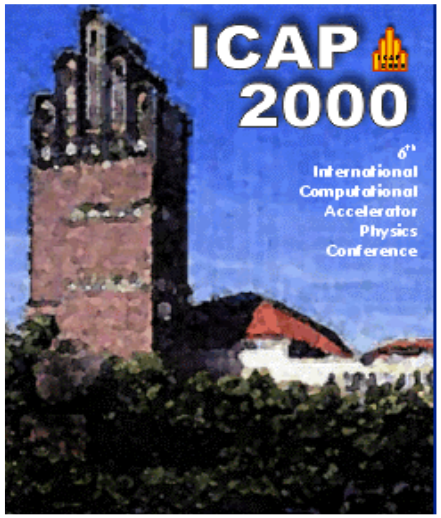
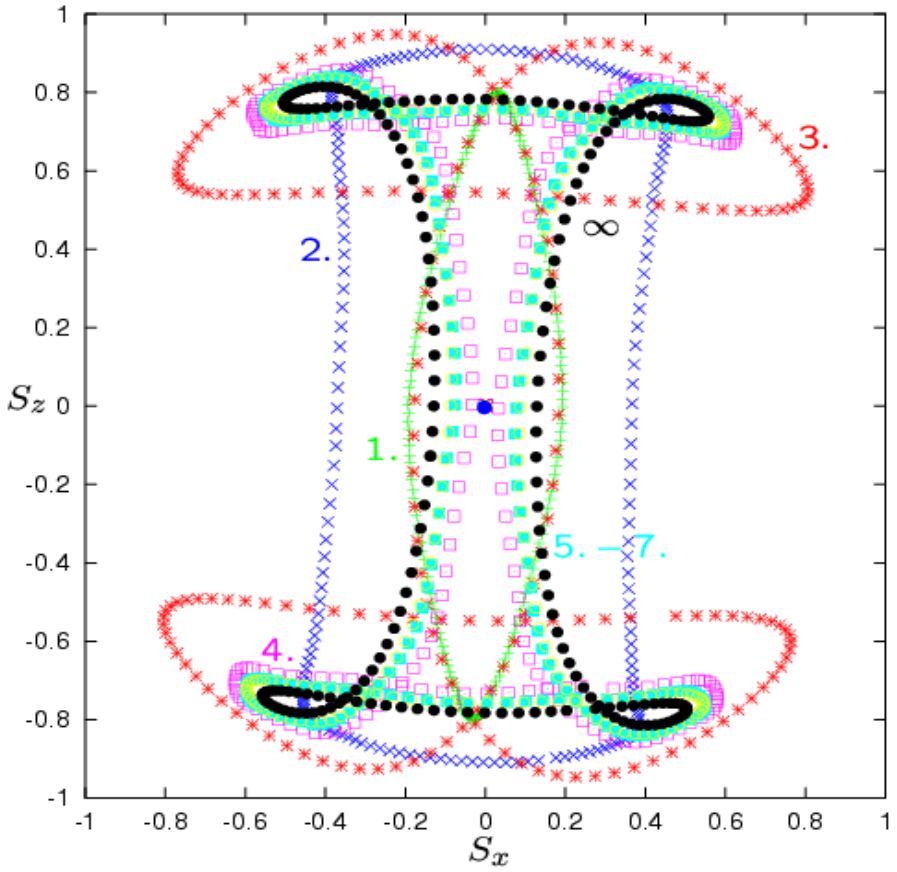
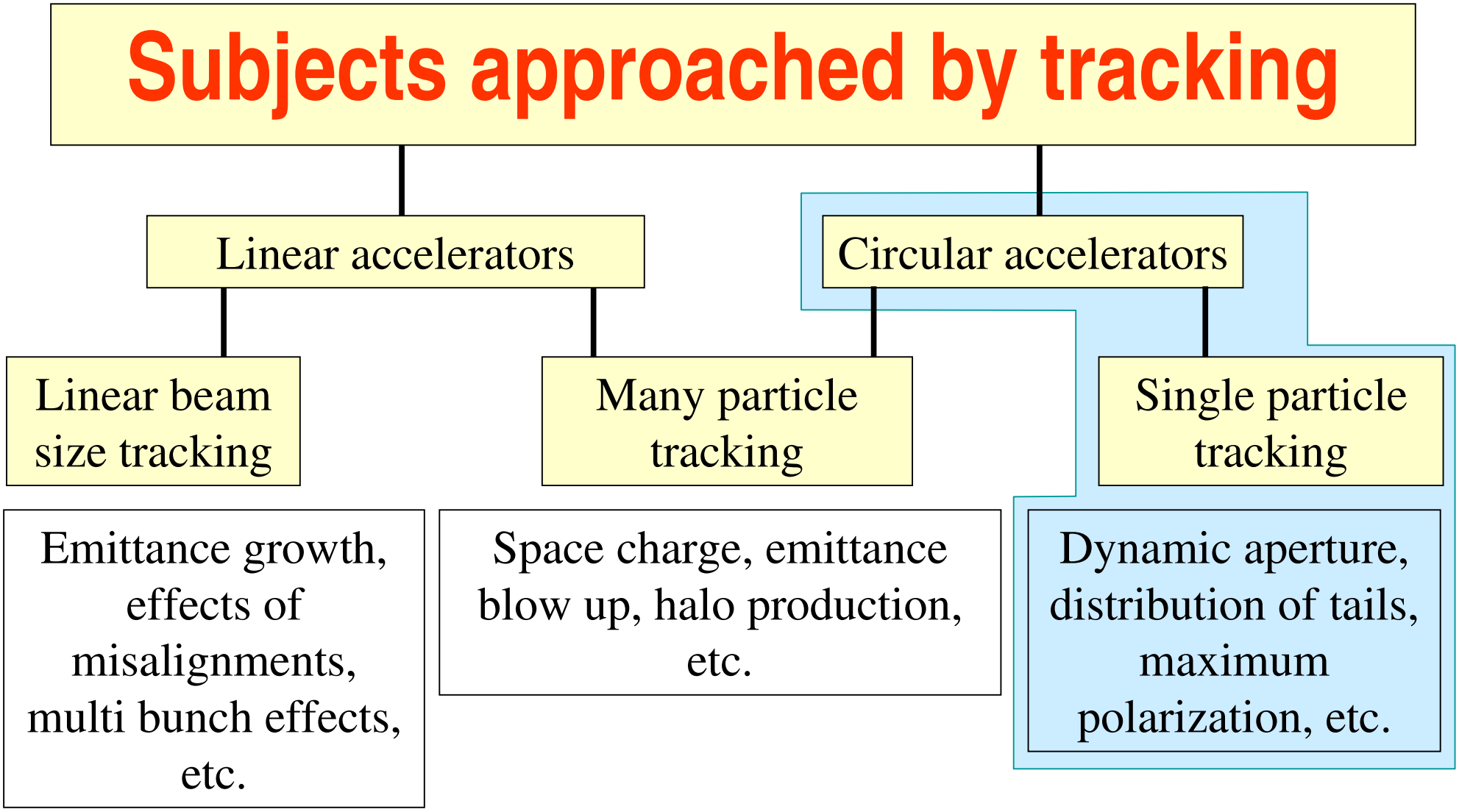


