A-Exam: Theory and phenomenology of Randall-Sundrum (RS) models

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In this section I will discuss the motivation for RS models, describe the initial RS model, explain the issues with flavor changing neutral currents (FCNC's) with the initially proposed model, and show how this problem can be solved and the solution leads to an explanation of fermion mass hierarchy.

Introduction: The Electroweak Hierarchy Problem

The Standard Model (SM) has been very successful at explaining experimental results over the last 50 years. There are however problems with the SM, one of which is known as the electroweak hierarchy problem. The problem is that corrections (fermion loops) to the Higgs scalar mass squared result in very large masses (of order our cut off, which is M_{Pl}). These divergences go as Λ^2 instead of the usual $log(\Lambda)$ divergence and thus do not vary slowly at high energies and the exact cut off becomes more important.

The Higgs mass without correction is given by:

$$m^2_{Higgs} = 2\mu^2 = 2\lambda v^2 \tag{1}$$

where λ is the yukawa coupling and v is the vacuum expectation value (VEV)for the Higgs. However from the mass of the W and Z bosons, and the electron charge we know that v=246 GeV. For perturbation theory to work we want a small dimensionless parameter, λ. This would imply a Higgs mass which is smaller than 246 GeV, but if we consider correction due to the diagram shown in Figure 1 we get an additive correction that goes as Λ^2. If we want this divergence to cancel when we apply renormalization we will want the bare mass of the Higgs to be of order our cut off. This cut off would be of order M_{Pl} which is much larger than the value of the Higgs mass that we would measure. This unexplained hierarchy is known as the electroweak hierarchy problem.

There are several phenomenological models that propose a solution to this problem such

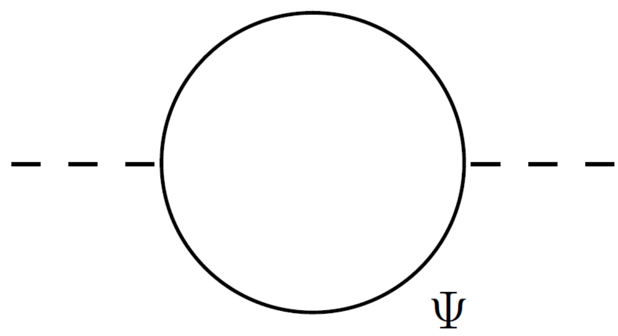

FIG. 1: Feynman Diagram for fermion loop correction to Higgs mass

as, super symmetry (SUSY), models with larger extra dimensions, and Randall-Sundrum models. Super symmetry aims at canceling the Λ^2 divergences with other diagrams that contain bosons in the loop. These bosons would be super symmetric partners of the fermions in the original diagram. Models with large extra dimensions have the SM fields contained on a brane while gravity propagates through several extra dimensions (beyond our four Minkowskian dimensions). These extra dimensions with large compactification radii cause a scaling of M_{Pl} down to the TeV range. The RS Model only contains one curved compactified extra dimension in which only gravity propagates while the SM fields are confined to a brane. The curvature of the extra dimension causes a scaling down of M_{Pl} which scales down the Higgs mass to our expected value on the order of the electroweak scale. I will focus on the RS Model and it's evolution in this paper.

Description of the RS Model

In the RS model one extra dimension is added to our 4D Minkowskian space-time. This dimension is compact (finite) by requiring the periodicity condition $\phi \to \phi + 2\pi$ for ϕ, the coordinate in the extra dimension which runs from $[-\pi, \pi]$ [1]. We also require that $\phi \to -\phi$. In this model there will be two branes, one at $\phi = 0$ and one at $\phi = \pi$, which will be able to support 4D (3 spacial, 1 time dimension) field theories. The SM fields will be localized to the $\phi = \pi$ brane while gravity will be allowed to propagate through the extra dimension. I will call the 5D space the bulk and any field that is present in all dimensions a bulk field, for now the only bulk field is gravity. We will also assume that there is a 5D bulk cosmological constant and require that the effective 4D Poincaré invariance (invariance under translation, rotation, and boosts) is respected. These requirements yield the following metric in 5D space:

$$ds^2 = e^{-A(\phi)}\eta_{\mu\nu}x^\mu x^\nu + r_c^2 d\phi^2 \tag{2}$$

Here r_c is a length that gives us the proper length in the new dimension when multiplied by ϕ before orbifolding (requiring $\phi \Rightarrow -\phi$), thus $r_c\phi$ gives us the arclength of the extra dimensional circle. The $e^{-A(\phi)}$ is known as the warp factor and $\eta_{\mu\nu}$ is the 4D metric. Solving the Einstein equation for two branes in 5D results in

$$A(\phi) = r_c|\phi|\frac{-\Lambda}{24M_*^3} \tag{3}$$

Λ is the bulk cosmological constant, which to give a reasonable solution must be negative. For notation, we will say: $k^2 = \frac{-\Lambda}{24M_*^3}$. M_* is the 5D Planck's mass and comes from the 5D Newton constant in the Einstein equations.

