On Maximum Brightness from X-ray Light Sources

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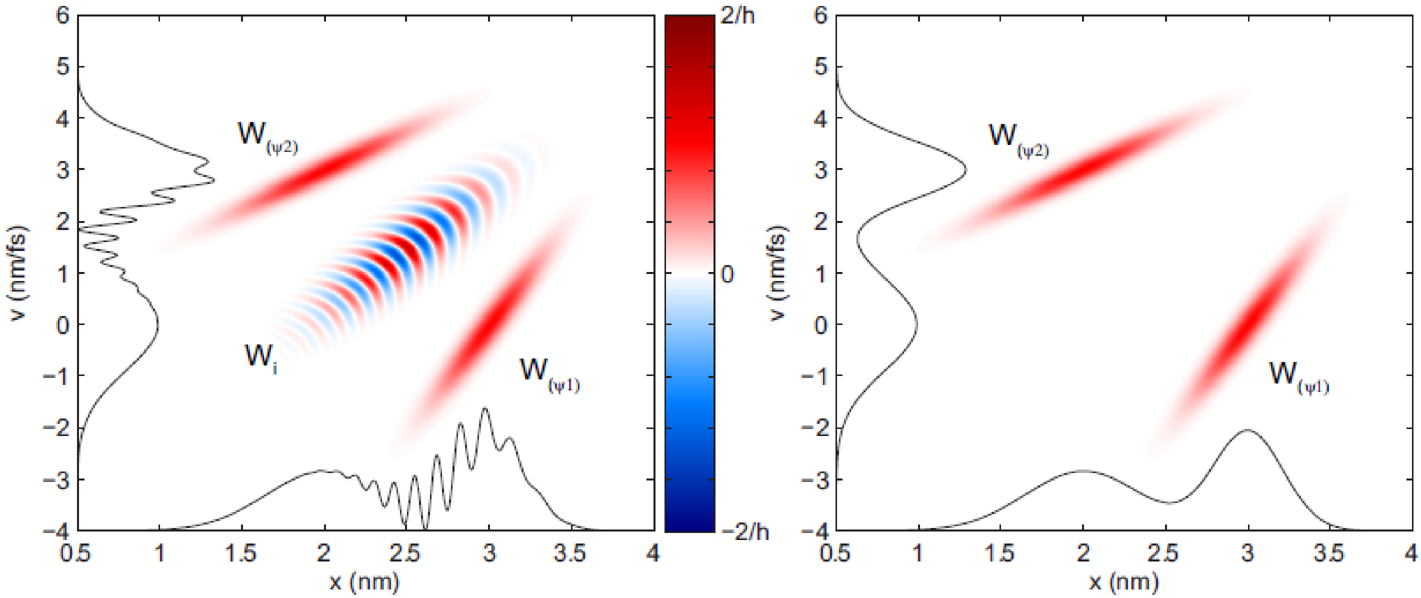

phase space of coherent (left) and incoherent (right) 2-state superposition

Some of today's talk points

- **Partially coherent radiation in phase space: revisiting what brightness really is**
- **Few words about rms emittance: introduction of a more appropriate metric (emittance vs. fraction)**
- **My view on SR vs ERL comparison**

Comparison metrics?

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- **Cost (capital, operational)**
- **Upgradeability**
- **Time structure (pulse length, rep rate)**
- **Brightness (maximize useful flux)**
  - **Efficient use of undulators (low beam spread, flexible matching)**
  - **Optics heat load (minimize total flux)**