State of the Art in Particle Tracking

Georg H. Hoffstaetter
DESY, Hamburg
11.Sep.2000



Subjects approached by tracking



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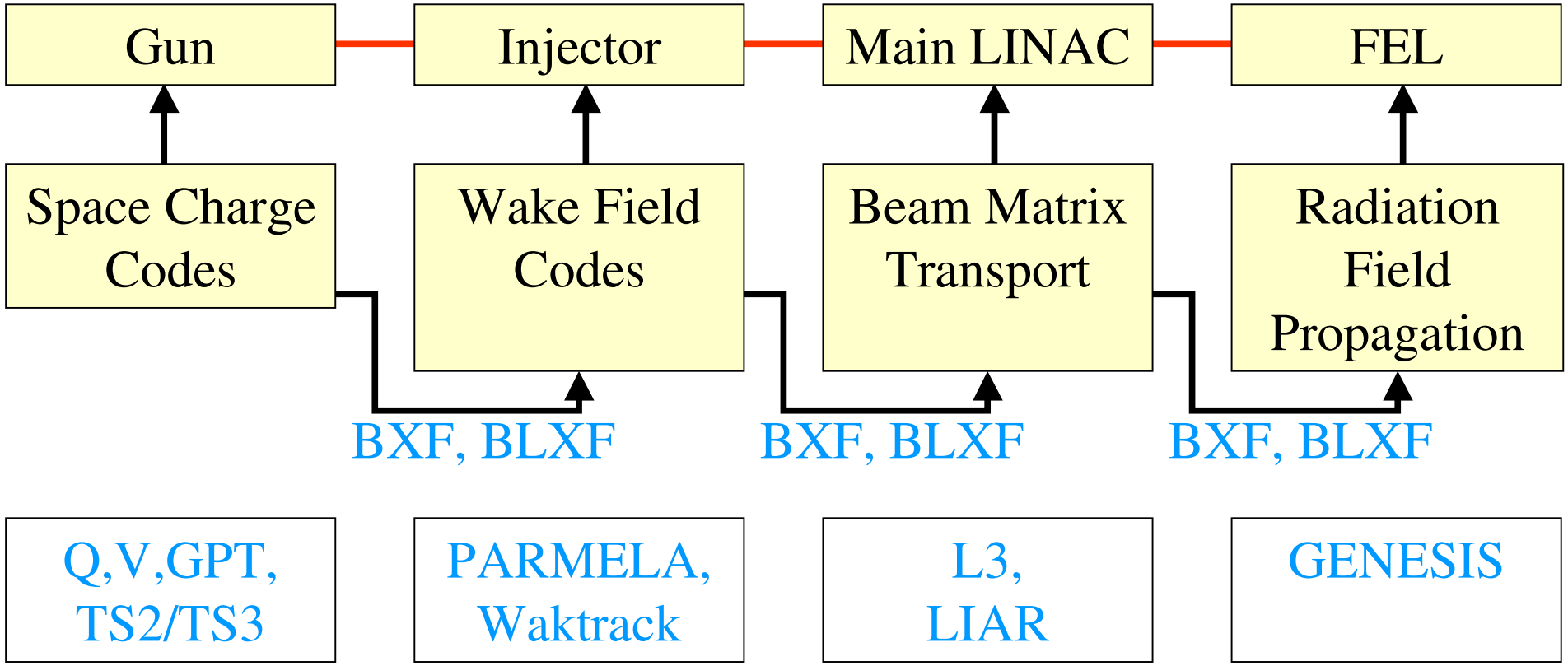
Example for Linear Accelerators: The *Track Project



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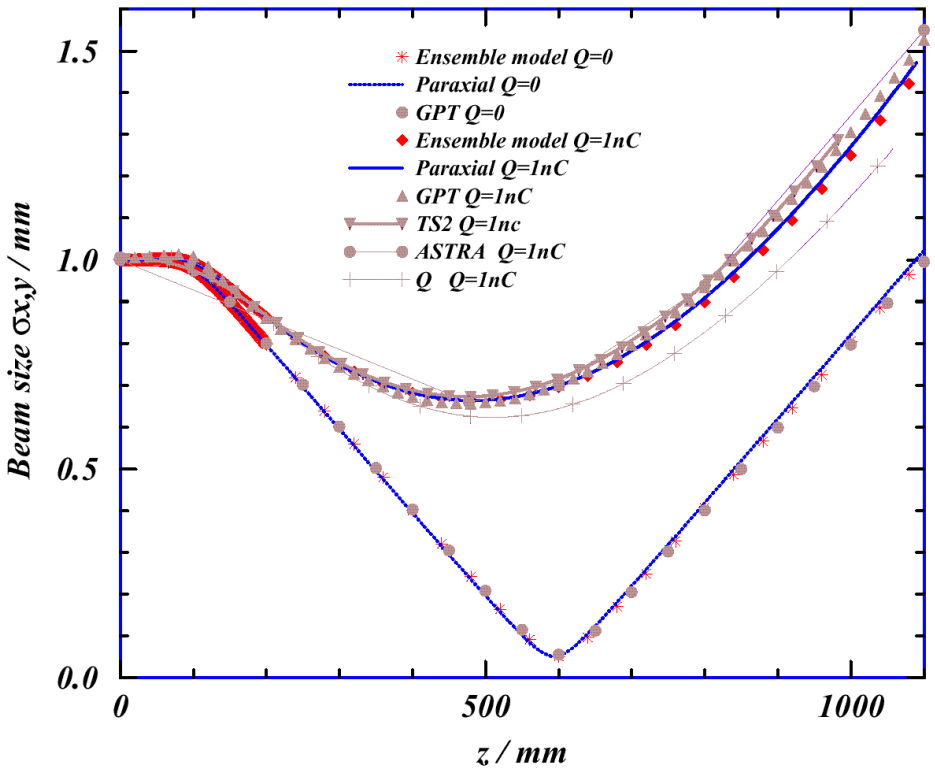
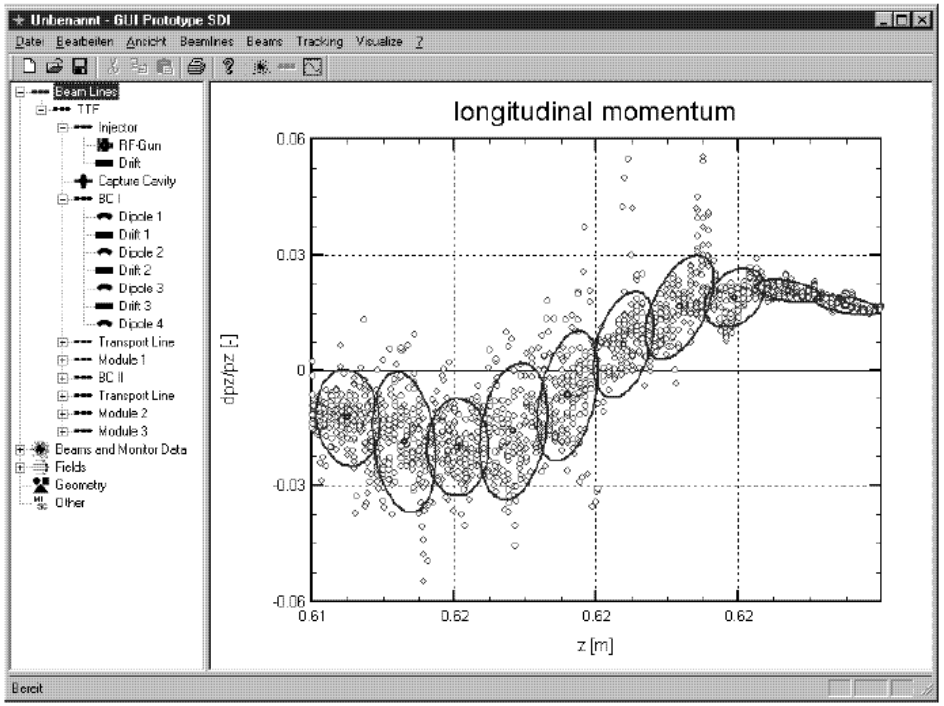
*Track example





