The effect of dipole and quadrupole HOMs on the performance of the Phase II ERL

Outline:

- Beam induced voltage: losses and kicks
- Beam breakup for quadrupole HOMs
- BBU statistics for “ERL in CESR tunnel”
- Discussion

Introduction

The “Big Four”

- Superposition
- Concept of normal modes
- Conservation of energy
- Causality

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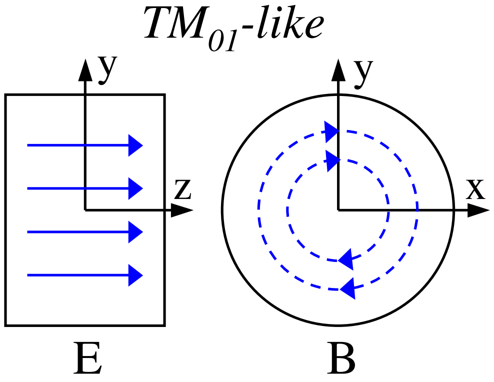
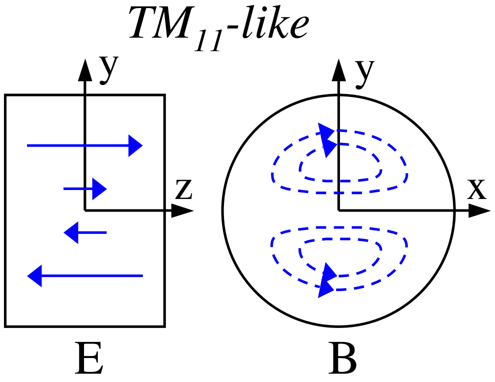
monopole ($m = 0$)



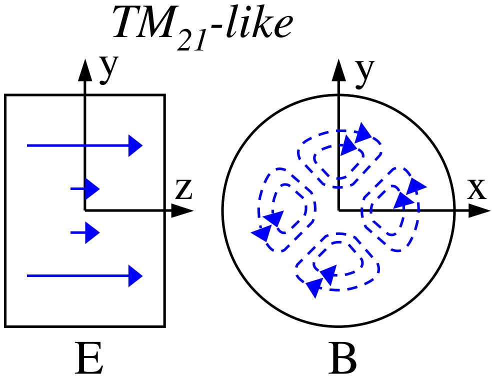
high losses, no kick

dipole ($m = 1$)



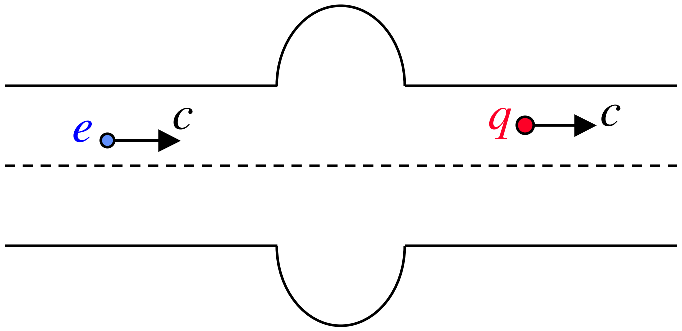
kick and losses when beam is not centered

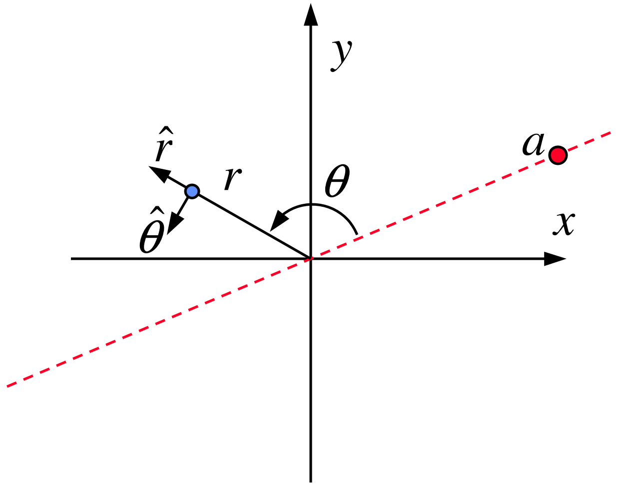
quadrupole ($m = 2$)



kick, coupling and losses when beam is not centered

Wake formalism for point charge





$$\int_{-L/2}^{L/2} \vec{F}_\perp ds = -eqa^m W_m(z) m r^{m-1} (\hat{r}\cos m\theta - \hat{\theta}\sin m\theta)$$

$$\int_{-L/2}^{L/2} F_\parallel ds = -eqa^m W_m'(z) r^m \cos m\theta$$

The above can be combined to say (Panofsky-Wenzel): $\nabla_\perp \int_{-L/2}^{L/2} F_\parallel ds = \frac{\partial}{\partial z} \int_{-L/2}^{L/2} \vec{F}_\perp ds$

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Wake function for resonant mode

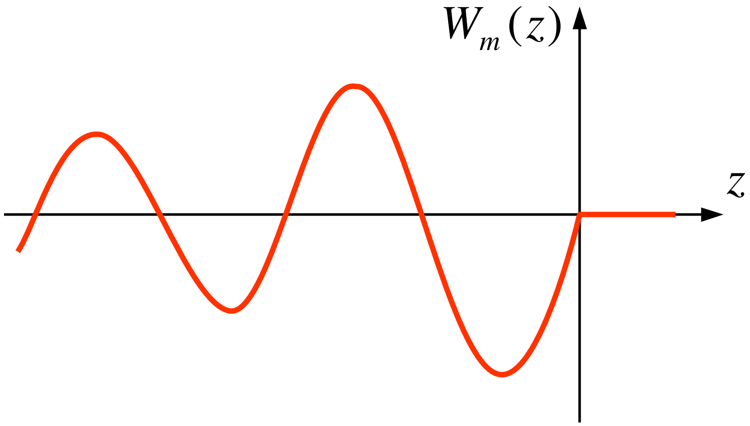
A mode characterized by $\left(\frac{R}{Q}\right)_m, Q, \omega$

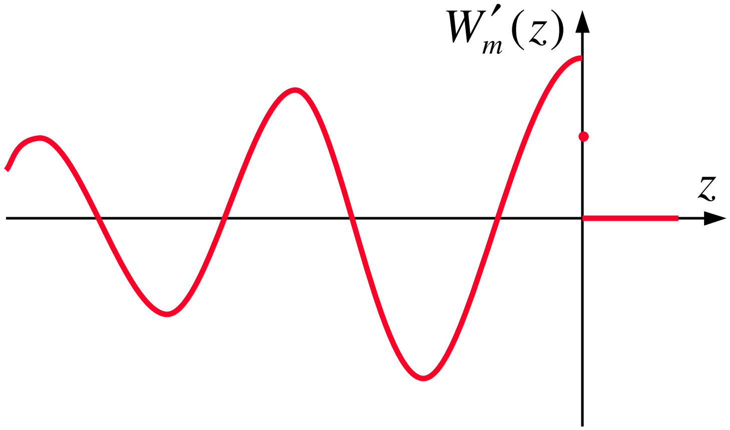
transverse

$$W_m = 2\frac{ck_m}{\omega} e^{\frac{kz}{2Q}} \sin kz$$

longitudinal

$$W'_m \approx 2k_m e^{\frac{kz}{2Q}} \cos kz$$





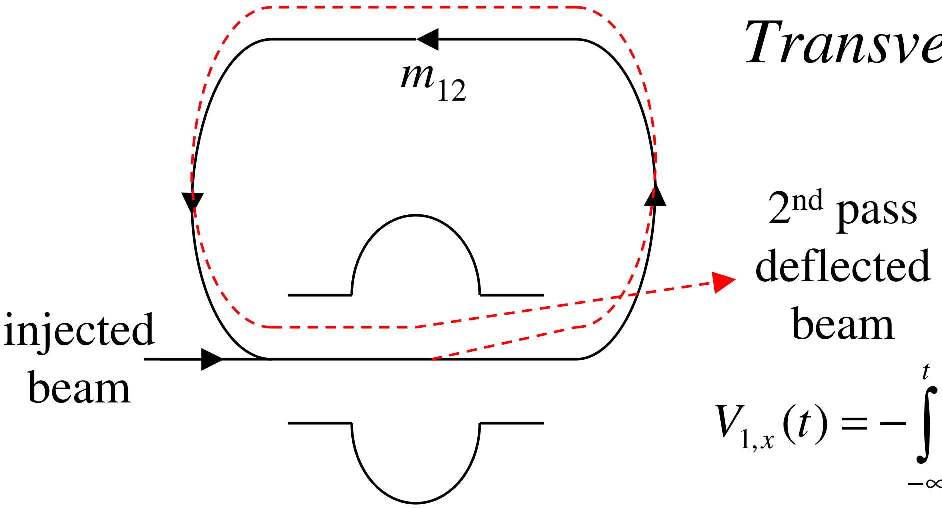
<u>loss factor</u> $k_m = \frac{\omega}{4}\left(\frac{R}{Q}\right)_m$

$\left(\frac{R}{Q}\right)_m$ in <u>units</u> of $\Omega \cdot \mathrm{m}^{-2m}$

$k = \omega / c$ is mode’s wave number

Beam breakup

Transverse



m_{12}

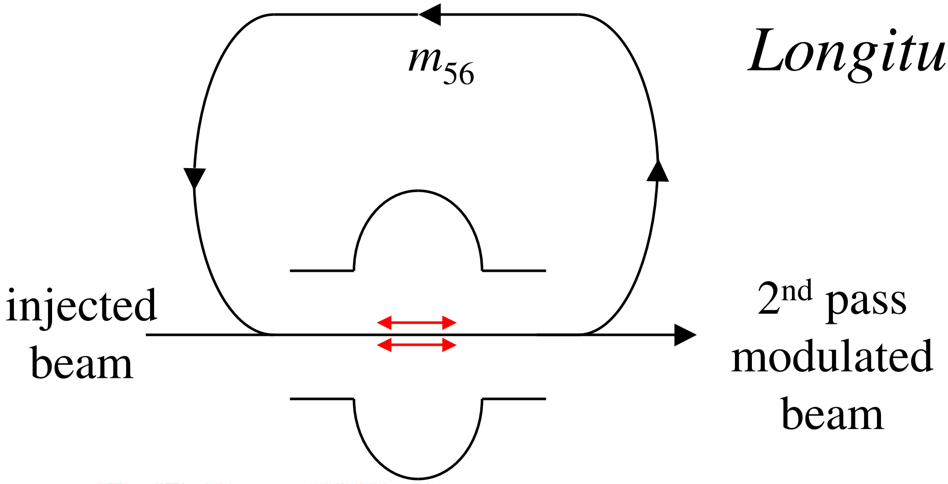
injected beam

2nd pass deflected beam

$$x^{(2)}(t) = \frac{e}{c} m_{12} V_x(t - t_r)$$

$$V_{1,x}(t) = -\int_{-\infty}^{t} W_1(t - t')[I^{(1)}(t')x^{(1)}(t') + I^{(2)}(t')x^{(2)}(t')]dt'$$