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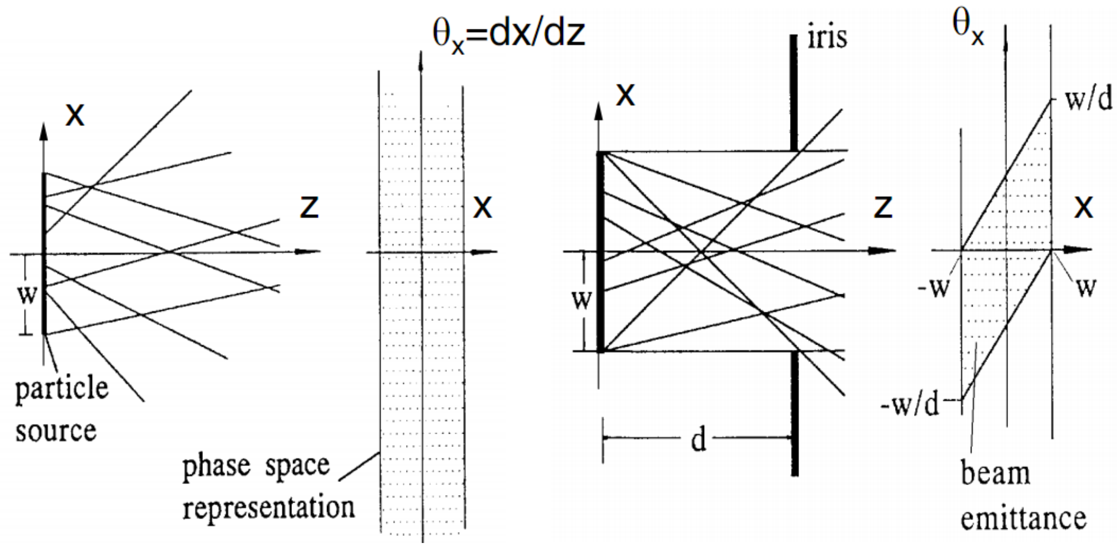
Do we understand physics of x-ray brightness?
Is *diffraction limit* same as *full transverse coherence*?
How to account for non-Gaussian beams (both e^- and γ)?

Brightness: geometric optics

- Rays moving in drifts and focusing elements



- Brightness = particle density in phase space (2D, 4D, or 6D)

Phase space in classical mechanics

- Classical: particle state (x, p)
- Evolves in time according to $\dot{p} = -\frac{\partial \mathcal{H}}{\partial x}$, $\dot{x} = \frac{\partial \mathcal{H}}{\partial p}$
- E.g. drift:

  $$\mathcal{H} = \frac{p^2}{2m}$$
  $$\dot{p} = 0, \quad \dot{x} = \frac{p}{m}$$

  linear restoring force:

  $$\mathcal{H} = \frac{p^2}{2m} + \frac{kx^2}{2}$$
  $$\dot{p} = -kx, \quad \dot{x} = \frac{p}{m}$$

- Liouville's theorem: phase space density stays const along particle trajectories

Phase space in quantum physics

- Quantum state:

$\psi(x)$	or	$\phi(p)$
Position space	$\leftarrow \mathcal{FT} \rightarrow$	momentum space

- If either $\psi(x)$ or $\phi(p)$ is known – can compute anything. Can evolve state using time evolution operator: $\exp(-\frac{i\mathcal{H}t}{\hbar})$
- $|\psi(x)|^2 dx$ - probability to measure a particle with $(x, x+dx)$
- $|\phi(p)|^2 dp$ - probability to measure a particle with $(p, p+dp)$

Wigner distribution

$$W(x,p) \equiv \int_{-\infty}^{+\infty} \langle\psi|x+\tfrac{x'}{2}\rangle\langle x+\tfrac{x'}{2}|p\rangle\langle p|x-\tfrac{x'}{2}\rangle\langle x-\tfrac{x'}{2}|\psi\rangle\, dx'$$

$$= \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \psi^*(x+\frac{x'}{2})\psi(x-\frac{x'}{2})e^{ipx'/\hbar}dx'$$

$$= \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \phi^*(p+\frac{p'}{2})\phi(p-\frac{p'}{2})e^{-ip'x/\hbar}dp'$$