- Comparison of different approaches is made simple
- Different approaches can be applied to the same data

Figures from: B. Krietenstein et al., Proceedings of FEL99

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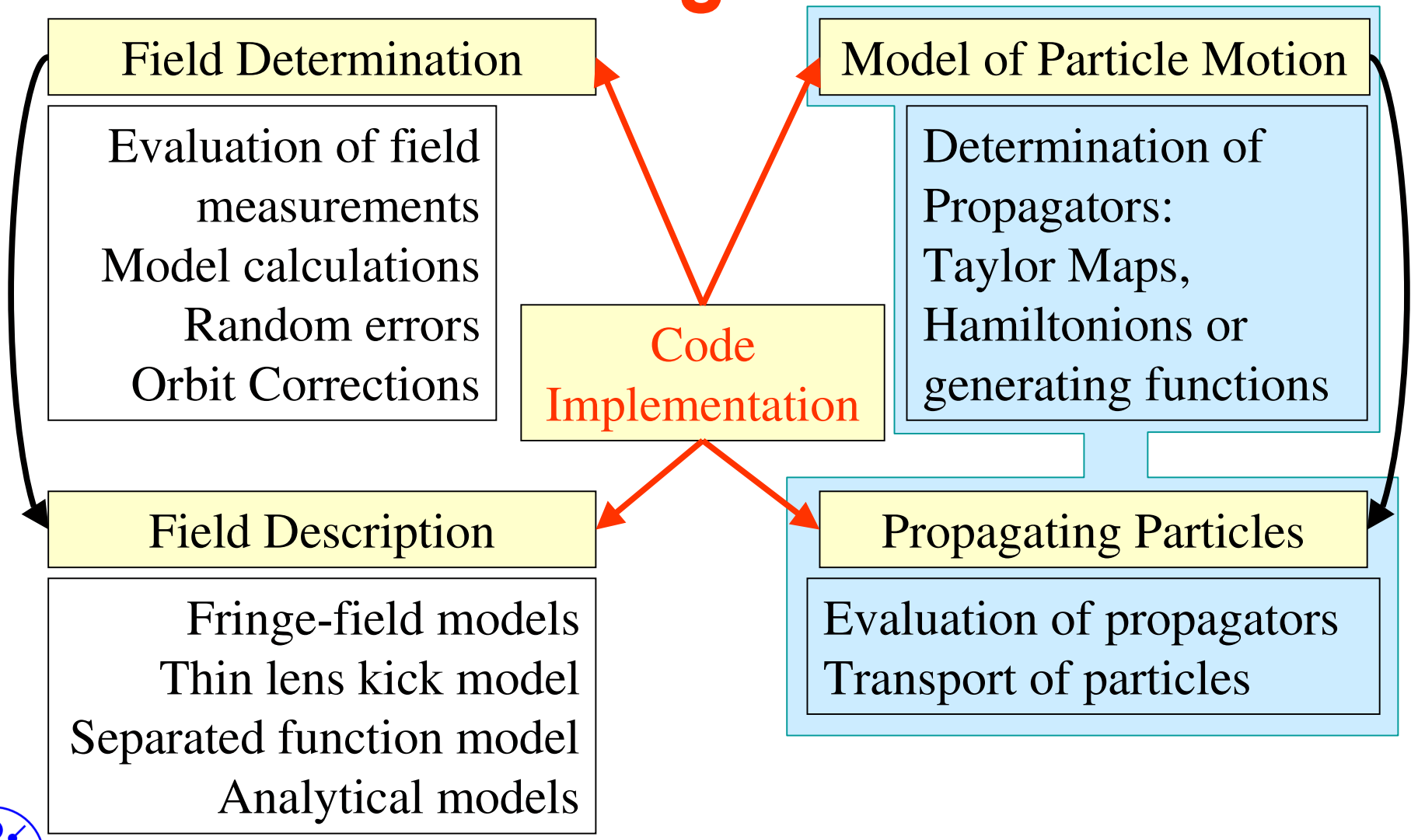
Tracking Codes