Longitudinal



m_{56}

injected beam

2nd pass modulated beam

$$I_{th} = \frac{2Ec}{e(R/Q)_\lambda Q_\lambda \omega_\lambda} \frac{1}{R_{56}}$$

$$I_{th} = \frac{2Ec^2}{e(R/Q)_\lambda Q_\lambda \omega_\lambda} \frac{1}{R_{12}}$$

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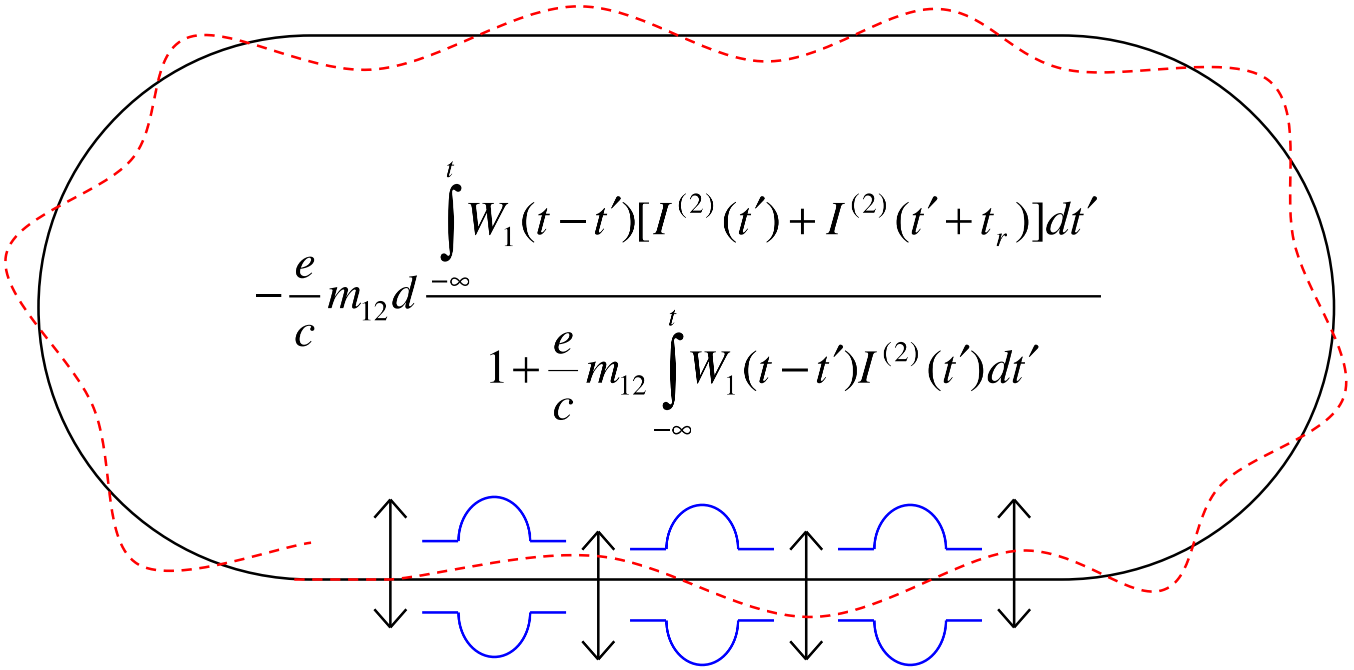
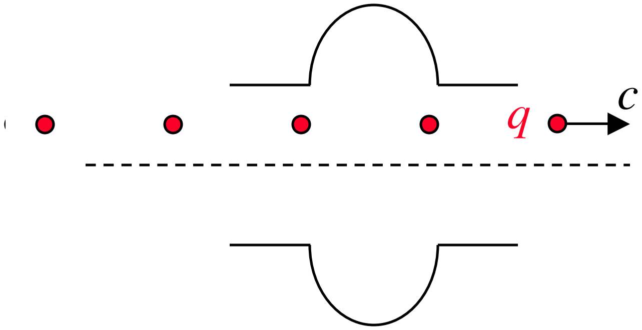
Transient and steady case parts of induced voltage

- Equation for transient part reproduces integral equation for BBU for aligned cavity
- Steady case part of beam induced voltage generates displaced orbit



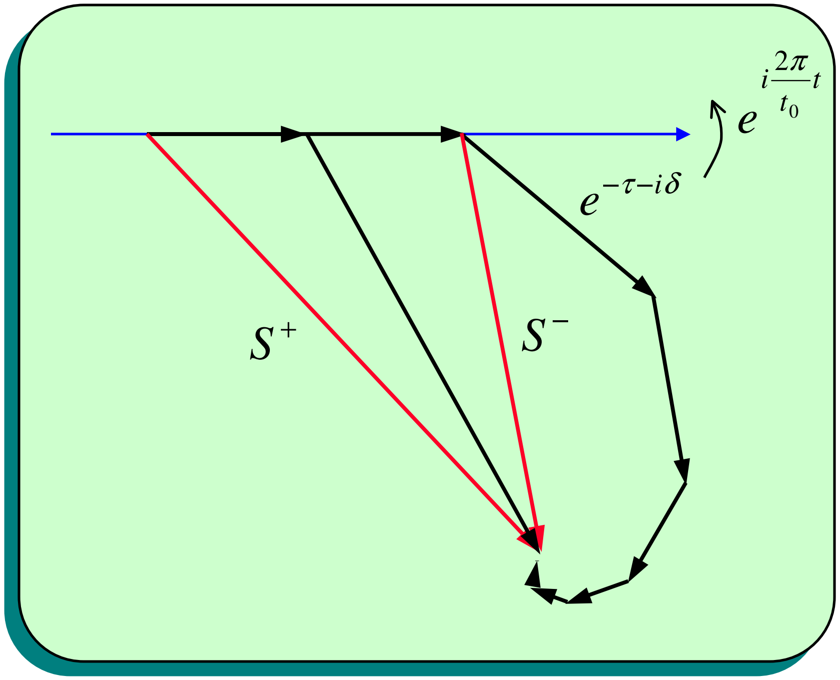
$$-\frac{e}{c} m_{12} d \frac{\int_{-\infty}^{t} W_1(t-t')[I^{(2)}(t')+I^{(2)}(t'+t_r)]dt'}{1+\frac{e}{c} m_{12} \int_{-\infty}^{t} W_1(t-t')I^{(2)}(t')dt'}$$

Beam induced voltage from infinite bunch train



$$V_{\parallel,m} = -qd^{2m}\frac{\omega}{2}\left(\frac{R}{Q}\right)_m \Re\left\{\sum_{n=0}^{\infty} e^{-n\frac{\omega t_0}{2Q}} e^{-in\omega t_0} - \frac{1}{2}\right\}$$

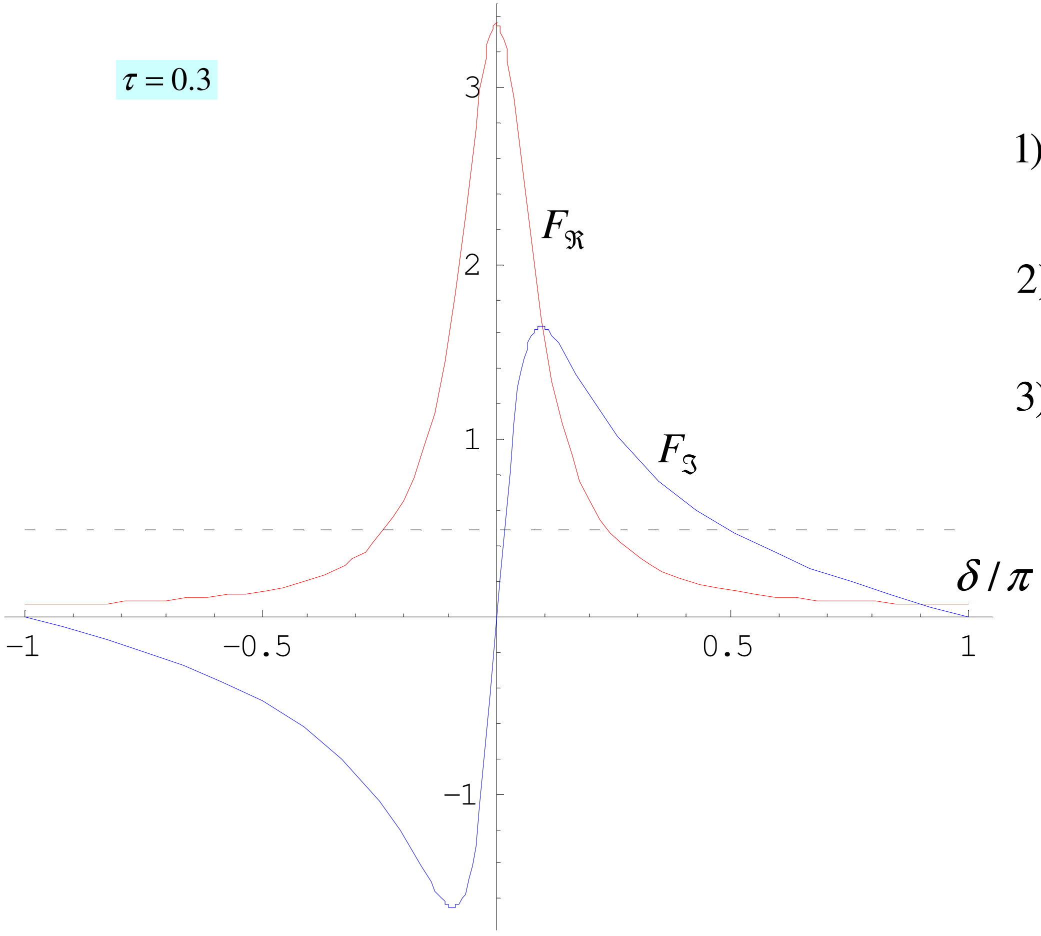
$$V_{\perp,m} = -\frac{m}{2}qd^{2m-1}c\left(\frac{R}{Q}\right)_m \Im\left\{\sum_{n=0}^{\infty} e^{-n\frac{\omega t_0}{2Q}} e^{-in\omega t_0} - \frac{1}{2}\right\}$$



$$\sum_{n=0}^{\infty} e^{-n\frac{\omega t_0}{2Q}} e^{-in\omega t_0} - \frac{1}{2} = S^+(\tau,\delta) - \frac{1}{2} = F_{\Re}(\tau,\delta) - iF_{\Im}(\tau,\delta)$$

$$\tau = \frac{\omega t_0}{2Q}, \ \delta = \omega t_0$$

Resonance function



Real part

1) $$\frac{1}{2\pi}\int_{-\pi}^{\pi} F_\Re(\tau,\delta)\,d\delta = \frac{1}{2}$$

2) $$\max F_\Re(\tau,\delta) = F_\Re(\tau,0) = \frac{1}{\tau}$$

3) $$F_\Re = \frac{1}{2} \text{ when } \delta = \pm\sqrt{2\tau} = \pm\sqrt{\frac{\omega t_0}{Q}}$$

