Coupler kicks in high $Q_{\text{ext}}$ linacs

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$$\kappa = \frac{\Delta P_y}{\Delta P_{||}}$$
FIG. 5: Orbit of one electron through the ERL lattice.

Especially if the coupler kick has 0 phase difference to the accelerating field.
Coupler Kicks cause emittance growth

Especially if the coupler kick has $90^\circ$ phase difference to the accelerating field.
Ways to cancel coupler kicks

(1) Kick cancellation by symmetry

(2) Kick cancellation by changing the distance between coupler and cavity to make the phase of the coupler kick $90^\circ$.

(3) Kick cancellation by a more symmetric coupler region
# Cavity parameters

**TABLE I: Parameters of accelerating cavities.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1300 MHz</td>
</tr>
<tr>
<td>Number of Cells</td>
<td>7</td>
</tr>
<tr>
<td>Cavity Shape</td>
<td>TESLA type</td>
</tr>
<tr>
<td>Accelerating Voltage</td>
<td>15 MV/m</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>$Q_{ext}$</td>
<td>$\in {2 \times 10^7, 10^8}$</td>
</tr>
<tr>
<td>Coupler Type</td>
<td>Coax</td>
</tr>
<tr>
<td>Coaxial Impedance</td>
<td>50 $\Omega$</td>
</tr>
</tbody>
</table>
Modeling the coupler kick

(1) Insert inner conductor of the coax coupler until the desired $Q_{\text{ext}}$ is obtained.

(2) Compute standing wave patterns $E^m, B^m$ for magnetic boundary conditions at the coupler boundary.

(3) Compute standing wave patterns $E^e$ and $B^e$ for electric boundary conditions at the coupler boundary.

These Boundary conditions are those of a traveling wave in the coax at two different times, $\frac{1}{4}$ oscillation apart.

\[
E^\pm(r, t) = Re\{ (\xi E^m(r) \pm i E^e(r)) e^{-i(\omega t - \phi \pm)} \},
\]

\[
B^\pm(r, t) = \pm Re\{ (B^e(r) \pm i \xi B^m(r)) e^{-i(\omega t - \phi \pm)} \}.
\]

\[
E^\pm(s, t) = Re\{ (\xi E^m_0(s) \pm i E^e_0(s)) e^{-i(\omega t - \phi \pm)} \},
\]

\[
B^\pm(s, t) = \pm Re\{ (B^e_0(s) \pm i \xi B^m_0(s)) e^{-i(\omega t - \phi \pm)} \}.
\]
Why are fields e and m fields equivalent?

\[ \mathbf{E}_0^\pm(s, t) = \text{Re}\{(\xi \mathbf{E}_0^m(s) \pm i \mathbf{E}_0^e(s))e^{-i(\omega t - \phi_\pm)}\} \]

\[ \mathbf{B}_0^\pm(s, t) = \pm \text{Re}\{(\mathbf{B}_0^e(s) \pm i \xi \mathbf{B}_0^m(s))e^{-i(\omega t - \phi_\pm)}\} \]

\[ \mathbf{E}_0^e(s) \approx s^e \mathbf{E}_0^m(s), \quad \mathbf{B}_0^e(s) \approx s^m \mathbf{B}_0^m(s) \]

(a) \( Q_{\text{ext}} = 2 \times 10^7 \)

(b) \( Q_{\text{ext}} = 10^8 \)
The standing wave approximation

Approximation:
Traveling waves in the coax excite standing waves in the cavity. For very large $Q_{ext}$, very little energy leaves the coupler, which validates this approximation.

$$E_0^\pm(s, t) = \text{Re}\{(\xi E_0^m(s) \pm i E_0^e(s))e^{-i(\omega t - \phi_\pm)}\}$$

$$B_0^\pm(s, t) = \pm \text{Re}\{(B_0^e(s) \pm i\xi B_0^m(s))e^{-i(\omega t - \phi_\pm)}\}$$

