The effect of dipole and quadrupole HOMs on the performance of the Phase II ERL

Outline:

• Beam induced voltage: losses and kicks
• Beam breakup for quadrupole HOMs
• BBU statistics for “ERL in CESR tunnel”
• Discussion
Introduction

The “Big Four”

- Superposition
- Conservation of energy
- Concept of normal modes
- Causality

monopole ($m = 0$)

$TM_{01}$-like

high losses, no kick

dipole ($m = 1$)

$TM_{11}$-like

kick and losses when beam is not centered

quadrupole ($m = 2$)

$TM_{21}$-like

kick, coupling and losses when beam is not centered
Wake formalism for point charge

The above can be combined to say (Panofsky-Wenzel):

\[ \nabla_\perp \int_{-L/2}^{L/2} F_\parallel ds = \frac{\partial}{\partial z} \int_{-L/2}^{L/2} \vec{F}_\perp ds \]
Wake function for resonant mode

A mode characterized by \( \left( \frac{R}{Q} \right)_m, Q, \omega \)

**Transverse**

\[
W_m = 2ck_m \frac{kz}{\omega} e^{2Q} \sin kz
\]

**Longitudinal**

\[
W'_m \approx 2k_m e^{2Q} \cos kz
\]

Loss factor \( k_m = \frac{\omega}{4} \left( \frac{R}{Q} \right)_m \)

\( \left( \frac{R}{Q} \right)_m \) in units of \( \Omega \cdot m^{-2m} \)

\( k = \omega / c \) is mode’s wave number
Beam breakup

**Transverse**

\[ x^{(2)}(t) = \frac{e}{c} m_{12} V_x(t - t_r) \]

\[ V_{1,x}(t) = -\int_{-\infty}^{t} W_1(t - t')[I^{(1)}(t')x^{(1)}(t') + I^{(2)}(t')x^{(2)}(t')]dt' \]

**Longitudinal**

\[ I_{th} = \frac{2Ec}{e(R/Q)\omega_\lambda Q_\lambda} \frac{1}{R_{56}} \]

\[ I_{th} = \frac{2Ec^2}{e(R/Q)\omega_\lambda Q_\lambda} \frac{1}{R_{12}} \]
Transient and steady case parts of induced voltage

- Equation for transient part reproduces integral equation for BBU for aligned cavity
- Steady case part of beam induced voltage generates displaced orbit

\[
-t \frac{\hbar}{c} \int_{-\infty}^{t} W_{1}(t-t') \left[ I^{(2)}(t') + I^{(2)}(t' + t_r) \right] dt'
- \frac{e}{c} \frac{m_{12} d}{1 + \frac{e}{c} \int_{-\infty}^{t} W_{1}(t-t') I^{(2)}(t') dt'}
\]
Beam induced voltage from infinite bunch train

\[ V_{\parallel,m} = -q d^{2m} \frac{\omega}{2} \left( \frac{R}{Q} \right)_m \Re \left\{ \sum_{n=0}^{\infty} e^{-n \frac{\omega t_0}{2Q}} e^{-in \omega t_0} - \frac{1}{2} \right\} \]

\[ V_{\perp,m} = -\frac{m}{2} q d^{2m-1} c \left( \frac{R}{Q} \right)_m \Im \left\{ \sum_{n=0}^{\infty} e^{-n \frac{\omega t_0}{2Q}} e^{-in \omega t_0} - \frac{1}{2} \right\} \]

\[ \sum_{n=0}^{\infty} e^{-n \frac{\omega t_0}{2Q}} e^{-in \omega t_0} = S^+ (\tau, \delta) - \frac{1}{2} = F_\Re (\tau, \delta) - i F_\Im (\tau, \delta) \]

\[ \tau = \frac{\omega t_0}{2Q}, \quad \delta = \omega t_0 \]
Resonance function

\[
\tau = 0.3
\]

\[
\begin{align*}
F_{\Re} &\quad \text{Real part} \\
F_{\Im} &\quad \text{Imaginary part}
\end{align*}
\]

1) \( \frac{1}{2\pi} \int_{-\pi}^{\pi} F_{\Re}(\tau, \delta) d\delta = \frac{1}{2} \)

2) \( \max F_{\Re}(\tau, \delta) = F_{\Re}(\tau, 0) = \frac{1}{\tau} \)

3) \( F_{\Re} = \frac{1}{2} \) when \( \delta = \pm \sqrt{2\tau} = \pm \sqrt{\frac{\alpha_0}{Q}} \)

1) \( \frac{1}{2\pi} \int_{-\pi}^{\pi} F_{\Im}(\tau, \delta) d\delta = 0 \)