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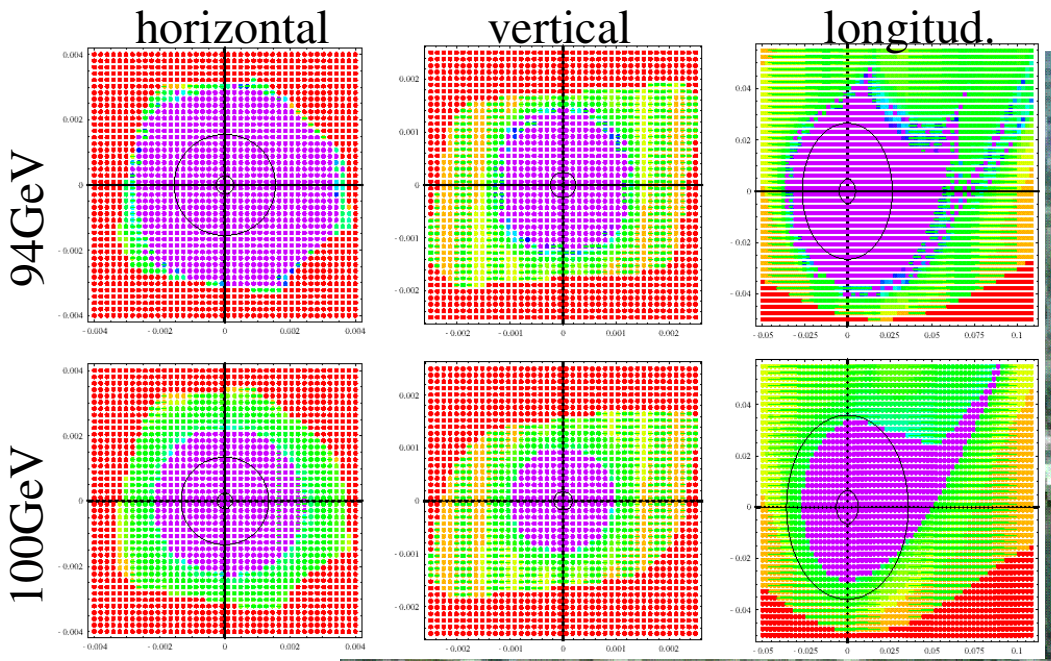
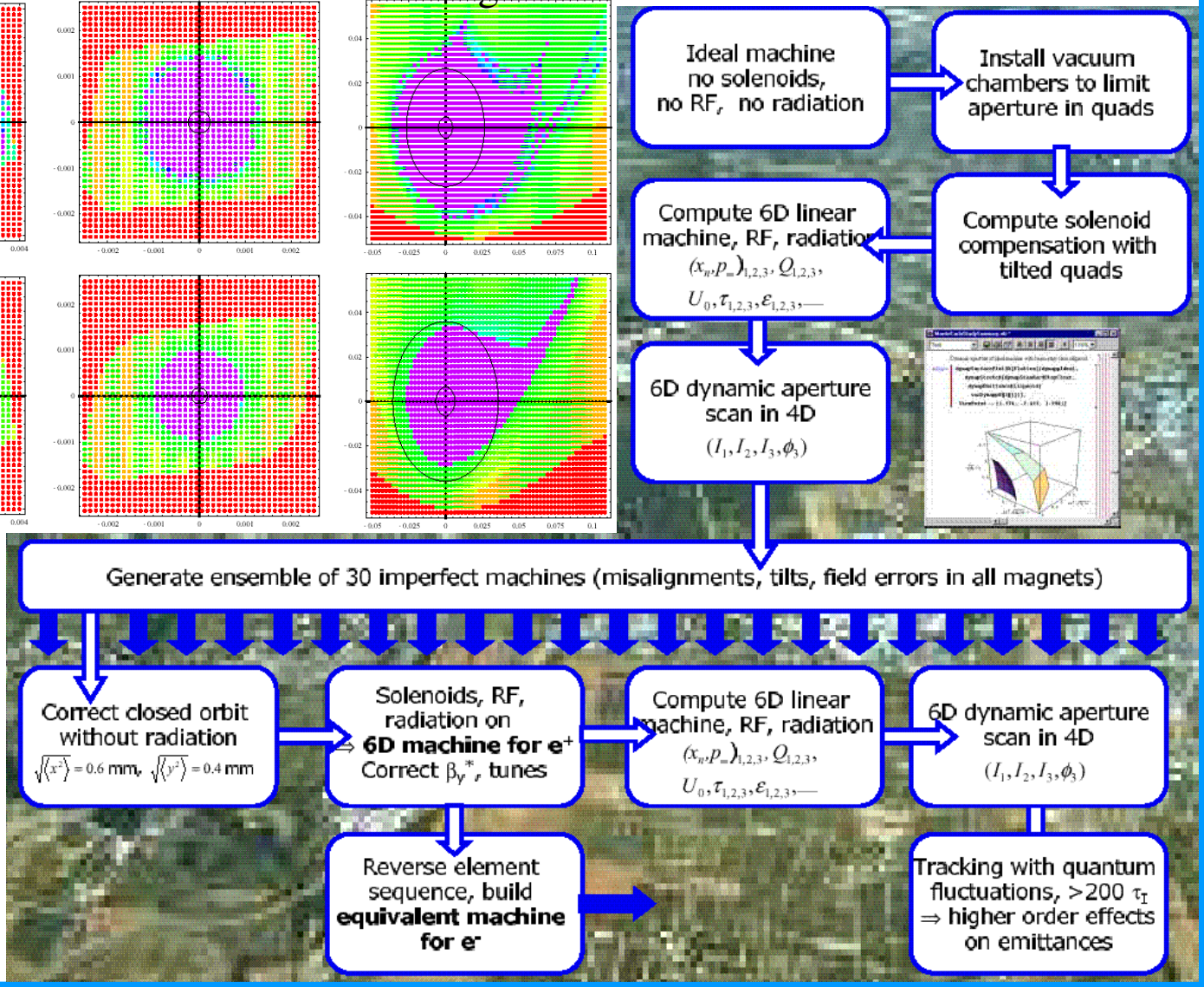
Example: DA Prediction for LEP





Performed by a MATHEMATICA environment driving MAD

Taken from J. Jowett, Contributions PAC99

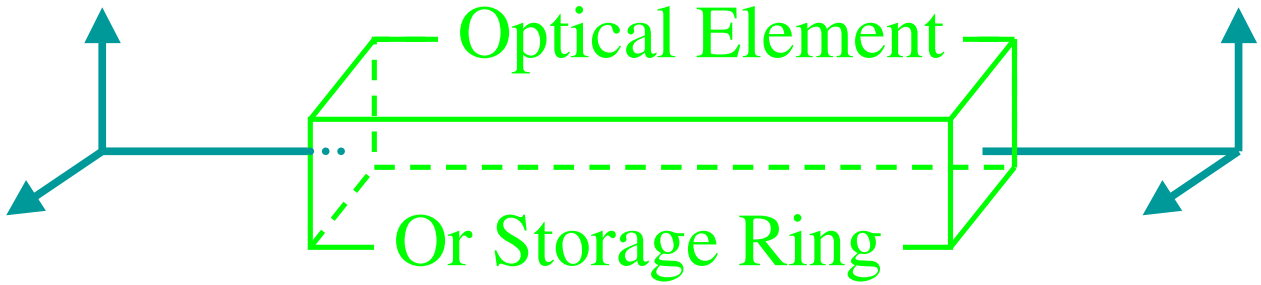
Phase Space Maps

$\vec{z}_i$

$$x, \frac{p_x}{p_0}, y, \frac{p_y}{p_0}, -t\frac{E_0}{p_0}, \frac{E}{E_0}$$

$$\vec{z}_f = \vec{M}(\vec{z}_i, \vec{\delta})$$

$\vec{z}_f$



Stochastic Maps

Electron distributions,
Polarization buildup,
Halo production by noise

Deterministic Maps

Symplectic motion:
Description by Hamiltonians,
Generating functions, Lie operators