- $W(x,p)dxdp$ – (quasi)probability of measuring quantum particle with $(x, x+dx)$ and $(p, p+dp)$

Classical electron motion in potential

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Animation: click to play

Same in phase space…

Animation: click to play

Going quantum in phase space…

Animation: click to play

Some basic WDF properties

- $W(x,p) \in \mathbb{R}$ (can be negative)
- $\iint W(x,p)dxdp = 1$
- $\int W(x,p)dp = |\psi(x)|^2$
- $\int W(x,p)dx = |\phi(p)|^2$
- Time evolution of $W(x,p)$ is *classical* in absence of forces or with linear forces

Connection to light

- **Quantum** – $\psi(x)$
- Linearly polarized **light** (1D) – $E(x)$
- Measurable $|\psi(x)|^2$ – **charge density**
- Measurable $|E(x)|^2$ – photon **flux density**
- **Quantum**: momentum representation
  $\phi(p)$ is FT of $\psi(x)$
- **Light**: far field (angle) representation
  $\mathcal{E}(\theta)$ is FT of $E(x)$

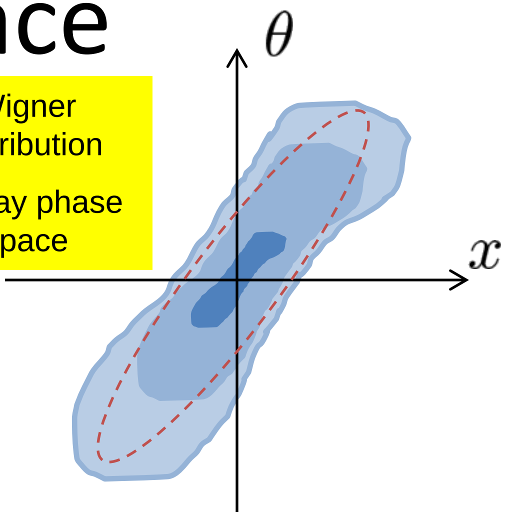
Connection to classical picture

- **Quantum**: $h \to 0$, recover classical behavior
- **Light**: $\lambda \to 0$, recover geometric optics
- $W(x,p)$ or $W(x,\theta)$ – phase space density (=brightness) of a quantum particle or light
- Wigner of a quantum state / light propagates classically in absence of forces or for linear forces
- **Wigner density function = brightness**

Extension of accelerator jargon to x-ray (wave) phase space

Wigner distribution

= x-ray phase space



- Σ -matrix

$$\Sigma = \begin{pmatrix} \langle x^2 \rangle & \langle x\theta \rangle \\ \langle \theta x \rangle & \langle \theta^2 \rangle \end{pmatrix}$$

- Twiss (equivalent ellipse) and emittance

$$\Sigma = \epsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \epsilon \mathbf{T}$$

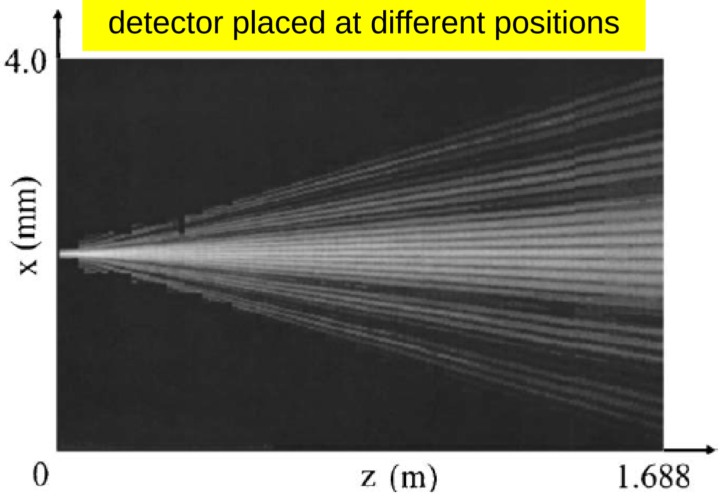
with $\det(\mathbf{T}) = 1$ and $\epsilon = \det(\Sigma)$ or

$$\epsilon = \sqrt{\langle x^2 \rangle \langle \theta^2 \rangle - \langle x\theta \rangle^2}$$