2) \( \max |F_{\Im}(\tau, \delta)| \approx |F_{\Im}(\tau, \pm \frac{\pi}{\sqrt{2}})| \approx \frac{1}{2\tau} \)

3) \( |F_{\Im}| \geq \frac{1}{2} \) when \( -\frac{\pi}{2} \leq \delta \leq \frac{\pi}{2} \)
A Simple Model: 1000 Monopoles with random f’s
Total HOM Monopole Power for random Sets of Frequencies

- All HOMs have $Q = 100$
- All HOMs have $Q = 1000$
- All HOMs have $Q = 10^4$
- All HOMs have $Q = 10^5$
Beam induced voltages

Typical

\[ \langle V_{\parallel,m} \rangle = -k_m q d^{2m} = -q d^{2m} \frac{\omega}{4} \left( \frac{R}{Q} \right)_m \]

At resonance

\[ |V_{\parallel,m}| = \left( \frac{R}{Q} \right)_m Q d^{2m} I \]

Typical

\[ \langle V_{\perp,m} \rangle = 0 \]

At resonance

\[ |V_{\perp,m}| = \frac{m}{2} \left( \frac{R}{Q} \right)_m Q k^{-1} d^{2m-1} I \]
### Dipole HOMs from TESLA TDR for 100 mA ERL

<table>
<thead>
<tr>
<th>$R/Q$, $\frac{\Omega}{\text{cm}^2}$</th>
<th>$Q$</th>
<th>$\max V_{\perp}$, $\frac{V}{\text{mm}}$</th>
<th>$\max V_{\parallel}$, $\frac{V}{\text{mm}^2}$</th>
<th>$\max P$, $\frac{W}{\text{mm}^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.21</td>
<td>50000</td>
<td>1.03E+04</td>
<td>1121</td>
<td>224.2</td>
</tr>
<tr>
<td>8.69</td>
<td>70000</td>
<td>1.12E+04</td>
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<td>243.32</td>
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<td>15</td>
<td>50000</td>
<td>1.38E+04</td>
<td>1500</td>
<td>300</td>
</tr>
<tr>
<td>15.51</td>
<td>20000</td>
<td>5.69E+03</td>
<td>620.4</td>
<td>124.08</td>
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<tr>
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<td>50000</td>
<td>6.00E+03</td>
<td>654</td>
<td>130.8</td>
</tr>
<tr>
<td>1.72</td>
<td>100000</td>
<td>3.16E+03</td>
<td>344</td>
<td>68.8</td>
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<tr>
<td>1.75</td>
<td>75000</td>
<td>2.41E+03</td>
<td>262.5</td>
<td>52.5</td>
</tr>
<tr>
<td>0.76</td>
<td>70000</td>
<td>9.76E+02</td>
<td>106.4</td>
<td>21.28</td>
</tr>
<tr>
<td>1.05</td>
<td>100000</td>
<td>1.93E+03</td>
<td>210</td>
<td>42</td>
</tr>
<tr>
<td>2.16</td>
<td>20000</td>
<td>7.93E+02</td>
<td>86.4</td>
<td>17.28</td>
</tr>
<tr>
<td>0.5</td>
<td>100000</td>
<td>9.18E+02</td>
<td>100</td>
<td>20</td>
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<tr>
<td>0.46</td>
<td>50000</td>
<td>4.22E+02</td>
<td>46</td>
<td>9.2</td>
</tr>
<tr>
<td>0.39</td>
<td>25000</td>
<td>1.79E+02</td>
<td>19.5</td>
<td>3.9</td>
</tr>
<tr>
<td>0.77</td>
<td>10000</td>
<td>1.41E+02</td>
<td>15.4</td>
<td>3.08</td>
</tr>
</tbody>
</table>
Transverse multipass beam breakup

dipole mode

\[ \Delta V_{1,x} = -qW_1(z)\langle x_a \rangle \]

\[ \Delta V_{1,y} = -qW_1(z)\langle y_a \rangle \]

\[ I_1 \approx \frac{2\alpha}{em_{12}(R/Q)_1Q} \]

\[ m_{12} = \sqrt{\frac{\beta^{(1)}_{x,a} \beta^{(2)}_{x,a}}{p^{(1)} \beta^{(2)}_{x,a}}} \sin \Delta \psi_x \]

quadrupole mode

focusing

\[ \Delta \frac{\partial}{\partial x} V_{2,x} = -\Delta \frac{\partial}{\partial y} V_{2,y} = \]

\[ -2qW_2(z)\left(\langle x_a \rangle^2 - \langle y_a \rangle^2 + \langle x_a^2 \rangle - \langle y_a^2 \rangle\right) \]

coupling

\[ \Delta \frac{\partial}{\partial y} V_{2,x} = \Delta \frac{\partial}{\partial x} V_{2,y} = \]

