Longitudinal Issues for a Full-Scale ERL

Topics for discussion

- first- and second-order correlation in longitudinal phase space
- requirements for momentum compaction of the ring
- lattice options
- second-order momentum compaction
- undulator performance

ERL mtg (Nov 25), then (Dec 4, 2002) with slight modifications

dump / energy recovery requirement on energy spread

$$\delta_{dump} = \frac{E_{dump}}{E_{ring}} = 2\times10^{-3}$$

require 6σ momentum clearing

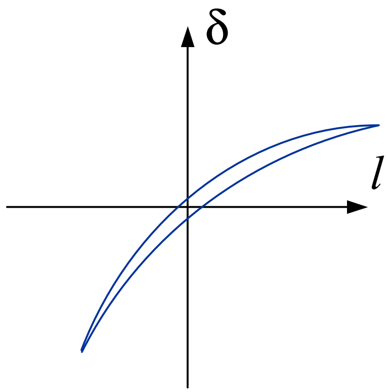
$$\sigma_{\delta_{dump}} = \frac{\delta_{dump}}{6} = 3.3\times10^{-4}$$ or 1.7 MeV

Sources for energy spread in the ring (rms)

- spontaneous radiation 4×10^{-5}
- CSR $1\text{-}6\times10^{-5}$
- wakes ??

Linac and optics only are treated in what is to follow …

1st- and 2nd-order correlation in longitudinal phase space



$$\delta = \delta_0 + \left.\frac{\partial \delta}{\partial l}\right|_{l=0} l + \frac{1}{2!}\left.\frac{\partial^2 \delta}{\partial l^2}\right|_{l=0} l^2 + \ldots$$

$$\delta \cong \delta_0 + \alpha_\delta l + \frac{1}{2}\beta_\delta l^2$$

energy spread:

$$\sigma_\delta = \sqrt{\sigma_{\delta_0}^2 + \alpha_\delta^2 \sigma_l^2 + \tfrac{1}{2}\beta_\delta^2 \sigma_l^4}$$

longitudinal emittance:

$$\varepsilon_{\delta\text{-}l} = \sigma_l \sqrt{\sigma_{\delta_0}^2 + \tfrac{1}{2}\beta_\delta^2 \sigma_l^4}$$

1st- and 2nd-order correlation after RF

$$\alpha_\delta = -\frac{E_{linac}}{E_{final}} k_{RF} \sin\varphi, \qquad \beta_\delta = -\frac{E_{linac}}{E_{final}} k_{RF}^2 \cos\varphi$$

$$k_{RF} = 2\pi / \lambda_{RF} = 27\ \mathrm{m}^{-1}$$

after acceleration

after the main linac:

$$\alpha_\delta \approx -k_{RF}\varphi$$

$$\beta_\delta \approx -k_{RF}^2$$

<u>energy spread</u>:

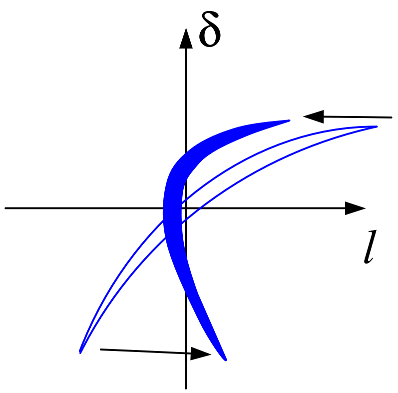
$$\sigma_\delta \approx \alpha_\delta \sigma_l \quad \text{for } |\varphi| > \tfrac{1}{\sqrt{2}} k_{RF}\sigma_l$$

$$\sigma_\delta \approx \tfrac{1}{\sqrt{2}} \beta_\delta \sigma_l^2 \quad \text{for } |\varphi| < \tfrac{1}{\sqrt{2}} k_{RF}\sigma_l$$