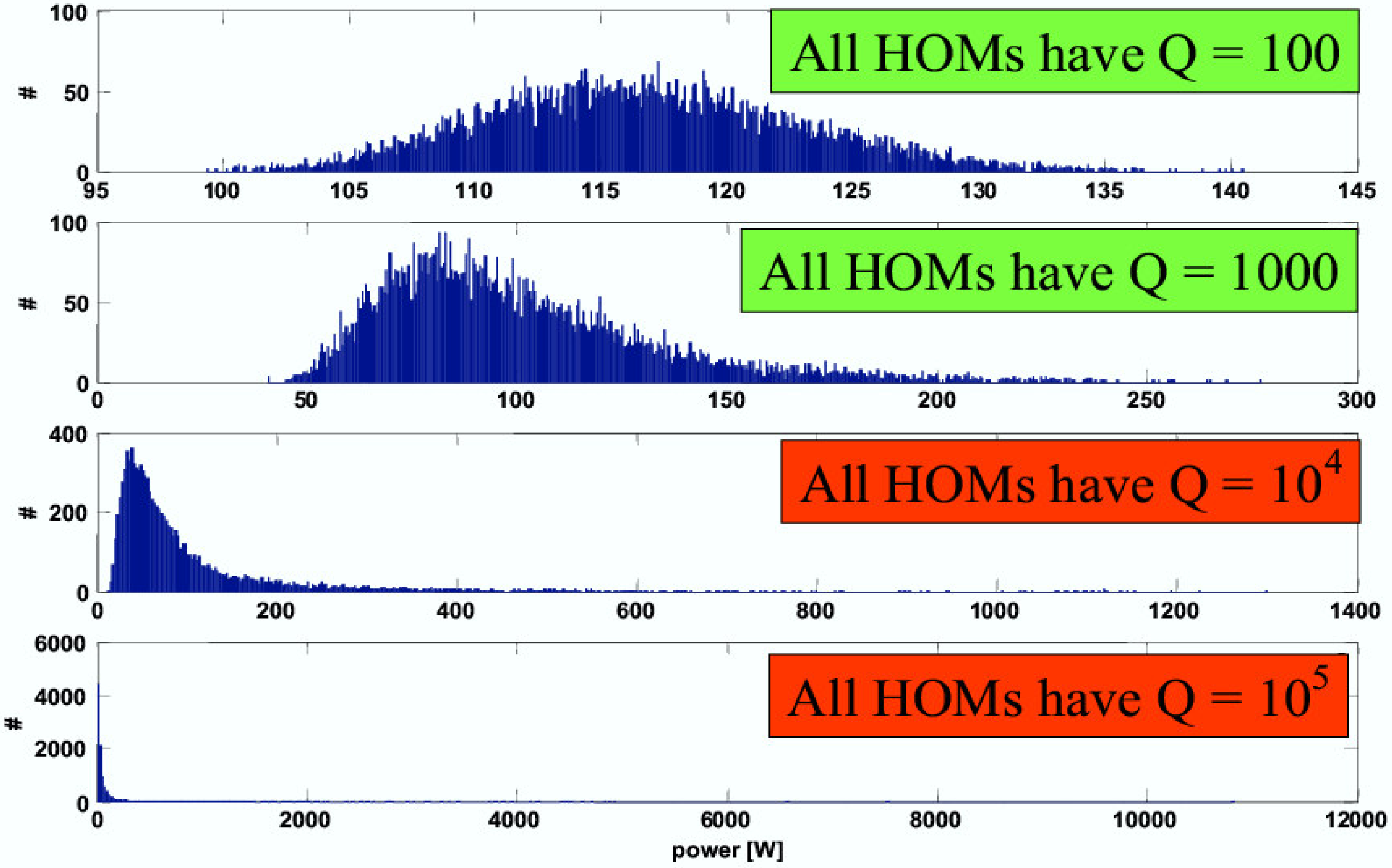
Imaginary part

1) $$\frac{1}{2\pi}\int_{-\pi}^{\pi} F_\Im(\tau,\delta)\,d\delta = 0$$

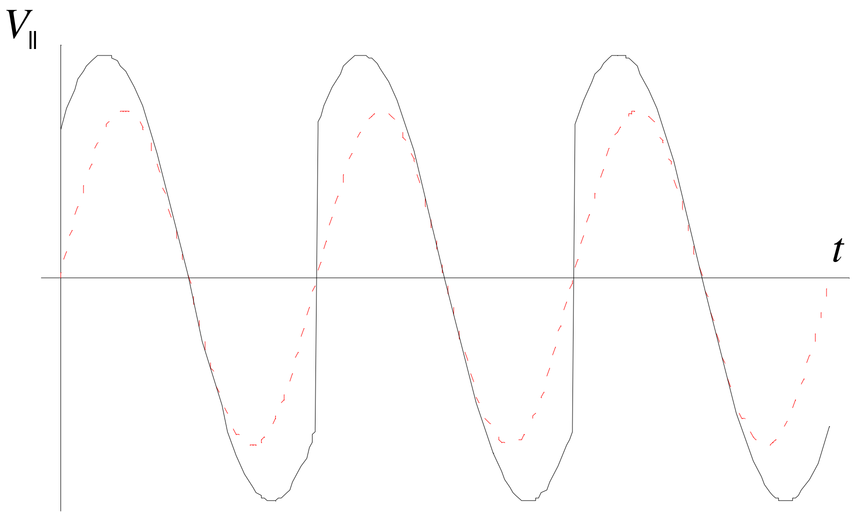
2) $$\max\left|F_\Im(\tau,\delta)\right| \approx \left|F_\Im(\tau,\pm\tfrac{\tau}{\sqrt{2}})\right| \approx \frac{1}{2\tau}$$

3) $$\left|F_\Im\right| \geq \frac{1}{2} \text{ when } -\frac{\pi}{2} \leq \delta \leq \frac{\pi}{2}$$

A Simple Model: 1000 Monopoles with random f's
Total HOM Monopole Power for random Sets of Frequencies



Beam induced voltages

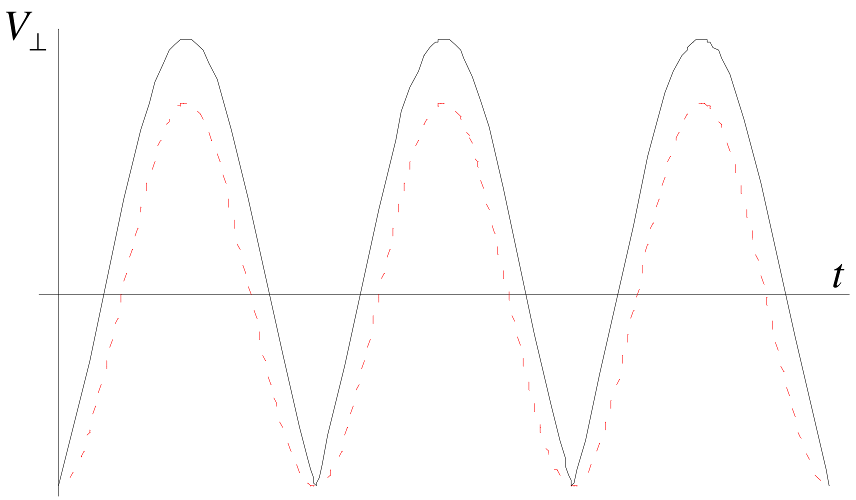


Typical

$$\langle V_{\parallel,m}\rangle = -k_m q d^{2m} = -q d^{2m}\frac{\omega}{4}\left(\frac{R}{Q}\right)_m$$

At resonance

$$\left|V_{\parallel,m}\right| = \left(\frac{R}{Q}\right)_m Q d^{2m} I$$



Typical

$$\langle V_{\perp,m}\rangle = 0$$

At resonance

$$\left|V_{\perp,m}\right| = \frac{m}{2}\left(\frac{R}{Q}\right)_m Q k^{-1} d^{2m-1} I$$

Dipole HOMs from TESLA TDR for 100 mA ERL

$R/Q, \frac{\Omega}{\text{cm}^2}$	Q	$\max V_\perp, \frac{\text{V}}{\text{mm}}$	$\max V_\parallel, \frac{\text{V}}{\text{mm}^2}$	$\max P, \frac{\text{W}}{\text{mm}^2}$
11.21	50000	1.03E+04	1121	224.2
8.69	70000	1.12E+04	1216.6	243.32
15	50000	1.38E+04	1500	300
15.51	20000	5.69E+03	620.4	124.08
6.54	50000	6.00E+03	654	130.8
1.72	100000	3.16E+03	344	68.8
1.75	75000	2.41E+03	262.5	52.5
0.76	70000	9.76E+02	106.4	21.28
1.05	100000	1.93E+03	210	42
2.16	20000	7.93E+02	86.4	17.28
0.5	100000	9.18E+02	100	20
0.46	50000	4.22E+02	46	9.2
0.39	25000	1.79E+02	19.5	3.9
0.77	10000	1.41E+02	15.4	3.08

Transverse multipass beam breakup

dipole mode

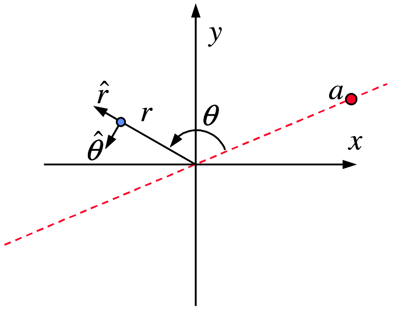
$$\Delta V_{1,x} = -qW_1(z)\langle x_a \rangle$$

$$\Delta V_{1,y} = -qW_1(z)\langle y_a \rangle$$

$$\Downarrow$$

$$I_1 \approx \frac{2\omega}{e m_{12} (R/Q)_1 Q}$$

$$m_{12} = \sqrt{\frac{\beta_{x,n}^{(1)}}{p^{(1)}} \frac{\beta_{x,n}^{(2)}}{p^{(2)}}} \sin \Delta\psi_x$$



quadrupole mode

focusing

$$\Delta \frac{\partial}{\partial x} V_{2,x} = -\Delta \frac{\partial}{\partial y} V_{2,y} =$$
$$-2qW_2(z)\left[\langle x_a \rangle^2 - \langle y_a \rangle^2 + \langle x_a^2 \rangle - \langle y_a^2 \rangle\right]$$

coupling

$$\Delta \frac{\partial}{\partial y} V_{2,x} = \Delta \frac{\partial}{\partial x} V_{2,y} =$$
$$-2qW_2(z)2\langle x_a \rangle \langle y_a \rangle$$