Another result from solving the Einstein equation is that the two branes will have equal but opposite tensions [2]:

$$V_{Planck} = -V_{SM} = 24M_*^3 k \tag{4}$$

More importantly our metric is now:

$$ds^2 = e^{-2r_c k|\phi|}\eta_{\mu\nu}x^\mu x^\nu + r_c^2 d\phi^2 \tag{5}$$

Radius Stabilization

In the original RS model the radius is arbitrarily chosen to solve the hierarchy problem. The perturbations of this radius however would call for the existence of a new scalar field, the radion. This scalar field would have to be massless due to the arbitrary choice of the radius. This would however cause conflicts with known effective 4D Newton's law. The radion therefor must attain a mass and this can be done with the Goldberger-Wise Mechanism (GW). This mechanism introduces a brane potential at each brane to essentially have a kinetic term (which prefers small derivatives and thus large radii) and a potential term (which prefers small radii) to equilibrate the radius at a non-trivial minimum for both kinetic and potential terms. The radion would have interactions similar to the Higgs and would have it's own Kaluza-Klein tower.

Quick Word on Kaluza-Klein Modes

Introducing extra dimensions usually introduces excited modes of fields that propagate into these extra dimensions. This can be shown by the Kaluza-Klein (KK) expansion. I will only show the KK expansion for 1 extra flat dimension as an illustration as to where these KK excited modes, which will be mentioned later in the paper, may come from. The KK modes are dependent on Bessel functions in the RS model and are more complex than with one extra flat space dimension. In this quick example we will have one extra dimension which is constrained by periodicity, that is $\phi \rightarrow \phi + 2\pi$. The KK expansions for a field $\phi(x, y)$, where x are the usual 4D variables and y describes the new dimension, is

$$\phi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi_n(x) e^{iny/R} \tag{6}$$

Plugging this into our action:

$$S = \int d^4x \int dy (|\partial_M \phi|^2 - m^2 |\phi|) \tag{7}$$

will yield a new 4D action of the form

$$S = \sum_{n=-\infty}^{\infty} \int d^4x (|\partial_\mu \phi|^2 - m_n^2 |\phi|) \tag{8}$$

where

$$m_n^2 = m^2 + \frac{n^2}{R^2} \tag{9}$$

This causes and infinite tower of new 4D effective fields to arise with new masses, equally spaced apart. Experimental results put limits on the mass and couplings of the excited states since we would have observed them if their couplings were strong (compared to gravity) and masses were below ∼1 TeV. This calculation is more involved with the RS model and results in mass spacing that are not equally spaced and depend on the solutions to Bessel functions [3].

Solving the Hierarchy Problem

Now let's consider how a scalar field at the SM brane ($\phi = \pi$) will be affected by our metric. The Lagrangian that involves the Higgs kinetic and mass term would be given by:

$$\sqrt{-g^{ind}} [g_{\mu\nu}^{ind} \partial^\mu H^\dagger \partial^\nu H - \lambda (H^\dagger H - v_0^2)^2] \tag{10}$$