X-ray phase space can be measured using tomography

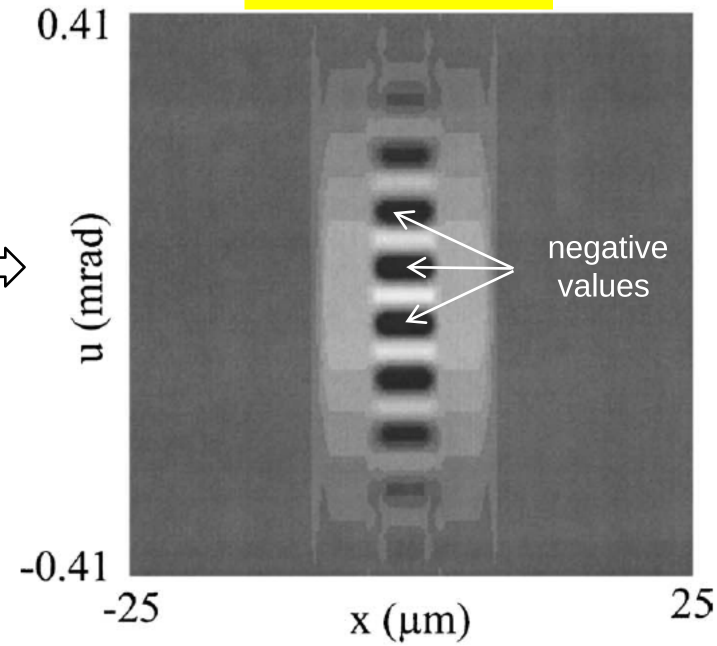
- **Same approach as phase space tomography in accelerators**
- **Except the phase space is now allowed to be locally negative**

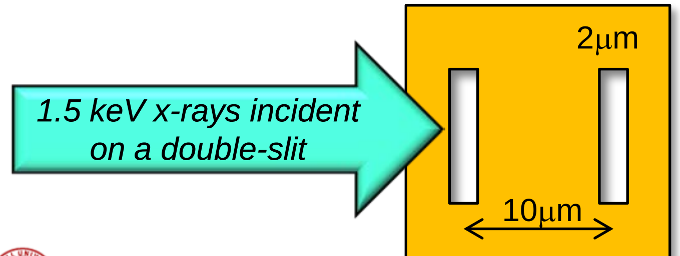
detector placed at different positions



Tomography

x-ray phase space





C.Q. Tran et al., JOSA A 22 (2005) 1691

Diffraction limit vs. coherence

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- **Diffraction limit (same as uncertainty principle)**

$$\sigma_p \sigma_x \geq \hbar/2 = h/4\pi \ \text{(QM)} \qquad \sigma_\theta \sigma_x \geq \lambda/4\pi \ \text{(light)}$$

$$M^2 = \frac{\epsilon_{\text{light}}}{\lambda/4\pi}$$

$M^2 \geq 1$ (ability to focus to a small spot)

  - **a classical counterpart exists (= e-beam emittance)**

- **Coherence (ability to form interference fringes)**
  - **Related to visibility or *spectral degree of coherence***

$$\mu_{12}(\omega) = \frac{\langle E_1(\omega) E_2^*(\omega) \rangle}{|E_1(\omega)|\,|E_2(\omega)|}$$