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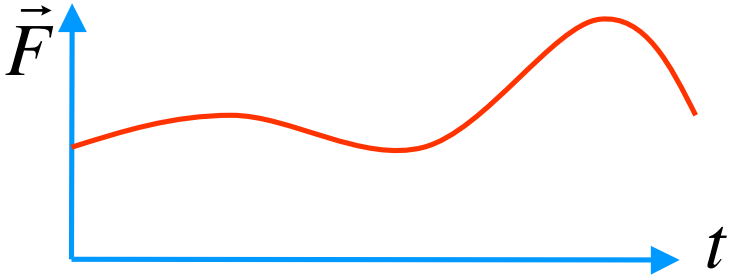
Simple Symplectic Integration

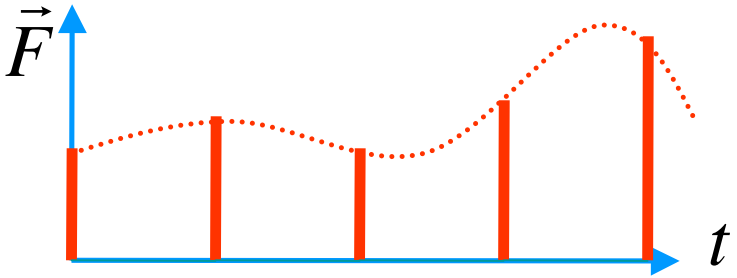
$$\frac{d}{dt}\vec{x} = \vec{p}$$
$$\frac{d}{dt}\vec{p} = \vec{F}(\vec{x},t) = -\vec{\nabla}V(\vec{x},t)$$

Symplectic map

$$\frac{d}{dt}\vec{x} = \vec{p}$$
$$\frac{d}{dt}\vec{p} = \sum_n \Delta t\,\delta(t - n\Delta t)\,\vec{F}(\vec{x}, n\Delta t)$$

Symplectic approximation for the map





Approximation of Kicks and Drifts

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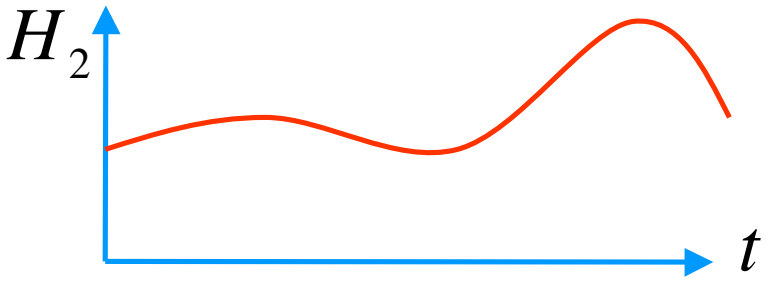
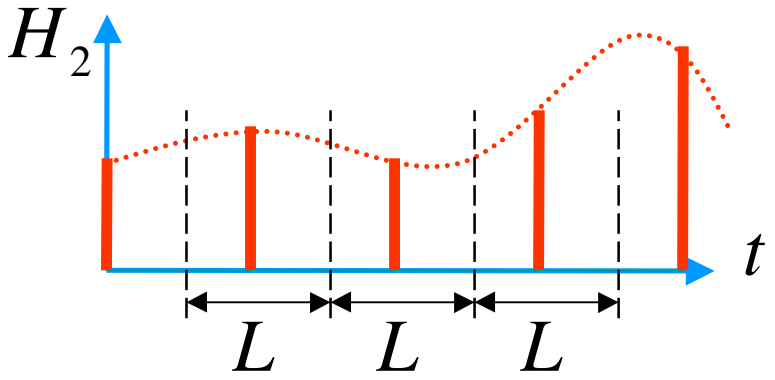
Symplectic Integration

For Hamiltonians with solvable parts: e.g. $H(\vec{z}) = T(\vec{p}) + V(\vec{x})$

$$H(\vec{z}) = H_1(\vec{z}) + H_2(\vec{z}) \approx H_1(\vec{z}) + L\delta(l - \tfrac{L}{2})H_2(\vec{z})$$





$$\vec{M}(\vec{z}, L) = e^{L:H(\vec{z}):}\vec{z}$$

Symplectic map

$$\approx e^{\frac{L}{2}:H_1(\vec{z}):} e^{L:H_2(\vec{z}):} e^{\frac{L}{2}:H_1(\vec{z}):}\vec{z}$$

Symplectic approximation for the map

Error is of 3rd order in L. Higher order integrators also exist.

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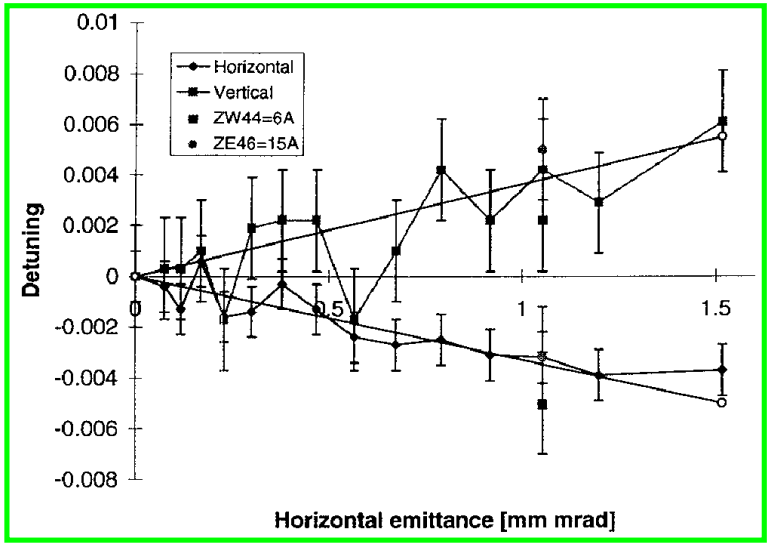
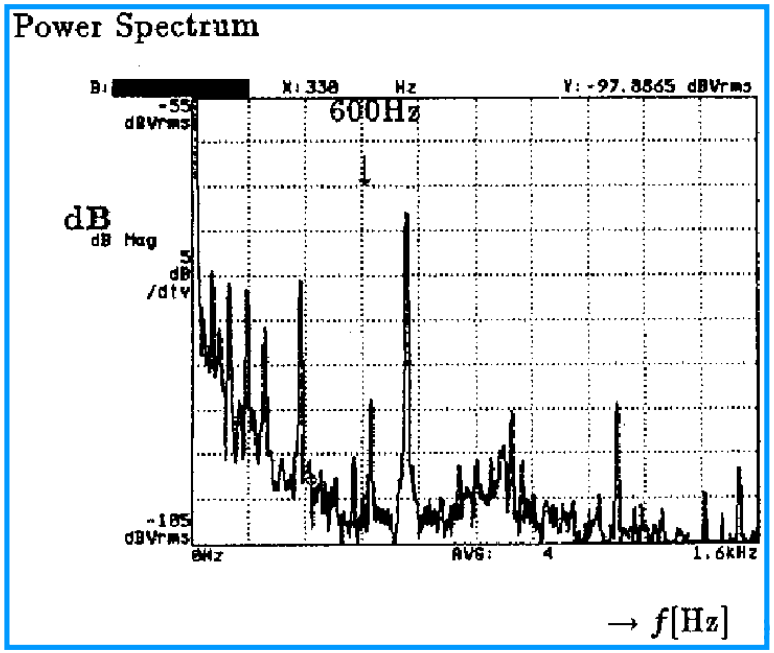
Example: DA Prediction for HERA