Quadrupole HOM induced voltage for 100 mA ERL

focusing effect:

$$\max \frac{\partial}{\partial x} V_{2,x} = \left(\frac{R}{Q} \right)_m Qk^{-1} I d^2$$

$$\max \frac{\partial}{\partial x} V_{2,x} \sim 10^4 \, \frac{\mathrm{V}}{\mathrm{m}} \cdot d^2 (\mathrm{mm}) \qquad \text{focus. length} \quad \frac{p}{\frac{e}{c} \frac{\partial}{\partial x} V_x} \geq 10^3 \ \mathrm{m} / d^2 (\mathrm{mm})$$

coupling effect:

$$\Delta \varepsilon_{n,c} = \frac{e}{mc^2} \sigma_x \sigma_y \frac{\partial}{\partial y} V_{2,x} \leq 1\% \cdot d^2 (\mathrm{mm})$$

$$\varepsilon_n^2 = \varepsilon_{n,0}^2 + \Delta \varepsilon_{n,c}^2$$

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Beam breakup due to quadrupole HOMs

a) aligned cavity $\sigma_{x,y} >> d$

$$I_{2a)} \approx \frac{2\omega}{e(R/Q)_2 Q} \frac{p^{(1)} p^{(2)}}{2m_e c[\varepsilon_{x,n} \beta^{(1)}_{x,n} \beta^{(2)}_{x,n} \sin 2\Delta\psi_x + \varepsilon_{y,n} \beta^{(1)}_{y,n} \beta^{(2)}_{y,n} \sin 2\Delta\psi_y]}$$

$$\frac{I_{2a)}}{I_1} \sim \frac{b^2}{4\sigma^2_{x,y}}$$

a) misaligned cavity $d >> \sigma_{x,y}$

$$I_{2b)} \approx \frac{2\omega}{e(R/Q)_2 Q m_{12}} \frac{1}{4d^2}$$

$$\frac{I_{2b)}}{I_1} \sim \frac{b^2}{4d^2}$$

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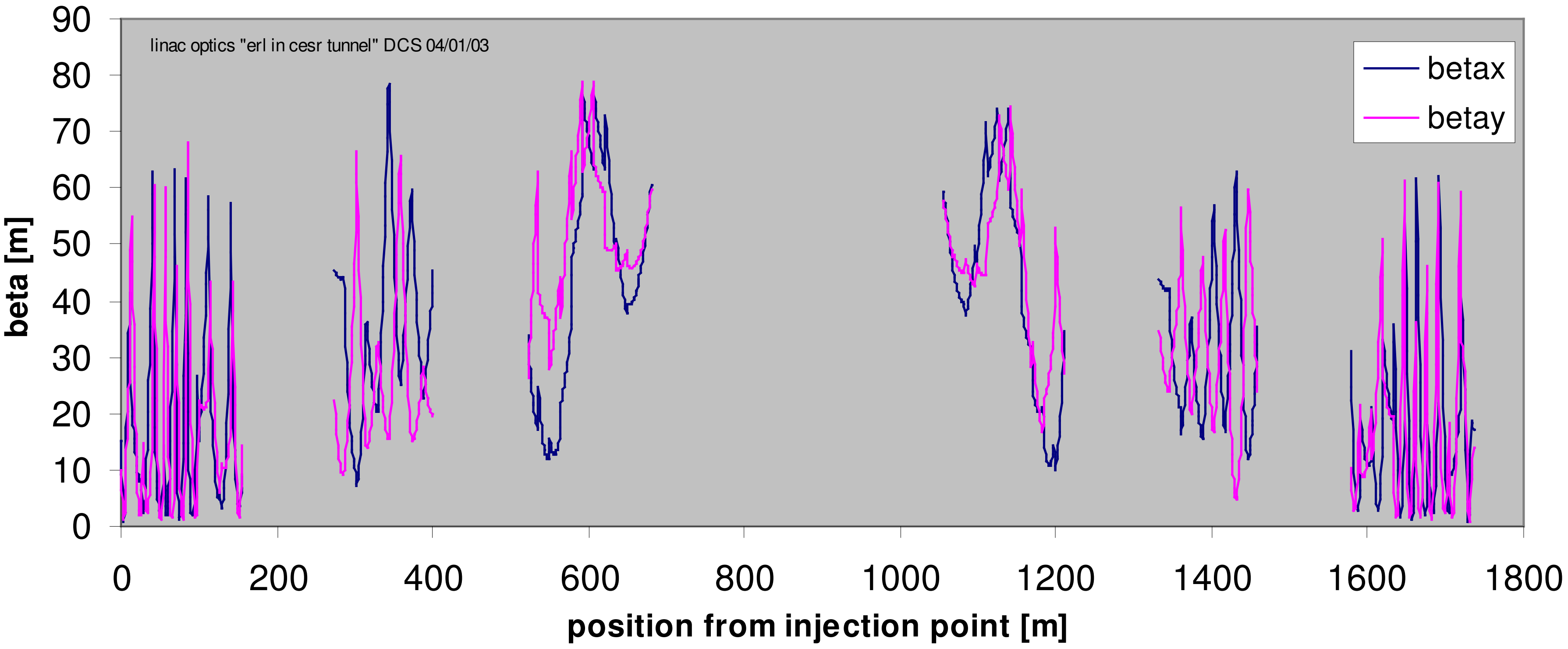
`bi` - numeric tool for transverse BBU

Features:

- written in C++
- cleaner algorithm than TDBBU
- faster than TDBBU (a single 5 GeV ERL run takes less than a minute; execution time is estimated to be 7-9 times faster when no coupling is present; with coupling it is estimated to be at least 4 times faster)
- allows any ERL topology
- easier to use

Linac optics (DCS, 04/01/03)



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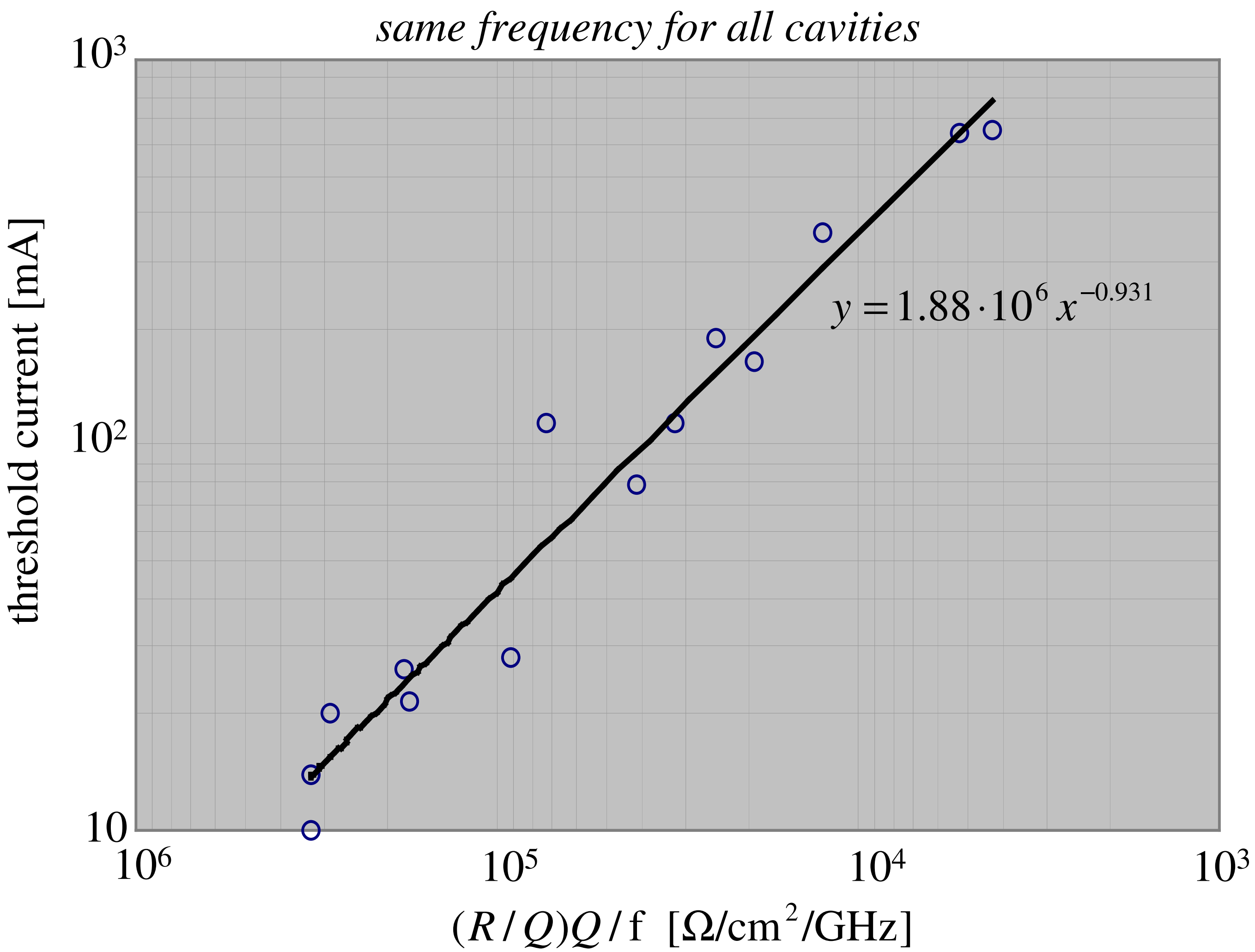
Worst 14 dipole modes from TESLA TDR