\[ -2qW_2(z)2\langle x_a \rangle\langle y_a \rangle \]
Quadrupole HOM induced voltage for 100 mA ERL

focusing effect:

\[
\max \frac{\partial}{\partial x} V_{2,x} = \left( \frac{R}{Q} \right)_m Qk^{-1} Id^2
\]

\[
\max \frac{\partial}{\partial x} V_{2,x} \sim 10^4 \frac{V}{m} \cdot d^2 \text{(mm)}
\]

focus. length \( \frac{p}{\frac{e}{c} \frac{\partial}{\partial x} V_x} \geq 10^3 \text{ m } / d^2 \text{(mm)} \)

coupling effect:

\[
\Delta \varepsilon_{n,c} = \frac{e}{mc^2} \sigma_x \sigma_y \frac{\partial}{\partial y} V_{2,x} \leq 1\% \cdot d^2 \text{(mm)}
\]

\[
\varepsilon_n^2 = \varepsilon_{n,0}^2 + \Delta \varepsilon_{n,c}^2
\]
Beam breakup due to quadrupole HOMs

a) aligned cavity $\sigma_{x,y} \gg d$

$$I_{2a}) \approx \frac{2\omega}{e(R/Q)Q} \frac{2m_e c [\varepsilon_{x,n} \beta_{x,n}^{(1)} \beta_{x,n}^{(2)} \sin 2\Delta \psi_x + \varepsilon_{y,n} \beta_{y,n}^{(1)} \beta_{y,n}^{(2)} \sin 2\Delta \psi_y]}{p^{(1)} p^{(2)}}$$

$$\frac{I_{2a})}{I_1} \sim \frac{b^2}{4\sigma_{x,y}^2}$$

a) misaligned cavity $d \gg \sigma_{x,y}$

$$I_{2b}) \approx \frac{2\omega}{e(R/Q)Qm_{12}} \frac{1}{4d^2}$$

$$\frac{I_{2b})}{I_1} \sim \frac{b^2}{4d^2}$$
**bi** - numeric tool for transverse BBU

Features:
- written in C++
- cleaner algorithm than TDBBU
- faster than TDBBU (a single 5 GeV ERL run takes less than a minute; execution time is estimated to be 7-9 times faster when no coupling is present; with coupling it is estimated to be at least 4 times faster)
- allows any ERL topology
- easier to use
Linac optics (DCS, 04/01/03)

- **Graph Title**: Linac optics "erl in cesr tunnel" DCS 04/01/03
- **Axes**:
  - Y-axis: beta [m] (0 to 90)
  - X-axis: position from injection point [m] (0 to 1800)
- **Lines**:
  - Blue line: betax
  - Pink line: betay

Ivan Bazarov, The effect of dipole and quadrupole HOMs on the performance of the Phase II ERL, ERL mtg, 23 June 2003
Worst 14 dipole modes from TESLA TDR

\[
y = 1.88 \cdot 10^6 x^{-0.931}
\]

\[\frac{(R/Q)Q}{f} \text{ [Ω/cm}\textsuperscript{2}/\text{GHz}]\]

threshold current [mA]
Uniform frequency spread of 10 MHz

\[ y = 2.59 \cdot 10^7 x^{-0.944} \]
Fixed current (30 mA)

rms = 0 Hz

rms = 33 kHz

rms = 42 kHz

rms = 46 kHz

rms = 53 kHz

rms = 67 kHz
Stabilizing effect of HOM frequency spread

- Effectively decreases $Q$ of the mode: $Q \sim \omega/\Delta \omega_{1/2}$
- Limited in its effect by $\sim \omega/Q$ of the HOM

Linearized solution for simplest case

\[
I_1 \approx \frac{-2c}{e(R/Q)Qkm_{12} \sin(\Omega t_r)},
\]

\[
\Delta \omega = \frac{\omega}{2Q} \cot(\Omega t_r)
\]

Periodic in $\Delta \omega = \Omega - \omega$ with $\omega_r = 2\pi/t_r$
Threshold vs. frequency spread

\[ f = 1699.1 \text{ MHz}, \ R/Q = 11.21 \ \Omega/cm^2, \ Q = 5 \times 10^4, \text{ same seed, uniform} \]
Threshold for different seeds (10 MHz uniform)

$f = 1699.1 \text{ MHz}, R/Q = 11.21 \, \Omega/\text{cm}^2, Q = 5 \times 10^4, 1000 \text{ seeds, uniform (2.9 MHz rms)}$

average: 204 mA  
rms: 58 mA
Spread in Q’s

\( f = 1699.1 \text{ MHz}, \frac{R}{Q} = 11.21 \text{ \Omega/cm}^2, \text{ uniform spread in } Q (2.5–7.5) \times 10^4, 200 \text{ seeds, uniform frequency spread } 2.9 \text{ MHz rms (same seed)} \)
Direction we are heading to?