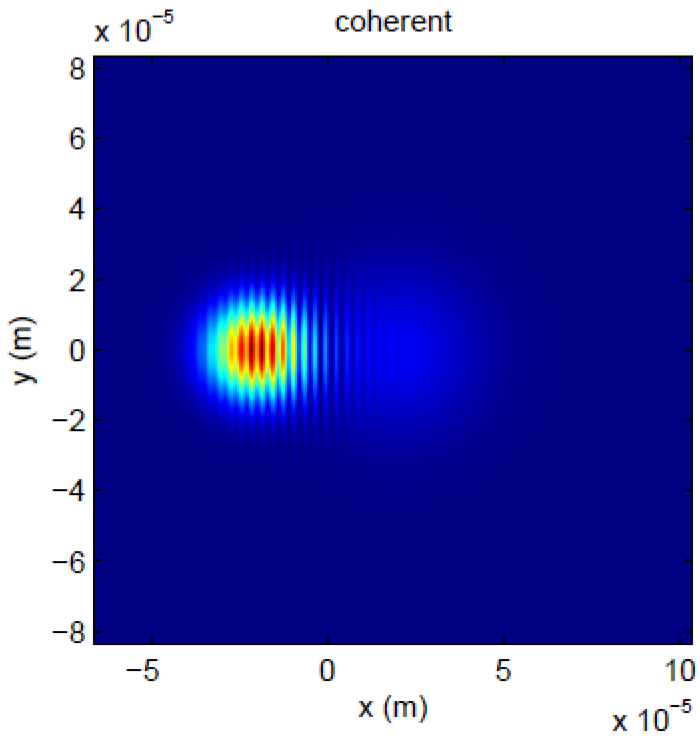
$0 \leq |\mu_{12}| \leq 1$

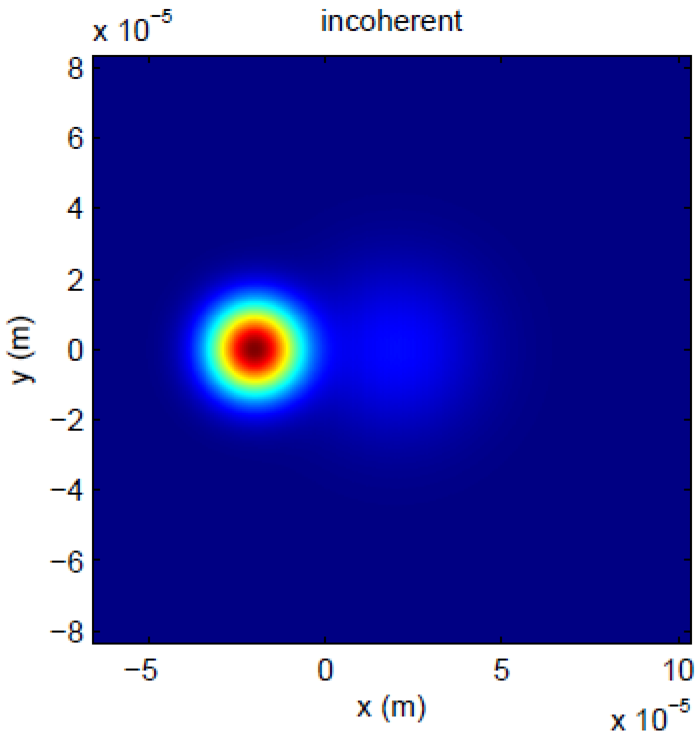
  - **quantum mechanical in nature – no classical counterpart exists**

- **Wigner distribution contains info about both!**

Example of combining sources (coherent vs incoherent)

