave}_{n,n_0}, it is now planned to implement a new version of the SODOM algorithm [26] [14]. This is much more mathematically sophisticated than SLIM in that it employs a Fourier representation to build the periodicities into the T-BMT solutions from the beginning. However, the SODOM algorithm will still require careful evaluation and checking, especially when studying the effects of the very slow synchrotron motion ($Q_s \approx 0.001$).

Both SODOM and SLIM allow $\vec{n}(\vec{u};s) - \vec{n}_0(s)$ and δ^{ave}_{n,n_0} to be extracted from one turn spin-orbit maps. Long term multi-turn tracking is not needed. Nevertheless, special multi-turn tracking algorithms will be employed for spot checks on the $\vec{n}(\vec{u};s)$ delivered by SODOM. For very low emittances SLIM and SODOM results should coincide.

It is obvious that δ^{ave}_{n,n_0} decreases as both the invariant emittance and the beam energy decrease. But reliable estimates of the potential gains will only be available when SODOM or an equivalent program is running. Since δ^{ave}_{n,n_0} increases with the strength of nearby resonances, it might be possible to reduce δ^{ave}_{n,n_0} at the interaction points by strong [3,4] or harmonic spin matching [27] –providing the optics is sufficiently flexible.

There are two points which limit the validity of our approach. Firstly, in practice a proton beam is never exactly in equilibrium and feedback systems are needed to damp embryonic coherent oscillations. Secondly, at HERA the beam size increases over a period of many hours due to noise in magnet power supplies, the beam-beam effect and field nonlinearities. So the orbital motion is not exactly integrable. But nevertheless we believe that the investigation of $\vec{n}$ provides a powerful way to understand the polarization properties of a beam. Indeed, since the beam blow-up due to beam-beam effect and power supply ripple is due to deterministic external electromagnetic forces and is adiabatic, and if we can neglect direct spin flip effects, knowledge of $\vec{n}(\vec{u};s)$ enables us to estimate the effect on the polarization: we speculate that the main result of beam blow-up will be an adiabatic increase of δ^{ave}_{n,n_0} resulting directly from an increase in the beam size and will lead to a decrease in the average polarization of the beam.

How would the polarization distribution at high energy arrive at equilibrium? The key to understanding this is to note that $\vec{n}(\vec{u};s)\cdot\vec{S}$ is an adiabatic invariant. At low energy δ^{ave}_{n,n_0} is small and so is the angle between $\vec{n}_0$ and $\vec{S}$ for each particle. Thus at low energy the average angle $\delta^{ave}_{n,\vec{S}}$, between $\vec{n}(\vec{u};s)$ and $\vec{S}$ is also small and if mishaps are avoided during the adiabatic acceleration it is still small at high energy. Then the polarization at high energy will be almost parallel to $\vec{n}$. [15].

Note that the equilibrium function $\vec{P}_{eq}(\vec{u};s)$ is by definition completely independent of the method used to polarize the beam. In particular, if δ^{ave}_{n,n_0} is very large, it could be that $\vec{P}_{eq}(\vec{u};s)$ at the edge of phase space is perpendicular to $\vec{n}_0$. This could have important implications for methods for polarization based either on the transverse or on the longitudinal Stern-Gerlach

[14]Containing better diagnostic facilities than the original version.

[15]Note that we could also have used the notion of adiabatic invariants when discussing the deliberate spin flip caused by Partial Snakes. When calculating $\vec{n}_0$ at *fixed* beam energy in the presence of imperfections one finds that $\vec{n}_0$ turns over on passing though integer values of ν_0. Then since during acceleration and in the absence of synchrobeta motion $\vec{n}_0(s)\cdot\vec{S}$ is an adiabatic invariant the spins follow $\vec{n}_0$ and flip too.

effect [16].

Polarimetry based on scattering protons out of the tails of the beam could also be difficult.

5 Summary and Conclusion

The HERA $e^{\pm}$-p colliding beam facility is unique: it provides a high energy, high quality and highly polarized electron (positron) beam for collisions with 820 GeV protons. One of the next steps should be to try to polarize the proton beam. Given the high electron polarization available it would surely be a tragedy not to try this and, in the not so humble opinion of the author, it would be inadvisable to rely for motivation solely on current fashions in elementary particle theory.

As has been pointed out above, a large amount of work has already been invested in thinking of ways to accelerate polarized protons up to high energy and most of those ways could be adapted for HERA. It is not necessary to reinvent the wheel!

But calculations based on concepts adapted from the theory of radiative electron polarization suggest that for proton beam energies of the order of 1 TeV and above, there could be a fundamental limitation on the polarization (averaged over the beam) attainable.

For HERA this might mean that both the beam energy and the emittance must be reduced.

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