- Refrigerator
  16.4 MW

- RF power
  1.1 MW

- Dumped power
  1 MW

- 7 undulators in the current design
Two-recirculation option

• Need half the linac
  ✓ linac cost
  ✓ refrigeration

• BBU is currently limited to ~ 20 mA

• Current in linac is now 400 mA (difficult even for storage rings), e.g. beam induced effects go up by a factor of 4
BBU in two-pass ERL: gruesome look [from ERL memo 02-4]

Table 1.
Results of TDBBU runs for 1-pass and 2-pass 5 GeV ERL.
HOM table (TESLA TDR 03/2001)

<table>
<thead>
<tr>
<th>f (MHz)</th>
<th>R/Q (Ohm)</th>
<th>Q</th>
<th>(R/Q)*Q</th>
<th>1-pass 5 GeV ERL BBU (mA)</th>
<th>2-pass 5 GeV ERL BBU (mA)</th>
<th>Improved by Q* a factor of</th>
</tr>
</thead>
<tbody>
<tr>
<td>1699</td>
<td>88.40</td>
<td>5.00E+04</td>
<td>4.42E+06</td>
<td>160</td>
<td>20</td>
<td>8.00E+02 62.5</td>
</tr>
<tr>
<td>1873</td>
<td>56.39</td>
<td>7.00E+04</td>
<td>3.95E+06</td>
<td>190</td>
<td>25</td>
<td>1.30E+03 53.8</td>
</tr>
<tr>
<td>2575</td>
<td>51.50</td>
<td>5.00E+04</td>
<td>2.57E+06</td>
<td>115</td>
<td>15</td>
<td>9.00E+02 55.6</td>
</tr>
<tr>
<td>1725</td>
<td>118.64</td>
<td>2.00E+04</td>
<td>2.37E+06</td>
<td>135</td>
<td>15</td>
<td>5.00E+02 40.0</td>
</tr>
<tr>
<td>1864</td>
<td>42.84</td>
<td>5.00E+04</td>
<td>2.14E+06</td>
<td>&gt; 200</td>
<td>40</td>
<td>2.00E+03 25.0</td>
</tr>
<tr>
<td>1880</td>
<td>11.08</td>
<td>1.00E+05</td>
<td>1.11E+06</td>
<td>&gt; 200</td>
<td>90</td>
<td>8.00E+04 1.3</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>&gt; 200</td>
<td>&gt; 100</td>
<td></td>
</tr>
</tbody>
</table>

* BBU th >=100 mA
Trapped in the ~ 20 MW ERL?

**Suggestion: drop the average current (~ 30 mA) and do two-recirculations**

**Pros:** real savings (both construction & operation)  
higher injection energy (~ 20 MeV)  
lower space charge (improved brilliance)  
more room for undulators

**Cons:** lower flux per meter of insertion device
Maxim time

Never say “never”
Flux: how many bulbs does it take?

It Takes How Many!

N×FLUX

BRILLIANCE

N
How much flux do people really care for?

- At 100 mA one has \( \approx 1 \text{kW} \) of X-ray flux per meter of undulator.

- At 30 mA, one would still have \( \approx 300 \text{W} \) per meter.

\[
\text{FLUX} \times (\Delta \omega/\omega) \times (\varepsilon_{ph}/\varepsilon_\perp)^2
\]

*spatial filtering with pin-hole for undulator does both*

- Flux-driven experiments don’t need small size and parallel beam – they need to be identified and dedicated IDs should be used for such applications.
Brightness degradation due electron angular spread

\[
\frac{\sigma'}{\theta_{\text{cen}}} = 0 \\
\frac{\sigma'}{\theta_{\text{cen}}} = \frac{1}{3} \\
\frac{\sigma'}{\theta_{\text{cen}}} = \frac{2}{3} \\
\frac{\sigma'}{\theta_{\text{cen}}} = 1 \\
\frac{\sigma'}{\theta_{\text{cen}}} = 2
\]

spectral flux

photon energy
Beam matching for max brilliance and flux

\[ \frac{dF_n}{d\Omega} = \frac{F_n}{2\pi \sqrt{\sigma_r^2 + \sigma_x^2 \sqrt{\sigma_r^2 + \sigma_y^2}}} \]

- 5 m long undulator
- 100 mA: 5 m ID
- 30 mA: 8 m ID
- same flux

optimizing flux

divergence

position

Cornell University
CHESS / LEPP
Explaining tracking curve