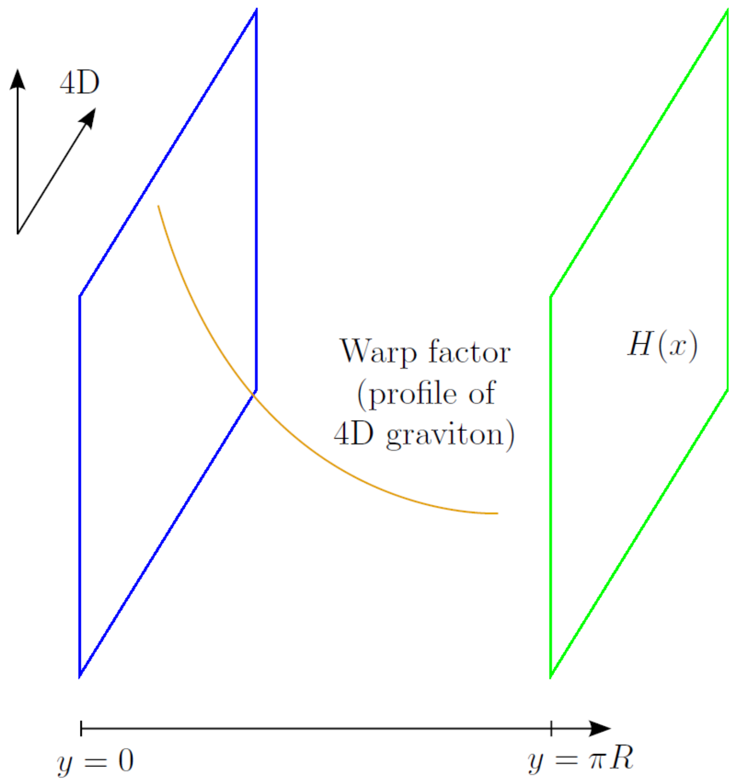

FIG. 2: The UV and IR brane separated πr_c

However from our 5D metric we know that the induced metric is given by

$$g^{ind}_{\mu\nu} = e^{-2kr_c\pi}\eta_{\mu\nu} \tag{11}$$

when we are at the SM brane ($\phi = \pi$). Here $\sqrt{-g^{ind}}$ is actually $\sqrt{\det(-g^{ind}_{\mu\nu})}$, and as a reminder $\det(cA) = c^n\det(A)$. After renormalization of the Higgs field ($H \to e^{-kr_c\pi}H$) this results in

$$\sqrt{-g}[\eta_{\mu\nu}\partial^\mu H^\dagger\partial^\nu H - \lambda(H^\dagger H - e^{-2kr_c\pi}v_0^2)^2] \tag{12}$$

Thus the weak scale Higgs VEV would be an exponentially suppressed VEV in a 5D theory. Even if v_0 is of scale M_{Pl} we can easily get the expected value of $\sim$ 250GeV by having $kr_c\pi \sim \mathcal{O}(10)$. This means that all 5D masses (such as the Higgs mass) will be warped down to their values on the order of the weak scale (Figure 2). This would explain why the bare mass of the Higgs could be so large compared to the electroweak scale.

Now however there may still be the same problem as with large extra dimensions: a new hierarchy is introduced involving the size of the extra dimension and M_*. Let's investigate this further by considering the zero modes of the gravitational fields and the 5D Einstein's equations. I will skip some of the details of the calculation and discuss the result which comes from the curvature term of the Lagrangian

$$M_{Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}] \tag{13}$$

which shows us that the 4D Planck scale depends weakly on r_c, the size of the extra dimension. Thus a large hierarchy is created by the exponential allowing a moderate radius without changing the Planck scale to a great degree, and thus not creating a new large hierarchy between $1/r_c$ and a 5D fundamental TeV Planck scale (as in ADD models). The fundamental energy scale for the 4D theory then becomes the TeV scale while the weakness of gravity can be explained by the small overlap of gravity's wave function with the TeV brane.

Flavor Changing Neutral Currents (FCNC) and the RS Model

In the original RS model all SM fields were localized on the SM brane (also called the IR brane) while gravity propagated through all dimensions. This however causes problems with calculating loop diagrams and renormalization. FCNC's in the SM only occur through loops involving W's and penguin diagrams, which result in operators that have dimensions greater than four. When calculating the cross sections for these diagram, divergences occur which require renormalization. When renormalizing, operators of dimension four or higher are suppressed as $\frac{1}{\Lambda}$ where Λ is our cut off, which in the SM was taken to be M_{Pl}.

$$\frac{\lambda}{\Lambda_{cutoff}}\psi\bar{\psi}\psi\bar{\psi} \tag{14}$$

However now that all of the SM fields are confined to the IR brane the relevant energy scale for cut offs has become the 1 TeV scale. This would prevent the suppression of these FCNC's and many other higher dimension operator processes needed to match strict experimental constraints. The measured $m_{K_L} - m_{K_s}$ ($\sim 10^{-12}$ MeV) value is an example of such an experimental constraint on FCNC's.

Bulk Fermions Fields and FCNC's

The suppression of FCNC's can be potentially achieved by moving the fermion and gauge fields to the bulk. This would allow us to suppress operators of dimension greater than four with a cut of order M_{Pl}. Since moving the fermions to the bulk will cause new fermionic KK modes we will focus on the zero modes, which we recognize as our SM fermions. From work in [4] we know that the 5D action for the fermions is

$$S_\Psi = \int d^4x \int dy \hat{\bar{\Psi}} \{ i\partial\!\!\!/ + e^{-k|y|}[k(\frac{\gamma_5}{2} - c)sgn(y) - \gamma_5 \partial_y]\} \hat{\Psi} \tag{15}$$