<u>longitudinal emittance</u>:

$$\varepsilon_{\delta\text{-}l} \approx \tfrac{1}{\sqrt{2}} \beta_\delta \sigma_l^3$$

longitudinal transform



$$l^* = l + R_{56}\delta + T_{566}\delta^2$$
$$\delta^* = \delta$$

$\Rightarrow$

$$\alpha_\delta^* = \frac{\alpha_\delta}{1 + R_{56}\alpha_\delta}$$

$$\beta_\delta^* = \frac{\beta_\delta - 2T_{566}\alpha_\delta^3}{(1 + R_{56}\alpha_\delta)^3}$$

$$L = \int \sqrt{(1 + x/\rho)^2 + x'^2 + y'^2}\, ds$$

momentum compaction (times the path length):

$$R_{56} = \int \frac{\eta}{\rho} ds$$

second-order momentum compaction:

$$T_{566} = \int \left[\frac{\eta_{(2)}}{\rho} + \frac{\eta^2}{2\rho} + \frac{\eta'^2}{2} \right] ds$$

first-order business

for maximum compression need $R_{56} = -\frac{1}{\alpha_\delta} \approx \frac{1}{k_{RF}\varphi}$

e.g. $R_{56} = 21\,\mathrm{cm}$ for $\varphi = 10°$

or $R_{56} = 14\,\mathrm{cm}$ for $\varphi = 15°$

accruing $R_{56} = N_d \langle\eta\rangle\theta_d$

1) DBA: $R_{56}\,/\,\mathrm{bend} = \frac{1}{6}\rho\theta_d^3$, e.g. $\rho = 50\,\mathrm{m},\;\; \theta_d = 4°$

$R_{56}\,/\,\mathrm{bend} = 3\,\mathrm{mm}$

2) TBA, QBA should be adequate

3) FODO: $R_{56}\,/\,\mathrm{bend} \approx \frac{L_{FODO/2}\theta_d^2}{\sin^2\mu_{FODO/2}}$ for a matched cell

e.g. $\langle\eta\rangle \sim 1\,\mathrm{m},\;\; \theta_d = 5° \;\;\Rightarrow\;\; R_{56}\,/\,\mathrm{bend} \sim 10\,\mathrm{cm}$

constraints for momentum compaction of the ring

$\left|\alpha_\delta^* - \alpha_\delta\right| l^* < \delta_{dump}$ * denotes values after the loop

tolerable deviation from the ideal isochronous condition:

$\left|\Delta R_{56}\right| < \frac{\delta_{dump}}{\alpha_\delta^2 l}$ or $\left|\frac{\Delta R_{56}}{R_{56}}\right| < \frac{\delta_{dump}}{\alpha_\delta l}$ half-ring value R_{56}

somewhat arbitrarily assuming $l = 3\sigma_l$ (e.g. can be 6σ)

e.g. for $\sigma_l = 1°$ $\varphi = 10°$ lattice should be within $\left|\Delta R_{56}\right| < 4.6\,\mathrm{cm}$

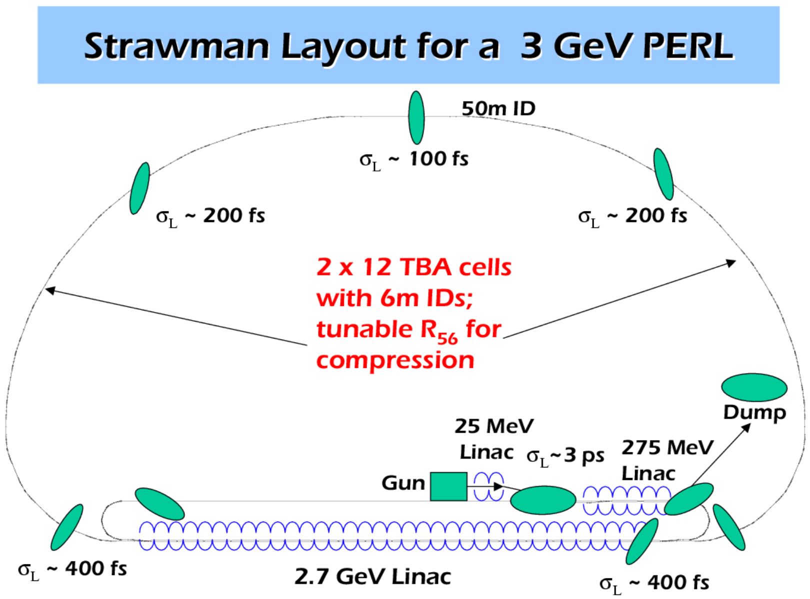
$\varphi = 15°$ $\left|\Delta R_{56}\right| < 2.0\,\mathrm{cm}$

need arcs with R_{56} and $-R_{56}$ unless we recover (π – φ) off-phase (need detuning of 118*φ [deg] Hz $\Rightarrow$ ~ 1 kHz)

e.g. these tricks won't work for our parameters

1) increasing the bunch length on the return pass through the linac over that of injection to mitigate HOM dissipation