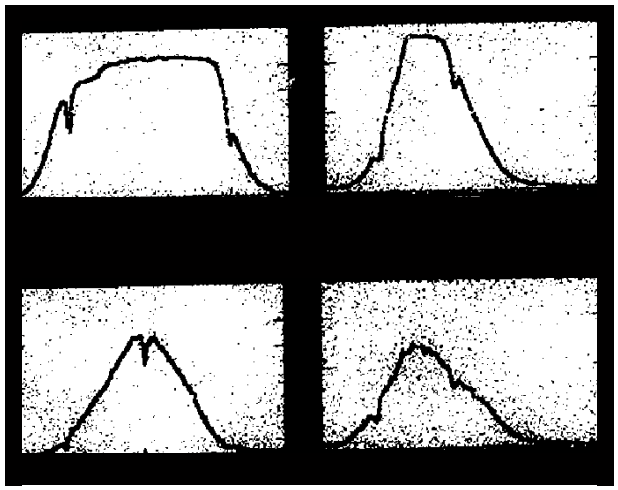
- Include measured multipole errors of magnets
- Adjust field errors to obtain measured detuning with amplitude
- Include measured tune ripple



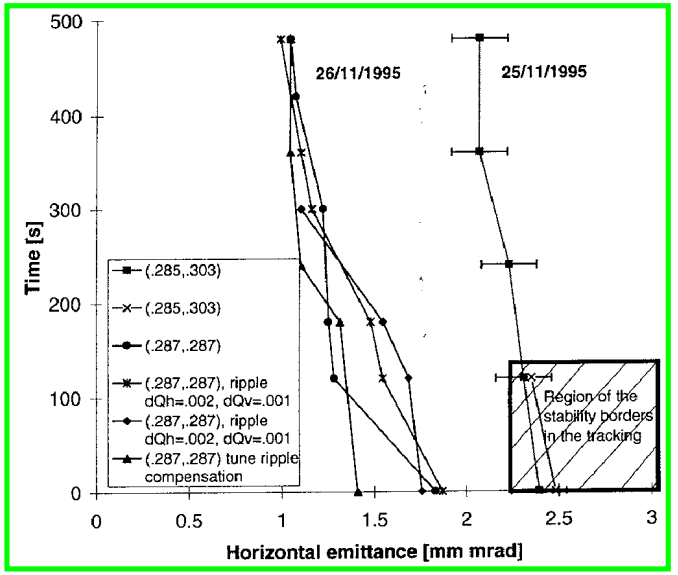


Taken from O. Brüning et al. Particle Accel. 54/96

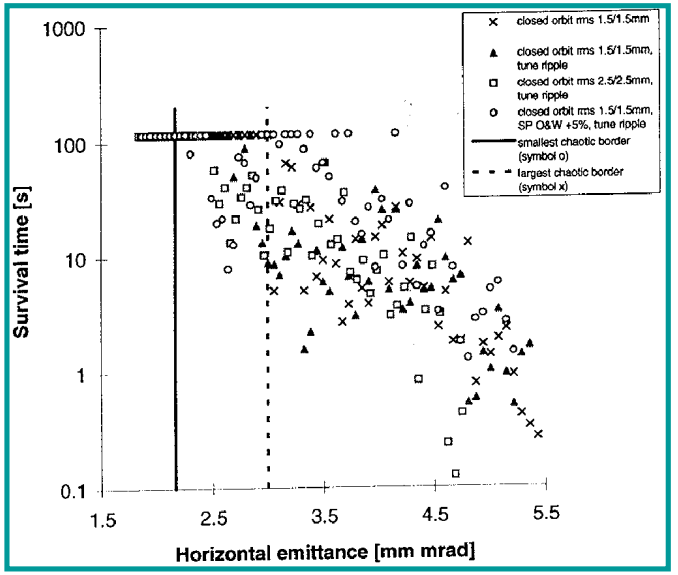
DA Measurement and Prediction for HERA



DA from profile measurements



DA from Tracking:
Performed with SIXTRACK
using Drift-Kick second order
symplectic integration per element



Taken from O. Brüning et al. Particle Accel. 54/96

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Computing Taylor Maps

$$g + h \longrightarrow \{g+h\} = \{g\}"+"\{h\}$$

$$g * h \longrightarrow \{g*h\} = \{g\}"*"\{h\}$$

$$e(g) \longrightarrow \{e(g)\} = "e"(\{g\})$$

Algorithm like a symplecti integrator

$$f = \vec{M}(\vec{z}) \longrightarrow \{f\} = "\vec{M}"(\{\vec{z}\})$$

Differential Algebra: property = Taylor map

$$\vec{M}(\vec{z}) =_n \vec{M}_n(\vec{z})$$

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Tracking through Taylor Maps

Problem: $\vec{M}_n(\vec{z})$ is not symplectic

Symmetric factorization ↔ Generating function tracking

$\vec{M}_n(\vec{z}_i) =_n \vec{z}_f$

$\vec{z} = (\vec{q}, \vec{p})$
$\vec{x} = (\vec{q}_i, \vec{p}_f)$
$\vec{y} = (\vec{p}_i, \vec{q}_f)$

Dif. Algebra

$$e^{L:H(\vec{z}):} =_n e^{L:\ldots+H_1(\vec{z})+H_2(\vec{z}):}\vec{z}$$

$$\approx \cdots e^{\frac{L}{2}:H_1(\vec{z}):} e^{L:H_2(\vec{z}):} e^{\frac{L}{2}:H_1(\vec{z}):}\vec{z}$$

… and higher order symplectic integrators

Dif. Algebra

$\partial_{\vec{y}} F_n(\vec{y}) =_n \vec{x}$ → $\partial_{\vec{y}^*} F_n(\vec{y}^*) = \vec{x}^*$

Numerical inversion → Symplectic map

nPB tracking ↔ Cremona maps

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Cremona Maps

(Symplectic polynomual maps)

- Kicks have Cremona maps:

$$\vec{q}_f = e^{:k(\vec{q}):}\,\vec{q} = \vec{q}$$

$$\vec{p}_f = e^{:k(\vec{q}):}\,\vec{p} = \vec{p} + \partial_{\vec{q}} k(\vec{q})$$

- Jolts have Cremona maps:

$$:g(\vec{z}):: g(\vec{z}):\vec{z} = 0$$

$$\vec{z}_f = e^{:g(\vec{z}):}\,\vec{z} = \vec{z} + :g(\vec{z}):\vec{z}$$

- Kick-, Jolt-, integrable polynomial-factorization:

$$\vec{M}_n(\vec{z}_i) =_n \cdots e^{:g_4(\vec{z}):} e^{:g_3(\vec{z}):} e^{:g_2(\vec{z}):} e^{:g_1(\vec{z}):}\,\vec{z}$$

Optimized codes are now available: e.g. J. Shi, D. Abell.