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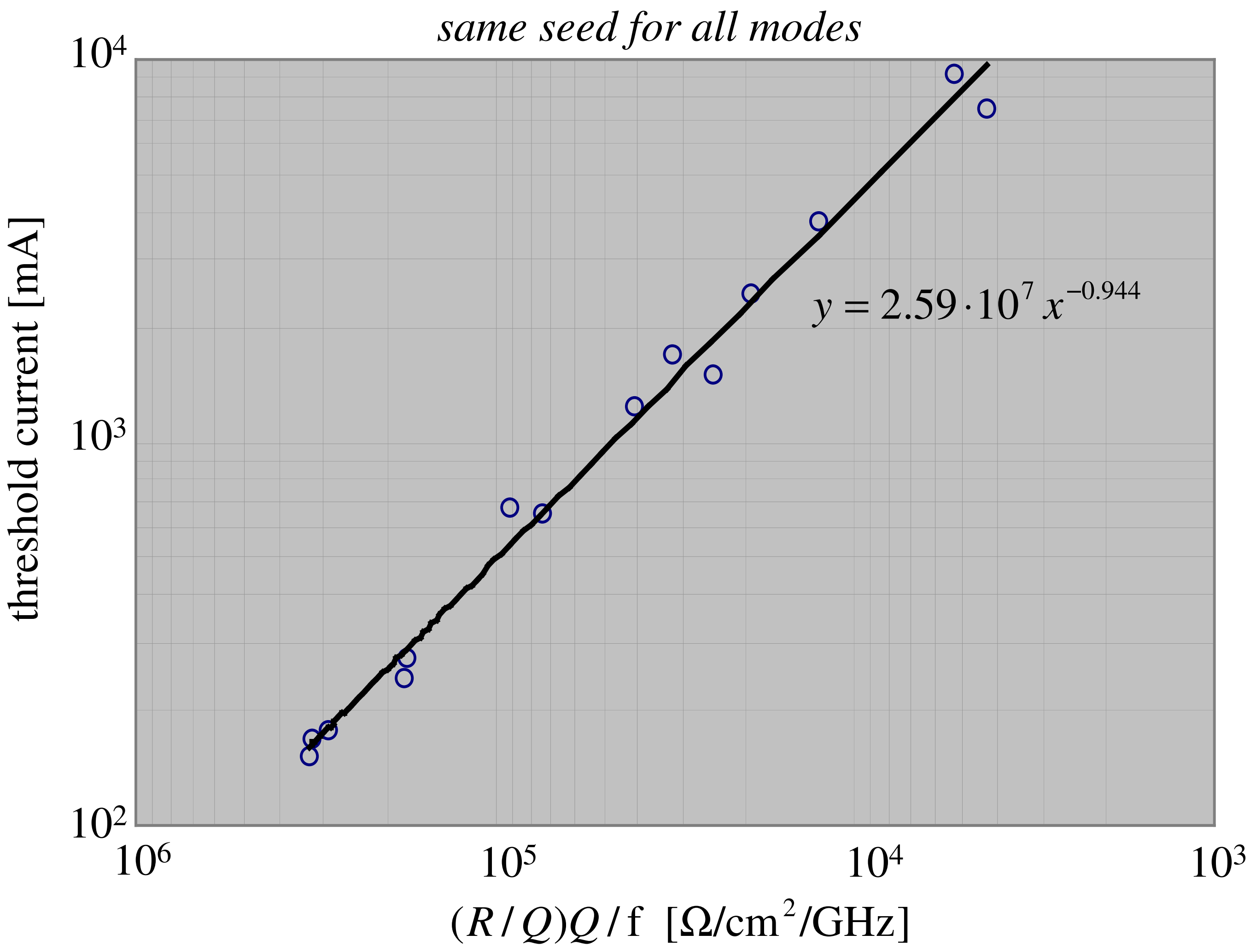
Uniform frequency spread of 10 MHz



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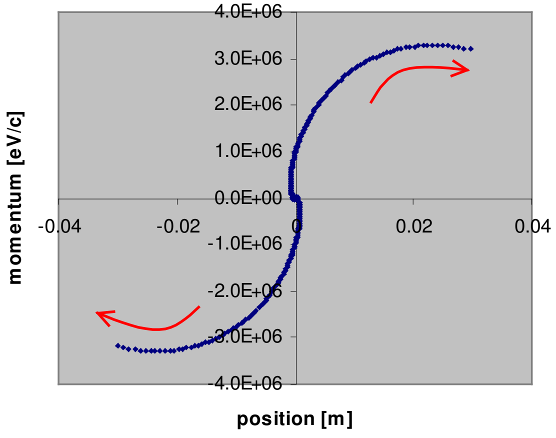
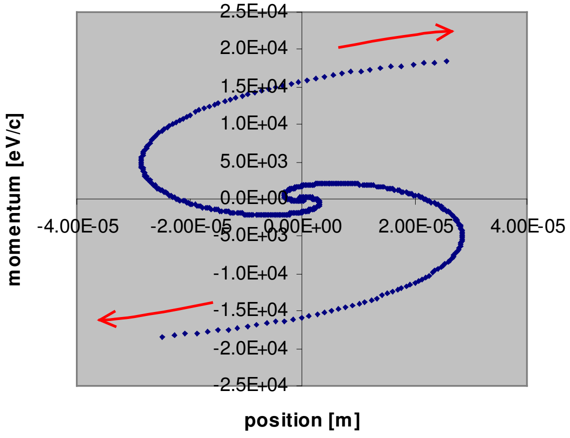
Fixed current (30 mA)

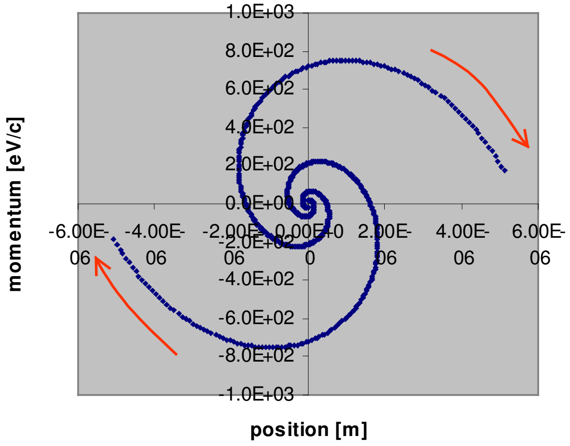
rms = 0 Hz



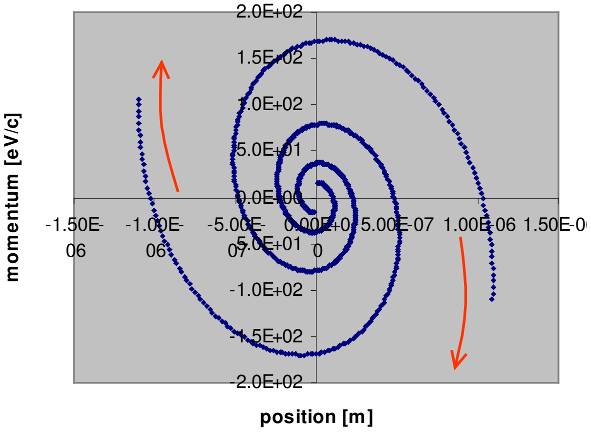
rms = 33 kHz



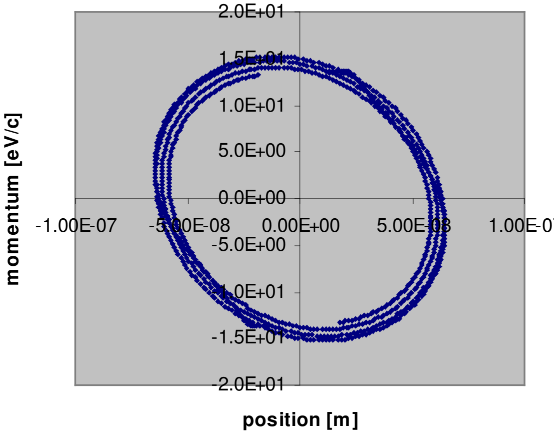
rms = 42 kHz



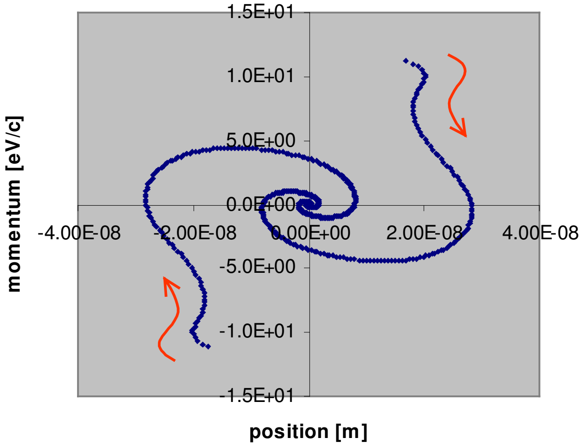
rms = 46 kHz



rms = 53 kHz



rms = 67 kHz



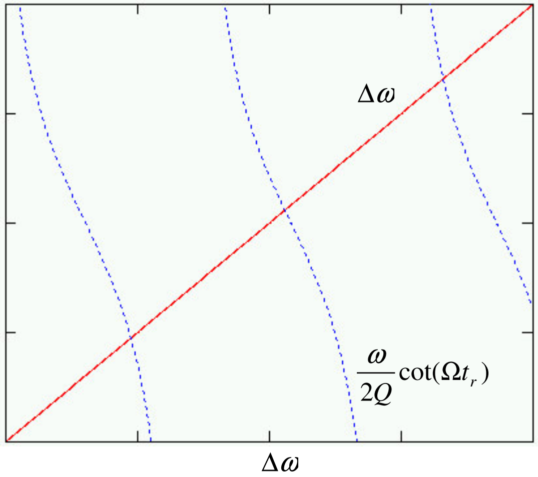
Stabilizing effect of HOM frequency spread

- Effectively decreases Q of the mode: $Q \sim \omega/\Delta\omega_{1/2}$
- Limited in its effect by $\sim \omega / Q$ of the HOM

Linearized solution for simplest case

$$I_1 \approx \frac{-2c}{e(R/Q)Qkm_{12}\sin(\Omega t_r)},$$

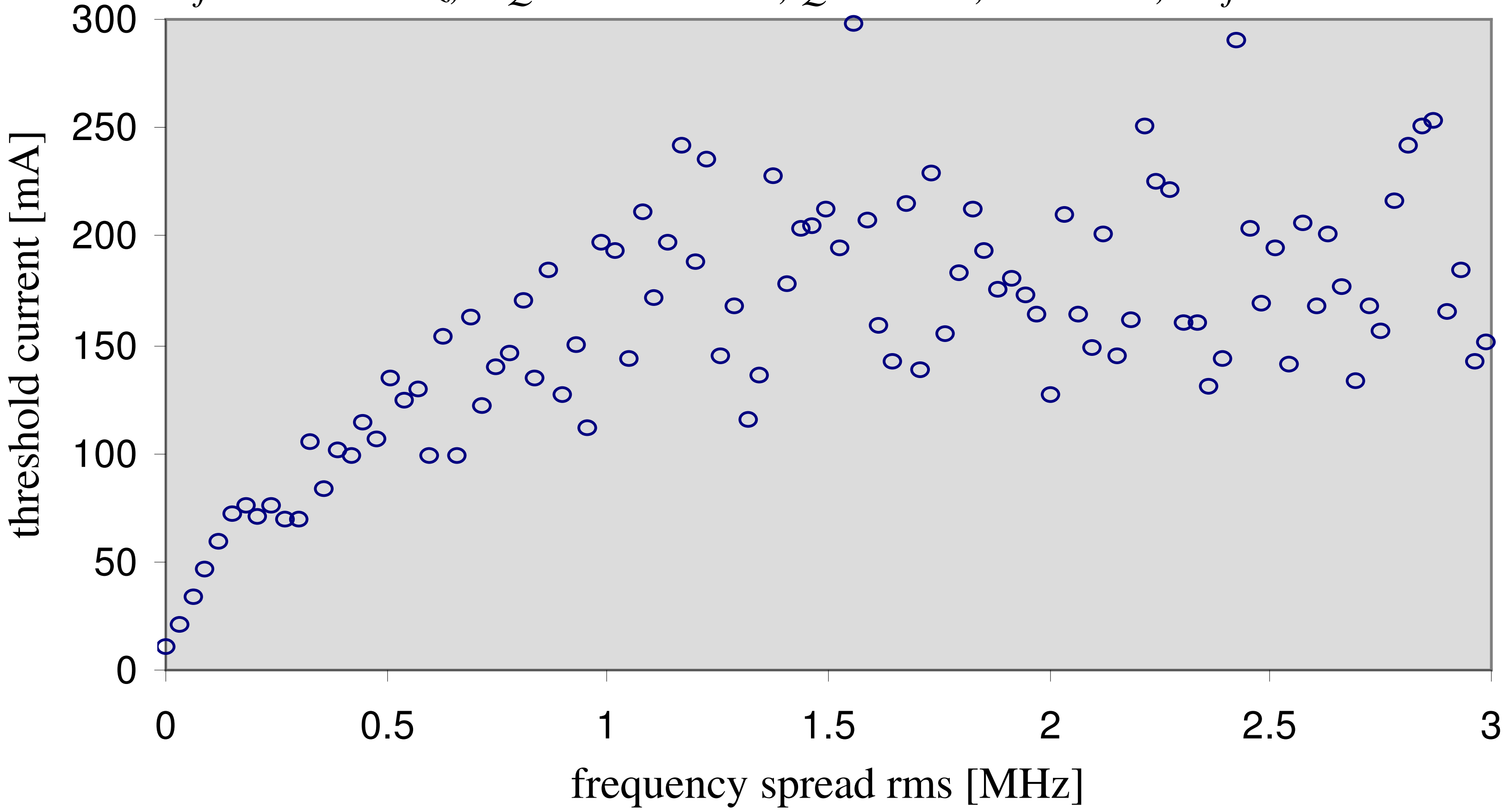
$$\Delta\omega = \frac{\omega}{2Q}\cot(\Omega t_r)$$