2) flipping longitudinal phase space could work only for $(\pi - \varphi)$ phase shift recovery



lattice options

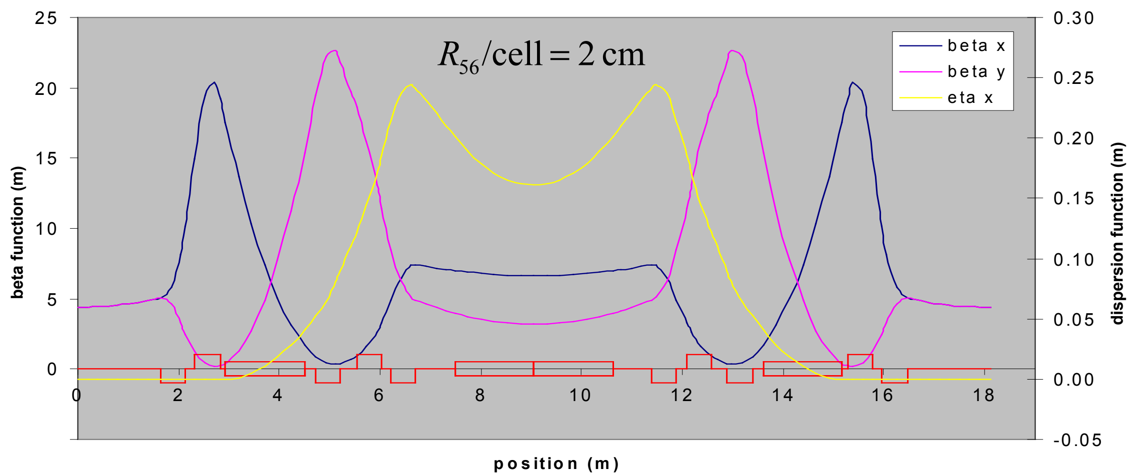
1) DBA won't work (would work if no compression desired)