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nPB Tracking

$$\vec{z} \Rightarrow \vec{J}, \vec{\phi}$$

Dif. Algebra

$$\vec{M}_n(\vec{z}) \Rightarrow e^{:h_T(\vec{J}):} \; e^{:h_R(\vec{J},\vec{\phi}):}$$

Analytic and very fast

$$e^{:h_R(\vec{J},\vec{\phi}):}(\vec{J},\vec{\phi}) \approx [1 + :h_R: + \tfrac{1}{2}:h_R:^2 + \tfrac{1}{6}:h_R:^3](\vec{J},\vec{\phi})$$

Not completely accurate and not completely symplectic but describes the main features of the ring and after further simplifications allows for extremely fast tracking studies.

Optimized codes are available: e.g. Y. Yan, J. Irwin.

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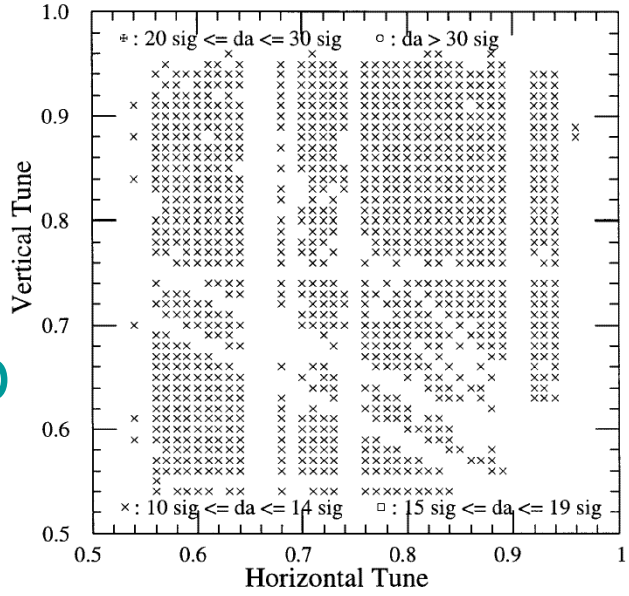
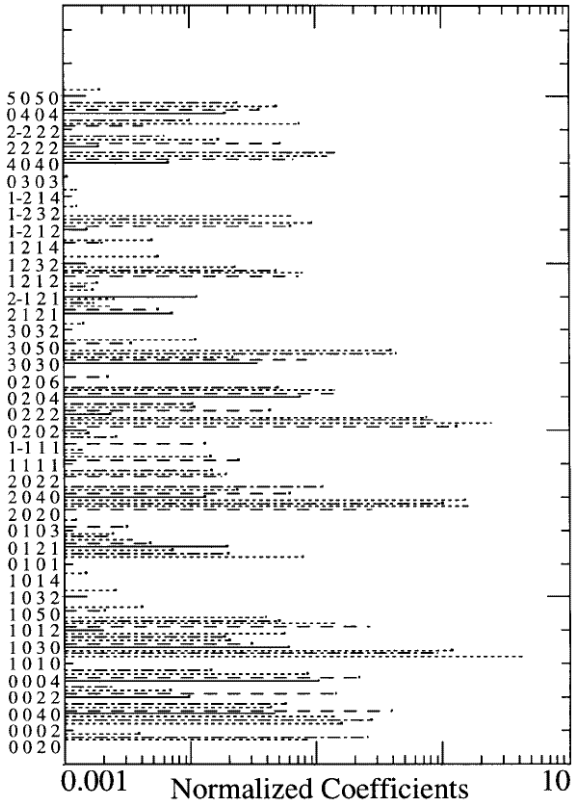
Simplifications in nPB Tracking

- Elimination of small coefficients in the expansion of $h_T(\vec{J})$ and $h_T(\vec{J})$
- Analysis of detuneing and resonanceterms by changing their strength
- Analysis of tune changes without computation of map





Figures from Y.Yan et al. SLAC/96

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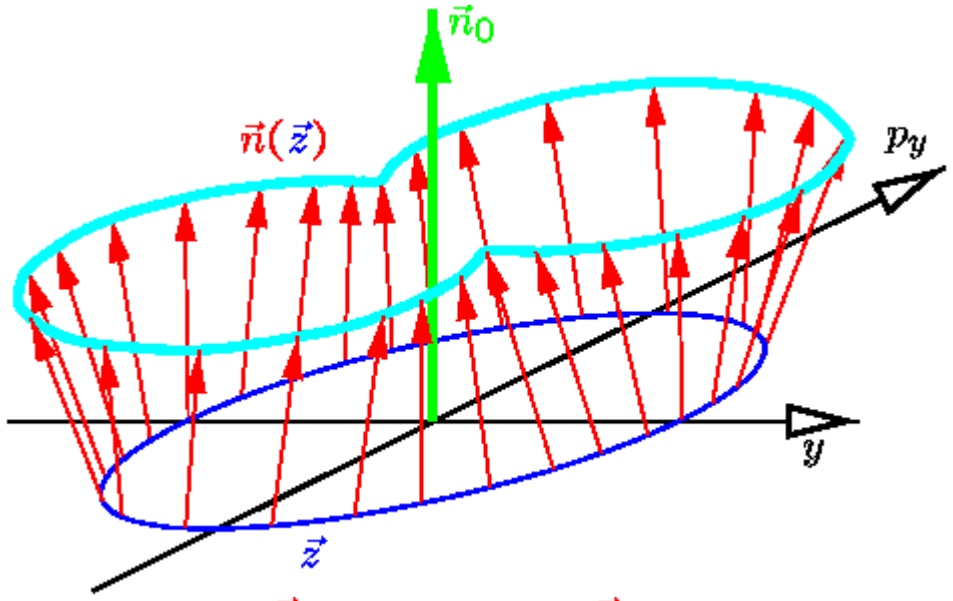
Spin-Orbit Tracking with Quaternions

Computation of the invariant spin field by analyzing tracking data:

- Fourier analysis
- Stroboscopic averaging
- Antidamping



$$\vec{S}_{n+1} = \underline{A}(\vec{z}_n)\vec{S}_n$$

defines the $\vec{n}$–axis

$$\vec{n}(\vec{z}_{n+1}) = \underline{A}(\vec{z}_n)\vec{n}(\vec{z}_n)$$

Computations perfomed in SPRINT, Hoffstaetter and Vogt, DESY/00

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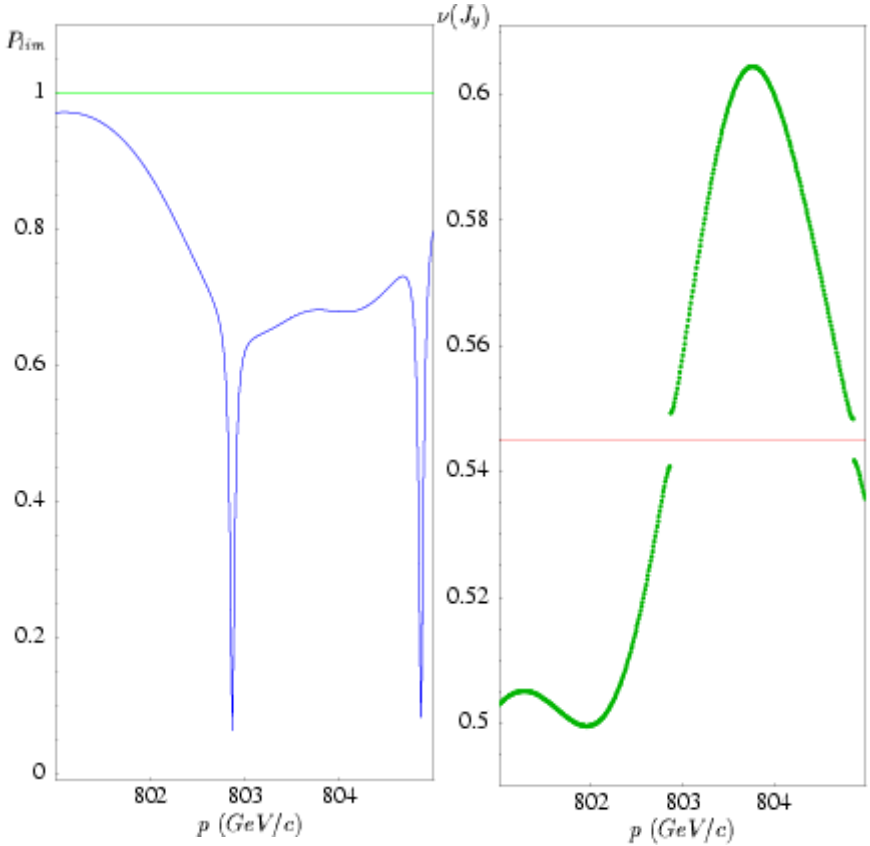
High Order Resonance Strength

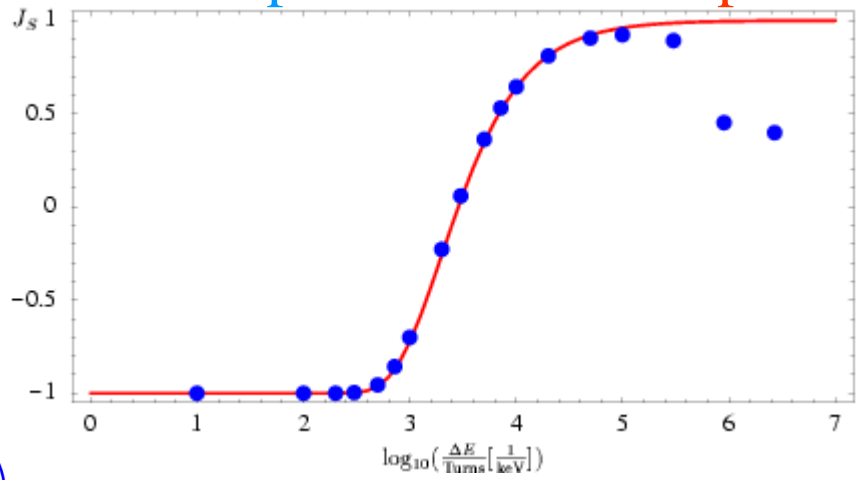
- Resonances up to 19th order can be observed
- Resonance strength can be determined from tune jump.

$P_{\mathrm{lim}} = < \vec{n}(\vec{z}) >$

Spin tune



Tracked depolarization as expected



Computations perfomed in SPRINT, Hoffstaetter and Vogt, DESY/00

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Computing Taylor Models

functions → Structure of properties

$$g + h \rightarrow \{g+h\} = \{g\}"+"\{h\}$$

$$g * h \rightarrow \{g*h\} = \{g\}"*"\{h\}$$

$$e(g) \rightarrow \{e(g)\} = "e"(\{g\})$$

$$f = \vec{M}(\vec{z}) \rightarrow \{f\} = "\vec{M}"(\{\vec{z}\})$$

Algorithm like a symplecti integrator

Remainder-enhanced Differential Algebra:
property = Taylor model

$$\vec{M}(\vec{z}) =_n \vec{M}_n(\vec{z}) \text{ and } \vec{M}(\vec{z}) \in \vec{M}_n(\vec{z}) + [\vec{I}_-, \vec{I}_+]$$

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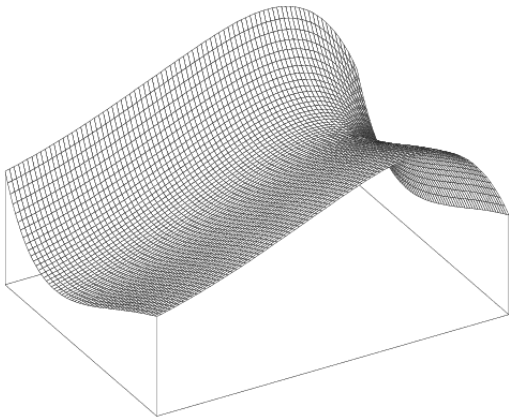
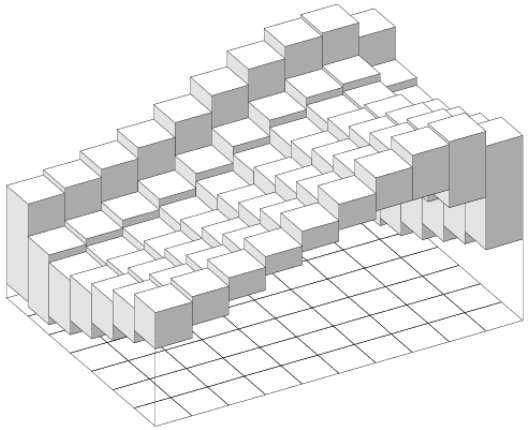
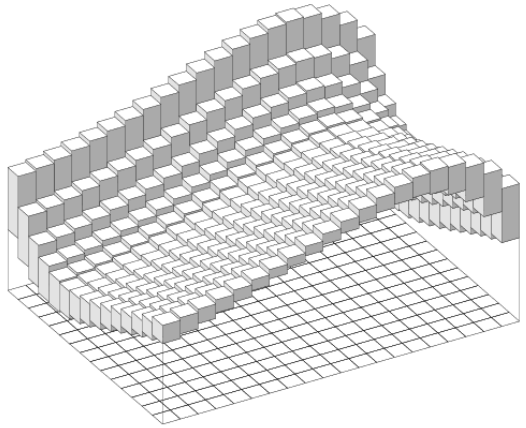
Tracking Intervals on Phase Space







- Rigorous statements about phase space regions can be obtained, since complete regions can be covered with intervals.

- Today still too slow for every day analysis

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