Periodic in $\Delta\omega = \Omega - \omega$ with $\omega_r = 2\pi / t_r$

Threshold vs. frequency spread

f = 1699.1 MHz, *R/Q* = 11.21 Ω/cm^2, *Q* = 5×10^4, same seed, uniform



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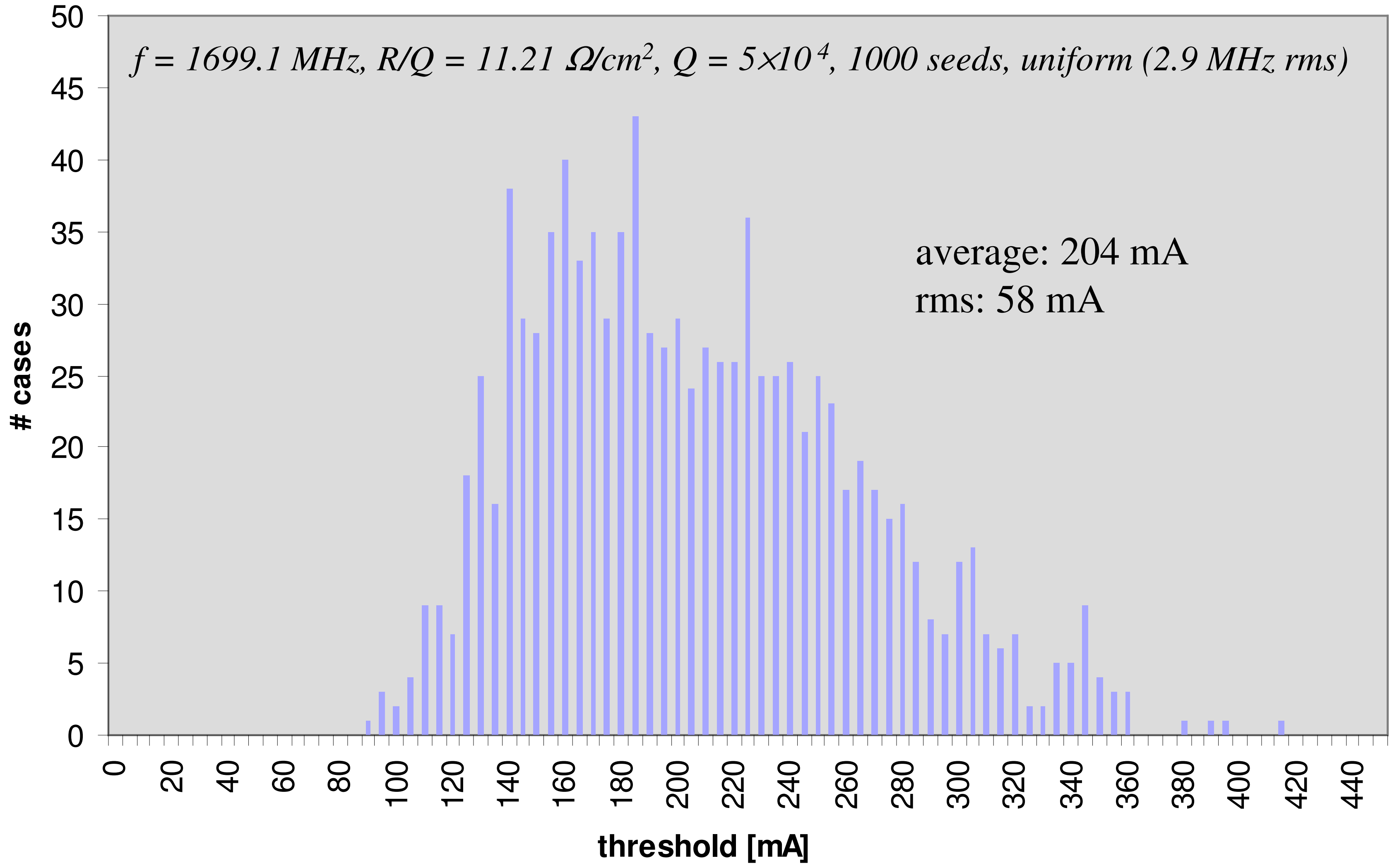
Threshold for different seeds (10 MHz uniform)



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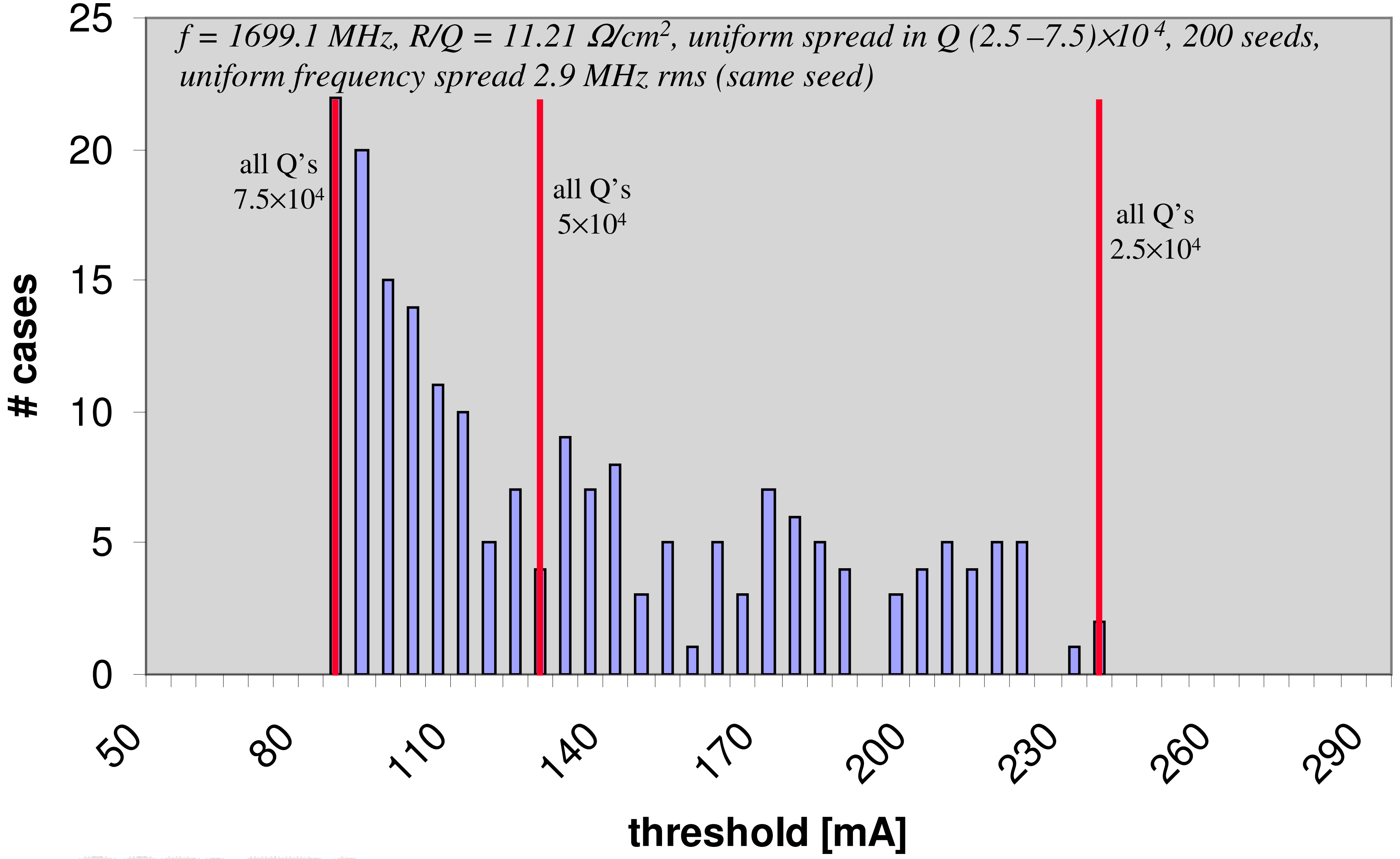
Spread in Q's