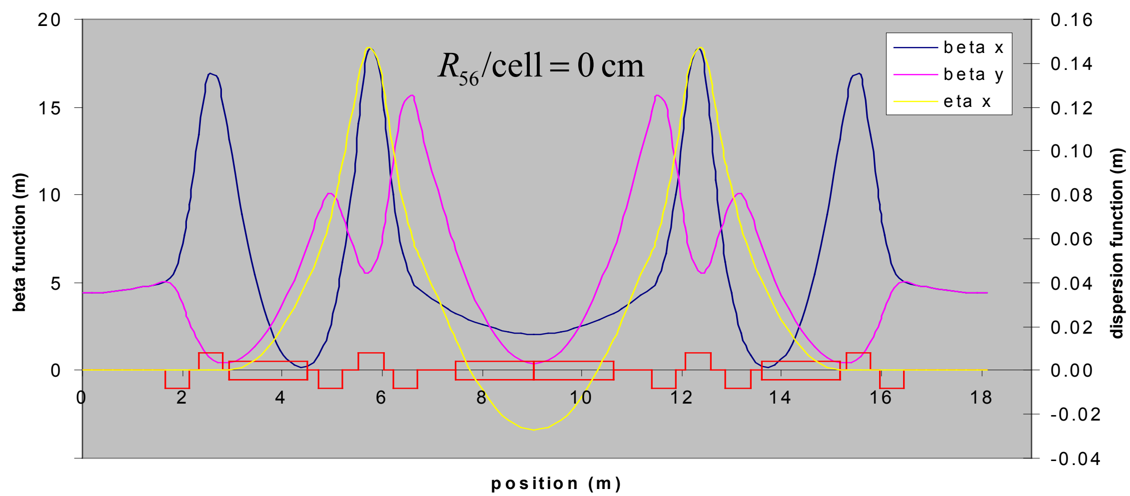
2) TBA, QBA, etc. is fine

TBA cell example
from ERL Study

adjustable

$R_{56}/\text{cell}\,(12°) = -1 \div 2\ \text{cm}$



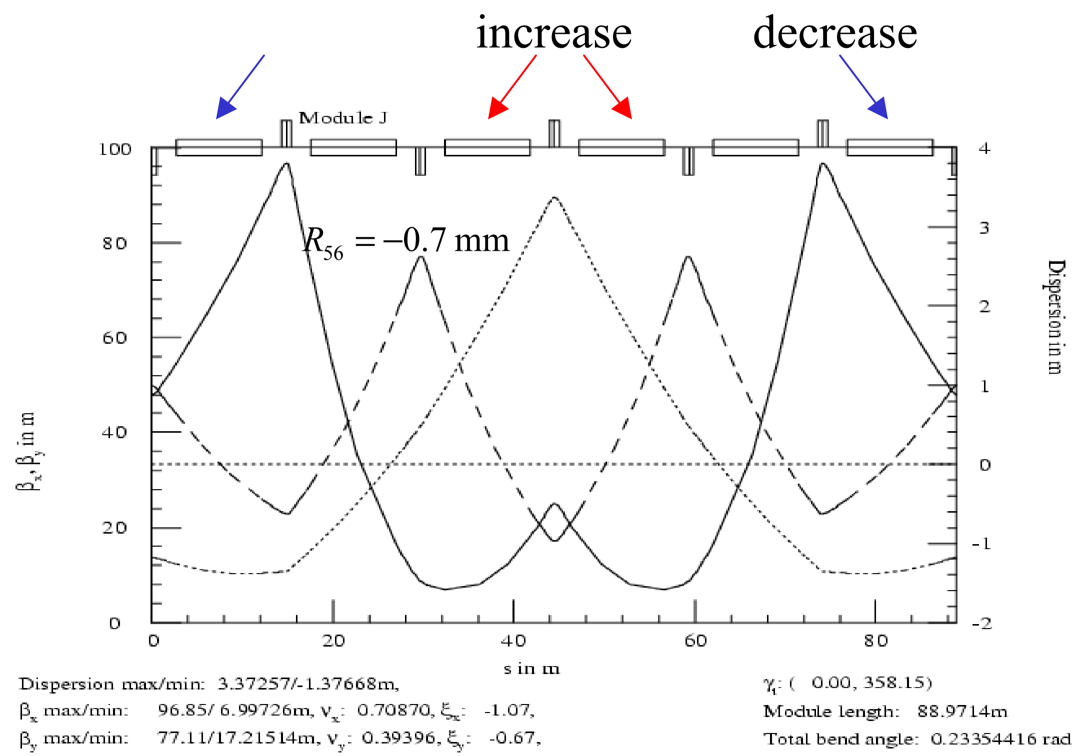


lattice options (contd.)

3) FODO should work at the expense of larger dispersion, beta (if $R_{56} < 0$) and need to have matching sections to create zero-dispersion sections for undulators

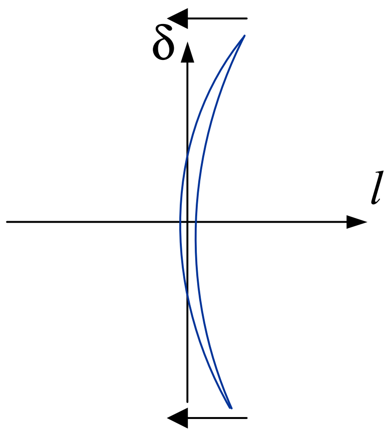
example:

RHIC 3 FODO cells perturbed to obtain negative momentum compaction [D. Trbojevic and E. Courant]



4) introduce negative bends

second-order business



for maximum compression need

$$T_{566} = \frac{\beta_\delta}{2\alpha_\delta^3} \approx \frac{1}{2k_{RF}\varphi^3}$$