where $\hat{\Psi} = e^{-3/2k|y|}\Psi$ is a redefinition of the 5D fermion field and $c \equiv \frac{m}{k}$. I will skip most of the mathematical details here and consider the results of doing the KK expansion on fermion fields:

$$\Psi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi_n(x) f_n(y) \tag{16}$$

which results in a zero mode solution for fermions

$$f_0 = \frac{e^{(2-c)ky}}{N_0} \tag{17}$$

where

$$N_0^2 = \frac{e^{2k\pi r_c(1/2-c)} - 1}{2k\pi r_c(1/2-c)} \tag{18}$$

Here N_0^2 is a normalization of the zero mode fermion states. Thus depending on c, which depends on k and the 5D fermion mass, fermions will either be suppressed ($c > 1/2$) at the IR brane or localized to the IR brane ($c < 1/2$). This is again in most parts due to the $e^{2k\pi r_c(1/2-c)}$ term, an exponential that with minor changes to a parameter (c in our case) can cause suppression along the extra dimension. This allows the GIM mechanism to work on the IR brane (since light fermions would be localized on the Planck brane).

Flavor Hierarchy Problem

In the Standard Model there are many different types of fermions (i.e. electrons, muons, ups, charms, tops, etc) with a wide range of masses. The electron is about .5 MeV while the top quark is 173 GeV, which is difference of five orders of magnitude. Even when you only consider quarks the difference between the lightest and the heaviest quark is of order 10^2. There is no obvious explanation for this mass hierarchy in the SM. If we now take a look at the Higgs-fermion interactions with bulk fermions we have the following action:

$$S = \int d^4x \int dy \sqrt{-g} \lambda_{ij}^{(5)} H(x)(\bar{\Psi}_{iL}(x,y)\Psi_{iR}(x,y) + h.c.)\delta(y - \pi r_c) \tag{19}$$

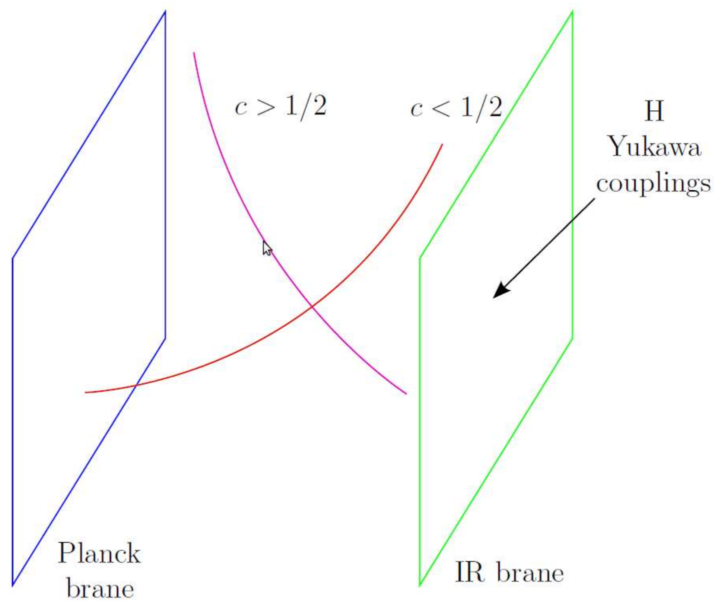

FIG. 3: Fermion localizations

Here $\lambda_{ij}^{(5)}$ are the 5D Yukawa couplings and the delta-function is due to the Higgs being constrained on the IR brane. This yields, for the zero mode fermions, the following relation between the diagonal 5D couplings and the 4D couplings seen on the IR brane (here we assume that there is a right-left symmetry)

$$\lambda = \frac{\lambda^{(5)}k}{N^2}e^{(1-2c)\pi kR} \tag{20}$$

where N is again a normalization of the form seen in the previous section. Thus a slightly varying parameter, c, can cause the localization of fermions to one of the branes in the RS model. To still solve the electroweak hierarchy problem we left the Higgs field localized on the IR brane. This however causes fermions that are localized on the Planck brane ($c > 1/2$) to have small overlap with the IR brane Higgs field and thus will have small Yukawa couplings (figure 3). Thus the c parameter offers an explanation to the flavor hierarchy problem without introducing a new hierarchy (we aren't extremely sensitive to the tuning of c).

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[2] Csaba Csaki. TASI lectures on extra dimensions and branes. 2004.

[3] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo. Phenomenology of the Randall-Sundrum Gauge Hierarchy Model. *Phys. Rev. Lett.*, 84:2080, 2000.

[4] Tony Gherghetta and Alex Pomarol. Bulk fields and supersymmetry in a slice of AdS. *Nucl. Phys.*, B586:141–162, 2000.