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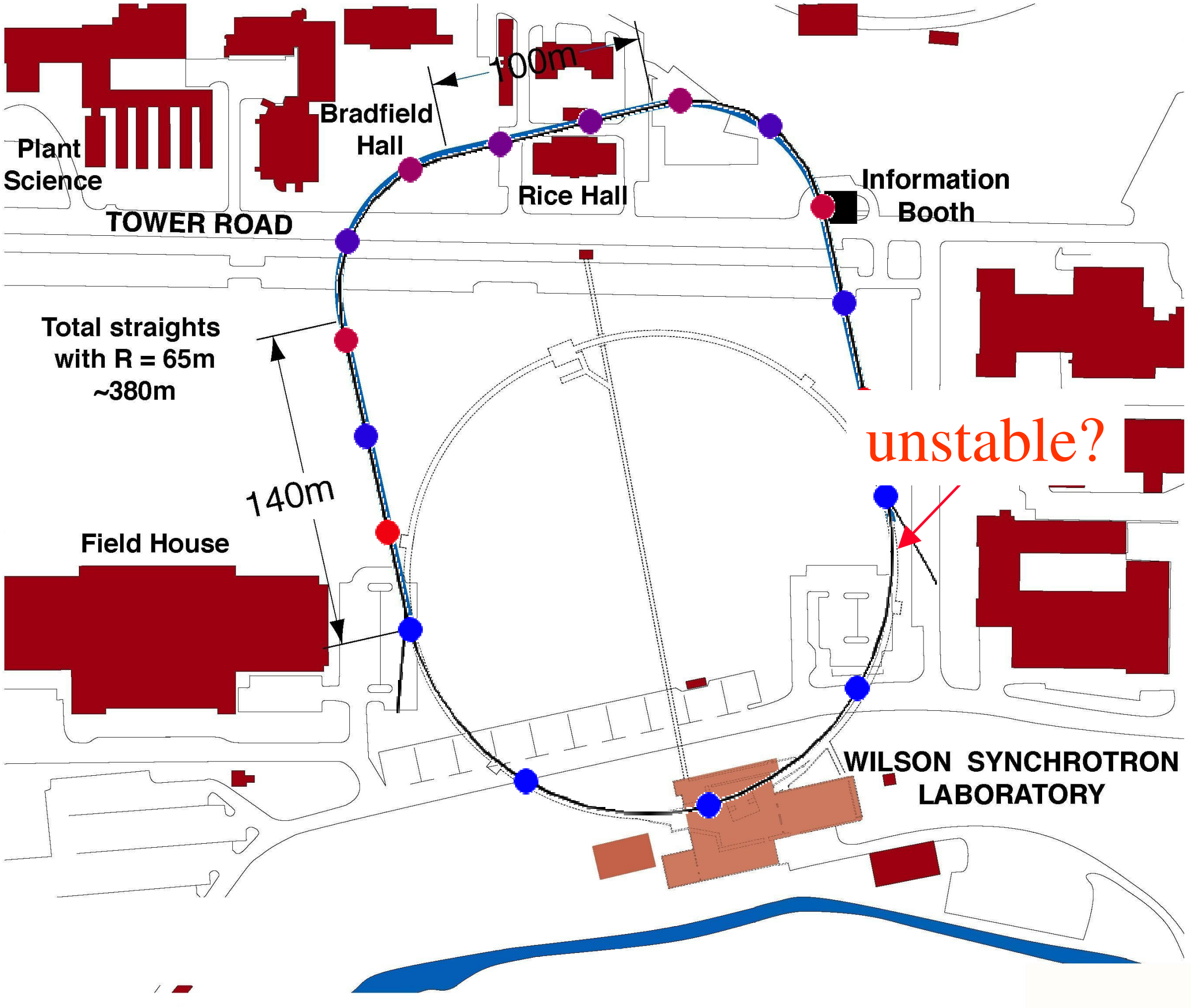
Direction we are heading to?

- Refrigerator 16.4 MW
- RF power 1.1 MW
- Dumped power 1 MW
- 7 undulators in the current design



Two-recirculation option

- Need half the linac
  - ✓ linac cost
  - ✓ refrigeration

- BBU is currently limited to ~ 20 mA

- Current in linac is now 400 mA (difficult even for storage rings), e.g. beam induced effects go up by a factor of 4

BBU in two-pass ERL: gruesome look [from ERL memo 02-4]

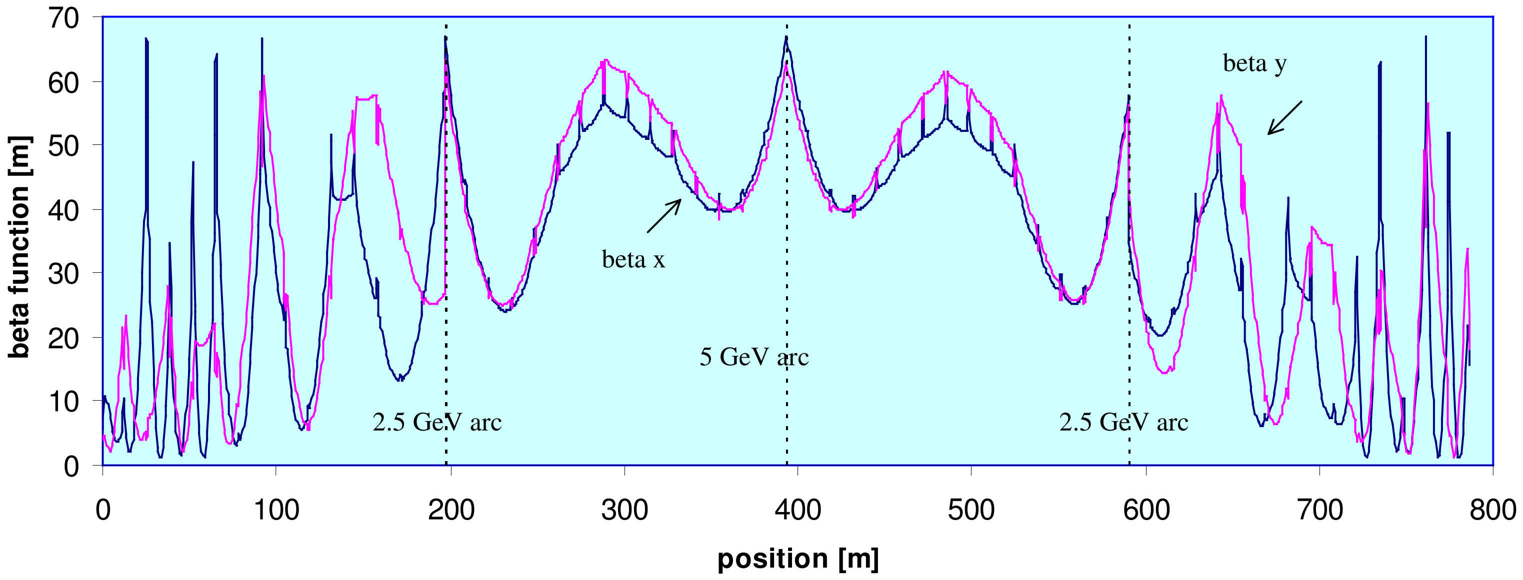


Table 1.
Results of TDBBU runs for 1-pass and 2-pass 5 GeV ERL.
HOM table (TESLA TDR 03/2001)

f (MHz)	R/Q (Ohm)	Q	(R/Q)*Q	1-pass 5 GeV ERL BBU (mA)	2-pass 5 GeV ERL BBU (mA)	Q*	Improved by a factor of
1699	88.40	5.00E+04	4.42E+06	160	20	8.00E+02	62.5
1873	56.39	7.00E+04	3.95E+06	190	25	1.30E+03	53.8
2575	51.50	5.00E+04	2.57E+06	115	15	9.00E+02	55.6
1725	118.64	2.00E+04	2.37E+06	135	15	5.00E+02	40.0
1864	42.84	5.00E+04	2.14E+06	> 200	40	2.00E+03	25.0
1880	11.08	1.00E+05	1.11E+06	> 200	90	8.00E+04	1.3
…	…	…	…	> 200	> 100		

* BBU th >=100 mA

Trapped in the ~ 20 MW ERL?

Suggestion: drop the average current (~ 30 mA) and do two-recirculations

Pros: real savings (both construction & operation)
higher injection energy (~ 20 MeV)
lower space charge (improved brilliance)
more room for undulators

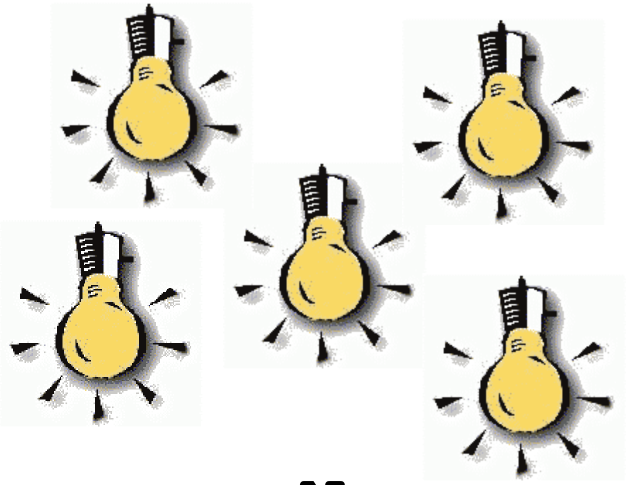
Cons: lower flux per meter of insertion device

Maxim time

Never say “never”

Flux: how many bulbs does it takes?

N×FLUX



BRILLIANCE

N

How much flux do people really care for?

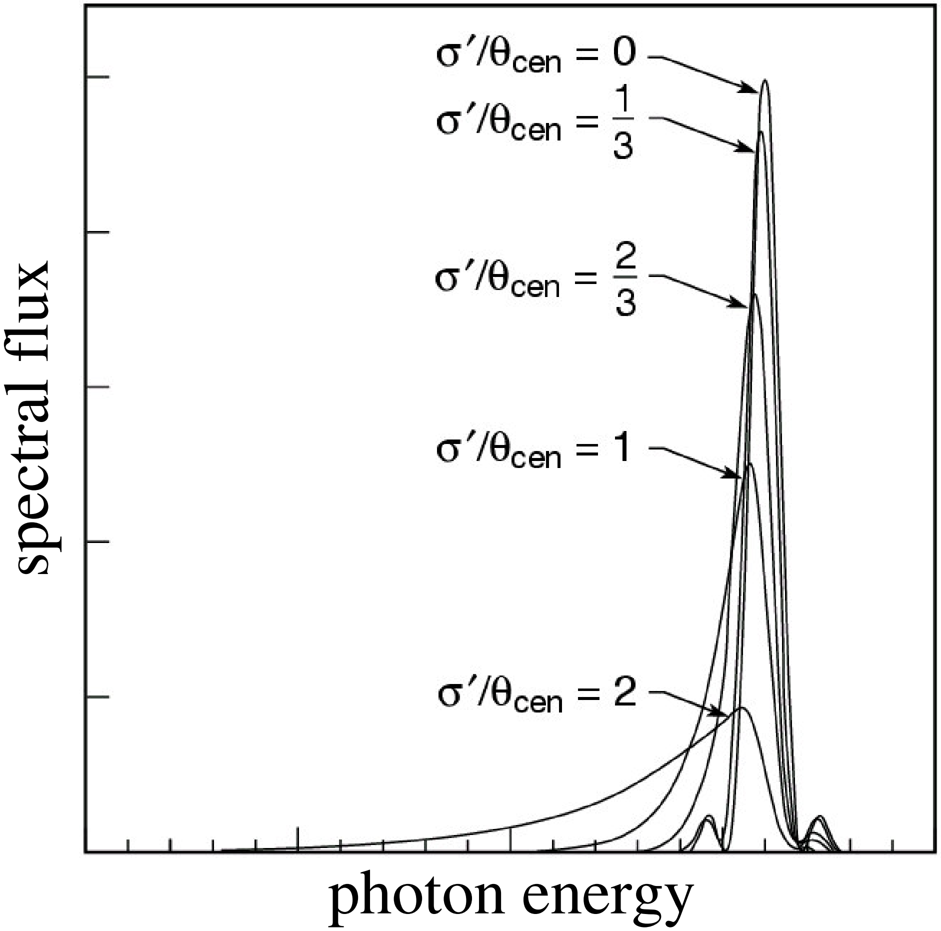
- At 100 mA one has ~ 1 kW of X-ray flux per meter of undulator
- At 30 mA, one would still have ~ 300 W per meter

$$\mathbf{FLUX} \times (\Delta\omega/\omega) \times (\varepsilon_{ph}/\varepsilon_{\perp})^2$$

spatial filtering with pin-hole for undulator does both

- Flux-driven experiments don’t need small size and parallel beam – they need to be identified and dedicated IDs should be used for such applications