actual (absolute) value of T_{566} can be smaller by

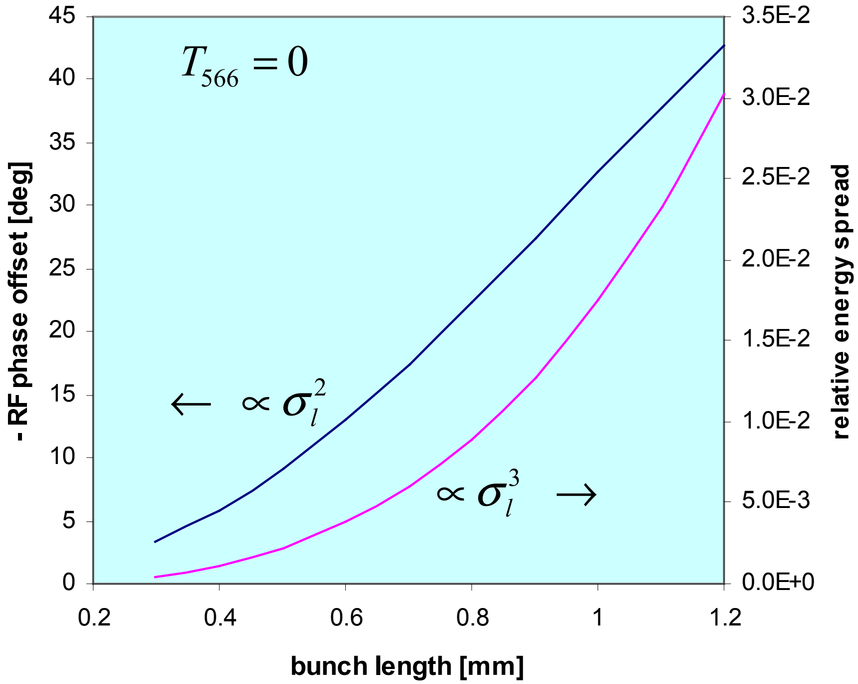
$$\Delta T_{566,\sigma_l^{comp}} = \frac{\sigma_l^{comp}}{\sqrt{2}\alpha_\delta^2\sigma_l^2} \approx \frac{\sigma_l^{comp}}{\sqrt{2}k_{RF}^2\sigma_l^2\varphi^2}$$

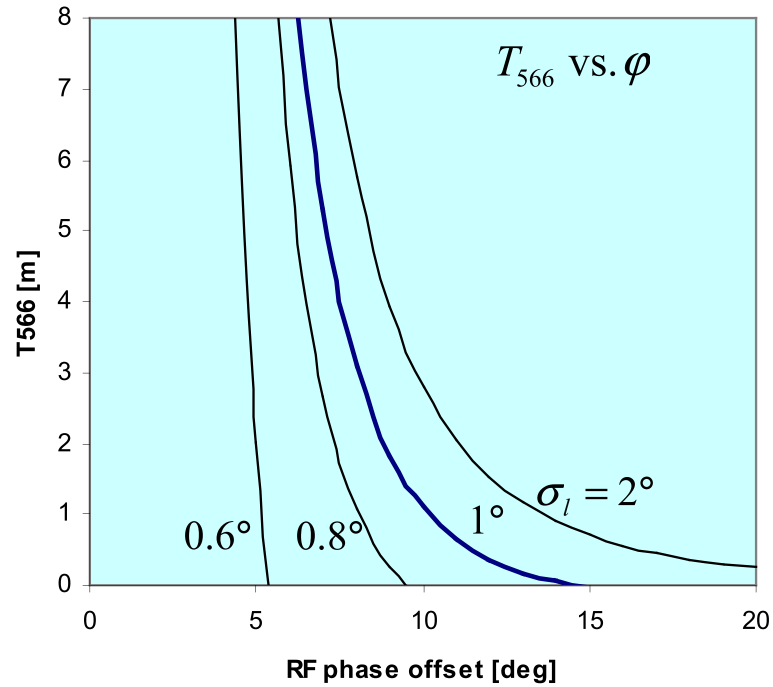
no T_{566} is needed beyond a certain off-crest phase given by

$$\varphi > \varphi_{T_{566=0}} = \frac{\sigma_l^2}{\sigma_l^{comp}}\frac{k_{RF}}{\sqrt{2}}$$

e.g. $\sigma_l = 1°$, $\sigma_l^{comp} = 100\text{ fs}$ $\qquad \varphi > 15°$

dependence on bunch length in the linac





constraints for second-order momentum compaction

$$\left|\beta_\delta^* - \beta_\delta\right| \frac{{l^*}^2}{\sqrt{2}} < \delta_{dump}$$

tolerable deviation from the ideal $T_{566} = 0$ (for π offset recovery):

$$\left|\Delta T_{566}\right| < \frac{\delta_{dump}}{\sqrt{2}\alpha_\delta^3 l^2} \approx \frac{\delta_{dump}}{\sqrt{2}k_{RF}^3 \varphi^3 l^2}$$