Standing wave:
The fields must be a function of $s$ times a function of $t$: $f(s) g(t)$
$\rightarrow E_0^m$ must be proportional to $E_0^e$
$\rightarrow B_0^m$ must be proportional to $B_0^e$

$$E_0^e(s) \approx s^e E_0^m(s), \quad B_0^e(s) \approx s^m B_0^m(s)$$

The fields are normalized to the same energy $\rightarrow |s^e| = |s^m| = 1$.

$$E_0^\pm(s, t) \approx \text{Re}\{E_0^m(s)(\xi \pm is^e)e^{-i(\omega t - \phi_\pm)}\},$$

$$B_0^\pm(s, t) \approx \pm \text{Re}\{B_0^m(s)(\pm i)(\xi \mp is^m)e^{-(i\omega t - \phi_\pm)}\}.$$ 

To satisfy Maxwell’s equations, both complex factors must be the same.
$\rightarrow s^m = 1, s^e = -1$, or $s^m = -1, s^e = 1$
Coupler kicks of reversed cavities

\[ \Delta P^+ = q \int_{t_i}^{t_f} \left[ E^+_0(s, t) + \nu e_s \times B^+_0(s, t) \right] dt \]

\[ \Delta P^+ = e^{i\phi_+} \frac{q}{v} \int_0^L \left[ \xi E^m_0(s) + iE^e_0(s) \right] \]

\[ + \nu e_s \times \left[ B^e_0(s) + i\xi B^m_0(s) \right] e^{-i\omega_0^s \frac{s}{v}} ds. \]

\[ \Delta P^{+'} = e^{i\phi_+} \frac{q}{v} \int_0^L \left[ \xi E^m_0(L-s) + iE^e_0(L-s) \right] \]

\[ - \nu e_s \times \left[ B^e_0(L-s) + i\xi B^m_0(L-s) \right] e^{-i\omega_0^s \frac{s}{v}} ds. \]

\[ \kappa^{+'} \approx (\kappa^+)^* \]

<table>
<thead>
<tr>
<th></th>
<th>( Q_{ext} = 2 \times 10^7 )</th>
<th>( Q_{ext} = 1 \times 10^8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before Cav</td>
<td>After Cav</td>
</tr>
<tr>
<td>(</td>
<td>\kappa</td>
<td>(10^{-4}) )</td>
</tr>
<tr>
<td>( \phi_c ) (rad)</td>
<td>2.838</td>
<td>-2.793</td>
</tr>
</tbody>
</table>
\[ \Delta y' \approx 2 \Delta y_0' - 2 \frac{\Delta E_0}{E} |\kappa| \omega \cos(\phi_c) \sin(\psi) \Delta t. \]
Detuning, reflection, and coupler kicks

\[ \kappa = \frac{\Delta P^+_y + \alpha \Delta P^-_y}{\Delta P^+_s + \alpha \Delta P^-_s} \]

\[ \kappa' \approx \kappa^* \] (even for all phases of the reflected wave)
Moving the coupler to have 0 phase kick

The coupler was moved from 4.5cm to 5.3cm off the first cell.

FIG. 8: Normalized emittance in the y direction for the six coupler configurations for $Q_{ext} = 2 \times 10^7$. (a)

FIG. 9: Normalized emittance in the y direction for the six coupler configurations for $Q_{ext} = 1 \times 10^8$. 
Symmetryzing the coupler region

![Graph showing the effect of different stub lengths on the normalized emittance. The graph has a log-log scale on the y-axis and a linear scale on the x-axis. Two curves are shown: one for a 1 cm stub and another for a 0.5 cm stub. The y-axis ranges from 1e-7 to 1.1e-7, and the x-axis ranges from 0 to 2500.]
Conclusion

(1) Kick cancellation by symmetry

(2) Kick cancellation by $0^0$ phase.

(3) Kick cancellation by a more symmetric coupler region.