Brightness degradation due electron angular spread



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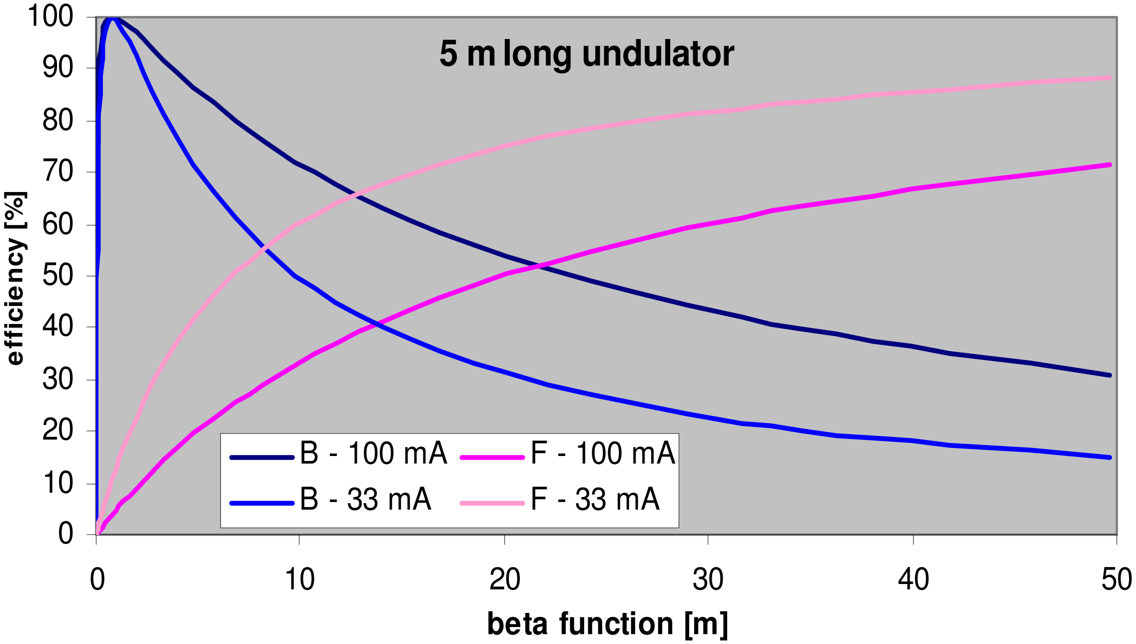
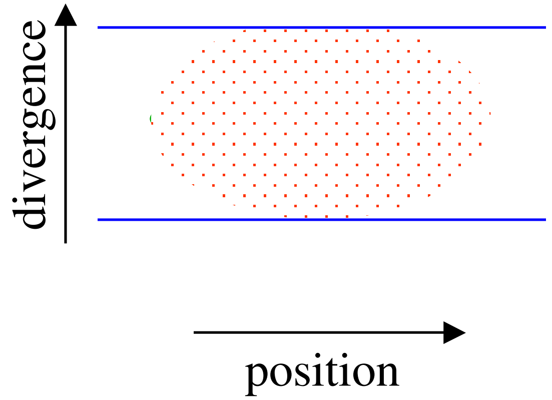
Beam matching for max brilliance and flux

$$\frac{dF_n}{d\Omega} = \frac{F_n}{2\pi\sqrt{\sigma_{r'}^2+\sigma_{x'}^2}\sqrt{\sigma_{r'}^2+\sigma_{y'}^2}}$$



optimizing flux



100 mA: 5 m ID

30 mA: 8 m ID

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same flux

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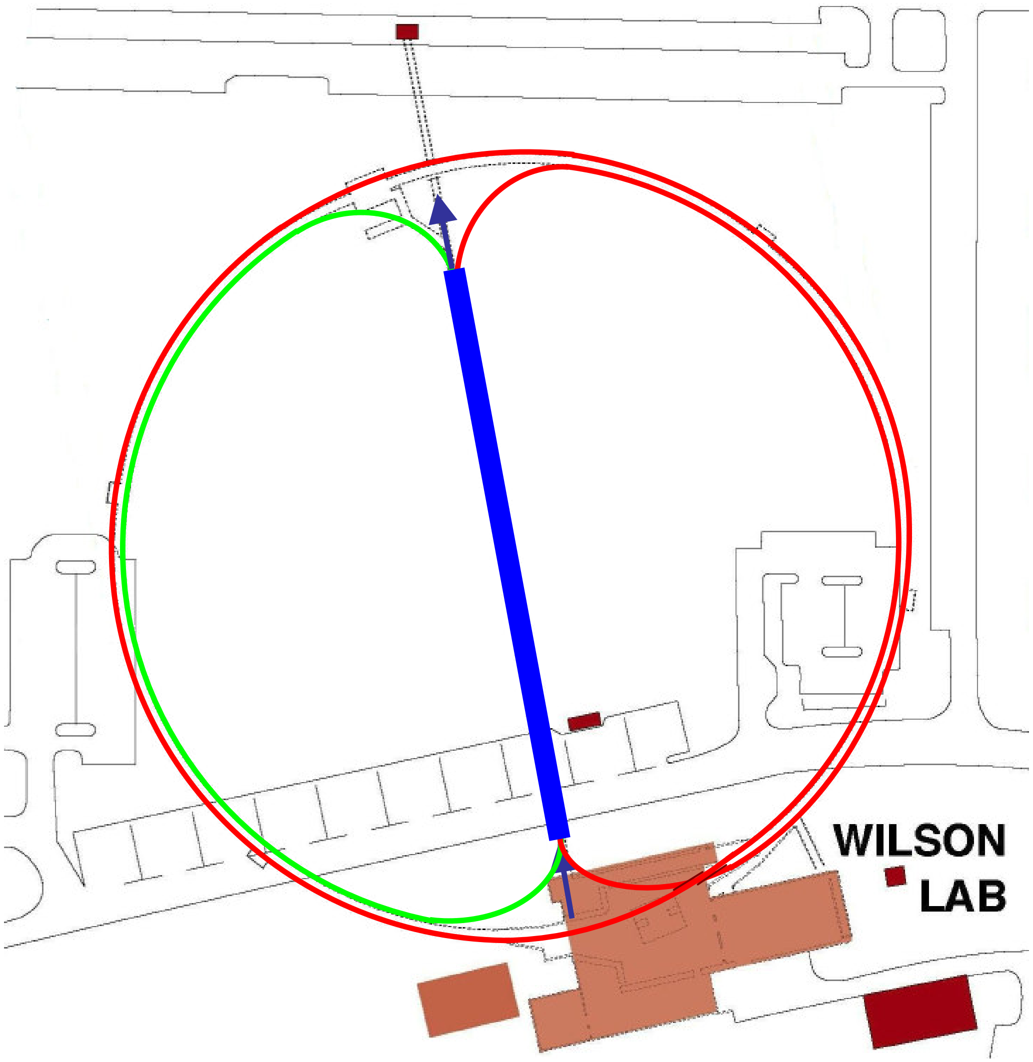
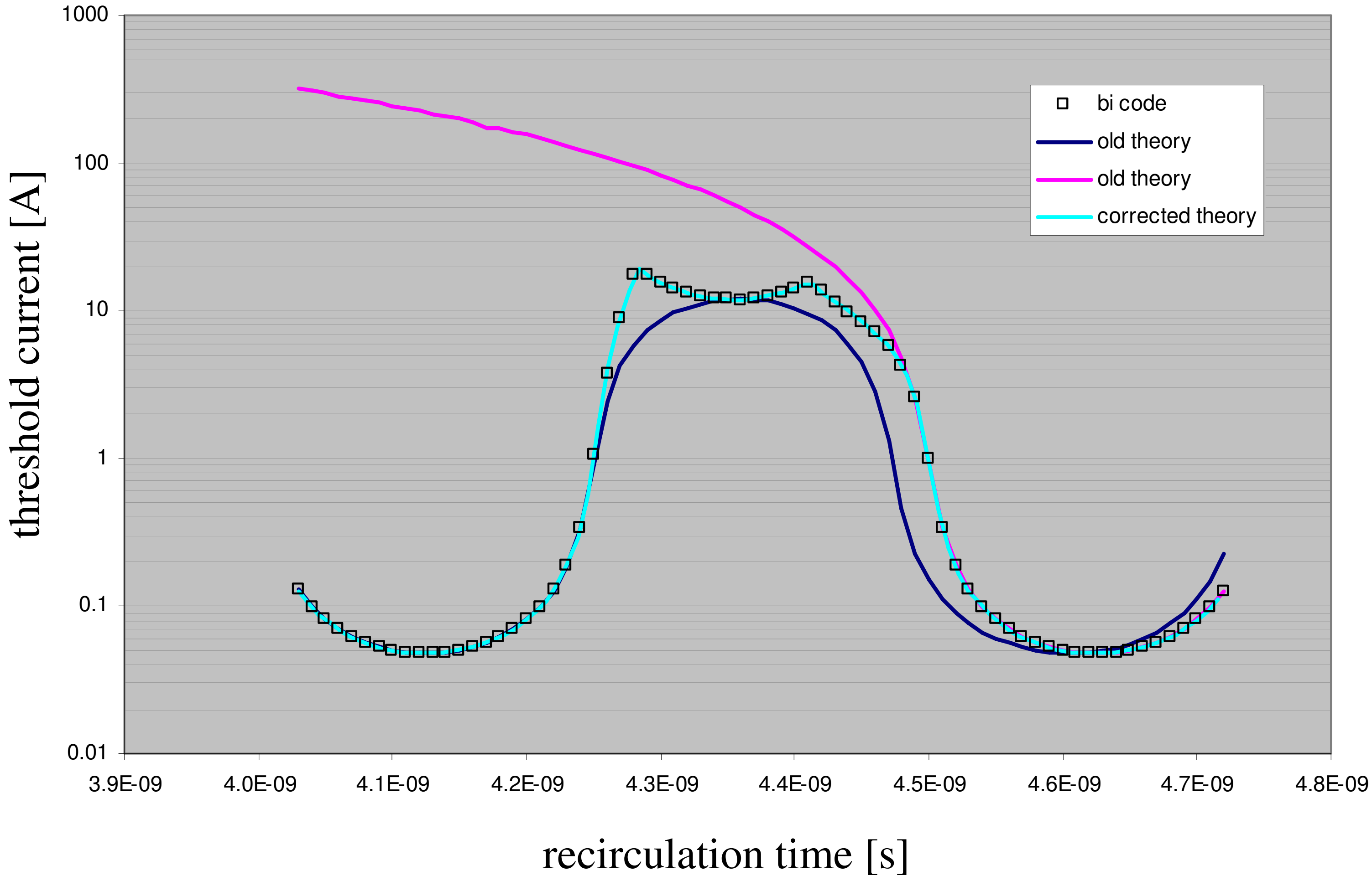
ERL in CESR tunnel II



Explaining tracking curve



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