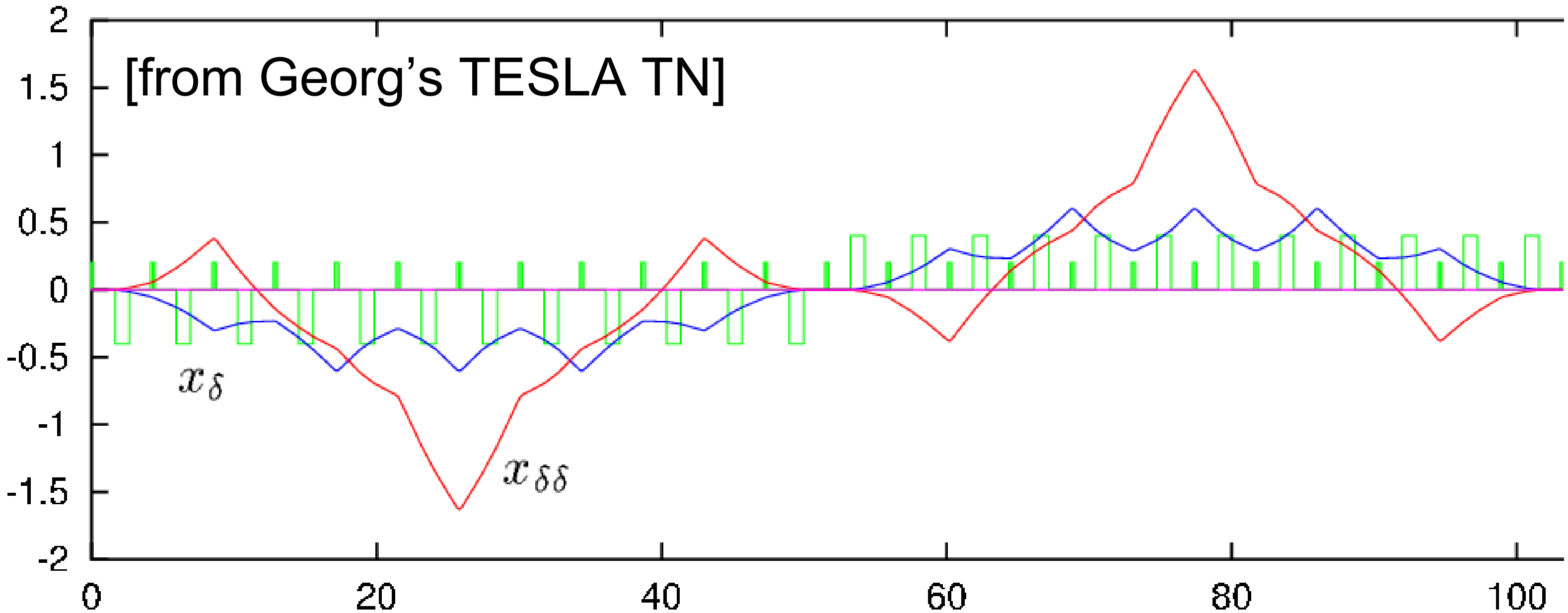
Summary for $\sigma_l = 1°$ and $\sigma_l^{comp} = 100\text{ fs}$, $l = 3\sigma_l$

	R_{56} [cm]	T_{566} [m]	ΔR_{56} [cm]	ΔT_{566} [m]
$\varphi = 10°$	21 ± 1	3.4 ± 2.2	< 4.6	< 3.6
$\varphi = 15°$	14.1 ± 0.7	1.0 ± 1.0	< 2.0	< 1.1

achieving the right value of T_{566}

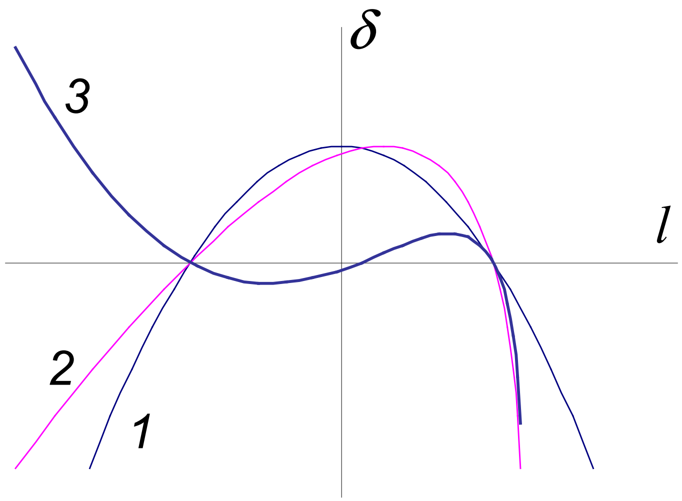
$$T_{566} = \int \left[\frac{\eta_{(2)}}{\rho} + \frac{\eta^2}{2\rho} + \frac{\eta'^2}{2} \right] ds$$

$$\eta''_{(2)} + K(s)\eta_{(2)} = -h + k_1\eta - \tfrac{1}{2}k_2\eta^2 + (h^3 + 2k_1 h)\eta^2 + \tfrac{1}{2}h\eta'^2 + h'\eta'\eta + 2h^2\eta$$



1) T_{566}/R_{56} ~ 10 seems feasible, higher values are difficult
2) proper solution with tolerable chromatic aberrations could not be found for energy spread of 10% (could be o.k. for 2%)

momentum compaction of the ring for on-crest running



$$\Delta\delta = \beta_\delta l \Delta l$$

$$\Delta l = R_{56}\delta \qquad \Delta l = T_{566}\delta^2$$

$$\Delta\delta < \delta_{dump}$$

$$R_{56} < \frac{2\delta_{dump}}{\beta_\delta^2 l^3} \qquad T_{566} < \frac{4\delta_{dump}}{\beta_\delta^3 l^5}$$

for our parameters this translates to successful energy recovery when momentum compaction of the ring is R_{56} < 1 m and T_{566} < 750 m (for $l = 6\sigma_l$, R_{56} < 13 cm and T_{566} < 24 m).

matching undulators to ERL parameters

scaling with number of periods

small detector: $\dfrac{\dot{N}_{ph}(d\Omega)}{d\Omega \cdot d\omega/\omega} \propto N_p^2$

brightness: $\dfrac{\dot{N}_{ph}(d\Omega_\theta)}{(r\theta)^2 \cdot d\omega/\omega} \propto N_p$

scaling rules above hold true only when spectral bandwidth is dominated by undulator natural broadening $\sim 1/N_p$

$$\text{b.w.}_{rms}^2 \approx (\gamma^* \sigma_{\theta_{beam}})^2 + \left(\frac{0.4}{N_p}\right)^2 + (2\sigma_\delta)^2$$

If $N_p \geq N_{p,\max} = \dfrac{0.2}{\sigma_\delta}$ then brightness stays approximately constant

energy spread effect on undulator performance

for ERL parameters $N_{p,\max} \sim \dfrac{0.2}{2\times10^{-4}} = 1000$ (performance $\frac{1}{\sqrt{2}}$ of the predicted)

lengthening undulator by 50 % beyond this point improves brightness by 15 %, i.e. coherent properties of radiation do not improve much, only flux does

when bunch is compressed, $N_{p,\max}^{comp} \sim 1000/(\sigma_l / \sigma_l^{comp})$

$N_p > N_{p,\max}^{comp}$ average brightness is decreased by $\sigma_l / \sigma_l^{comp}$
peak brightness stays constant

$N_p < N_{p,\max}^{comp}$ average brightness stays constant
peak brightness is increased by $\sigma_l / \sigma_l^{comp}$

preliminary conclusions

- ERL with current parameters requires elaborate optics with controlled first- and second-order momentum compaction
- we would need arcs with something like $R_{56} = \pm 20$ cm, $T_{566}/ R_{56} = 1/2\varphi^2 \sim 15$ for an off-crest phase of 10°
- high brightness favors uncompressed bunches not only because of smaller transverse emittance, but because of smaller energy spread as well (true for all but very short undulators)
- on-crest operation without compression is rather forgiving of optics, T_{566} in particular