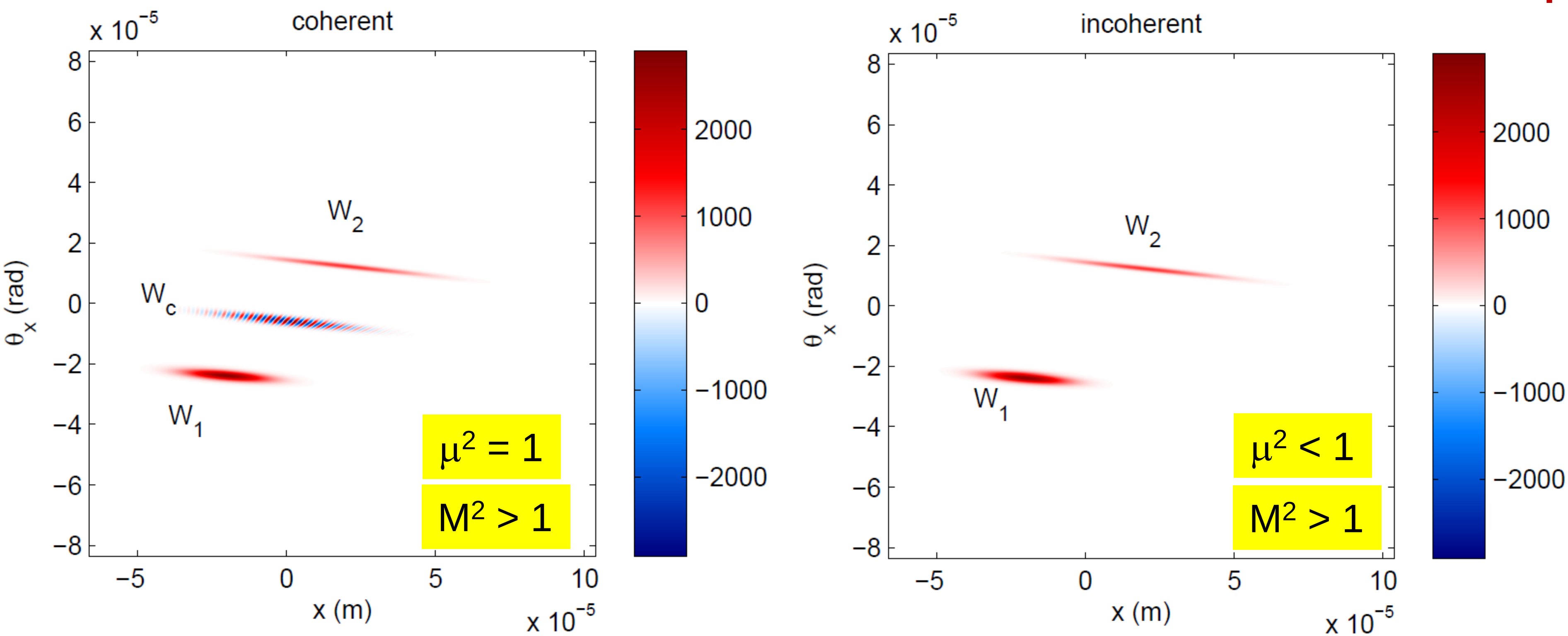
two laser Gaussian beams





Same picture in the phase space

two laser Gaussian beams



Facts of life

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- **Undulator radiation (single electron) is fully coherent ($\mu^2 = 1$)**

$$\mu^2 \equiv \lambda^2 \frac{\int W^2 d^2r d^2\theta}{(\int W d^2r d^2\theta)^2}$$

- **But is not diffraction limited $M^2 > 1$**
- **X-ray phase space of undulator's central cone is not Gaussian**
- **Old (Gaussian) metrics are not suitable for (almost) fully coherent sources**
- **For more on the subject refer to**

IVB, arXiV 1112.4047 (2011) (submitted to PRST-AB)

But the undulator radiation in central cone is Gaussian… or is it?

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animation: scanning around 1st harm. ~6keV (zero emittance)

Spectral flux (ph/s/0.1%BW/mm^2) at 50m from undulator (5GeV, 100mA, λ_p = 2cm)

click to play

Light in phase space

animation: scanning around 1st harm. ~6keV (zero emittance)

Phase space near middle of the undulator (5GeV, 100mA, λ_p = 2cm)

click to play

Emittance vs. fraction for light

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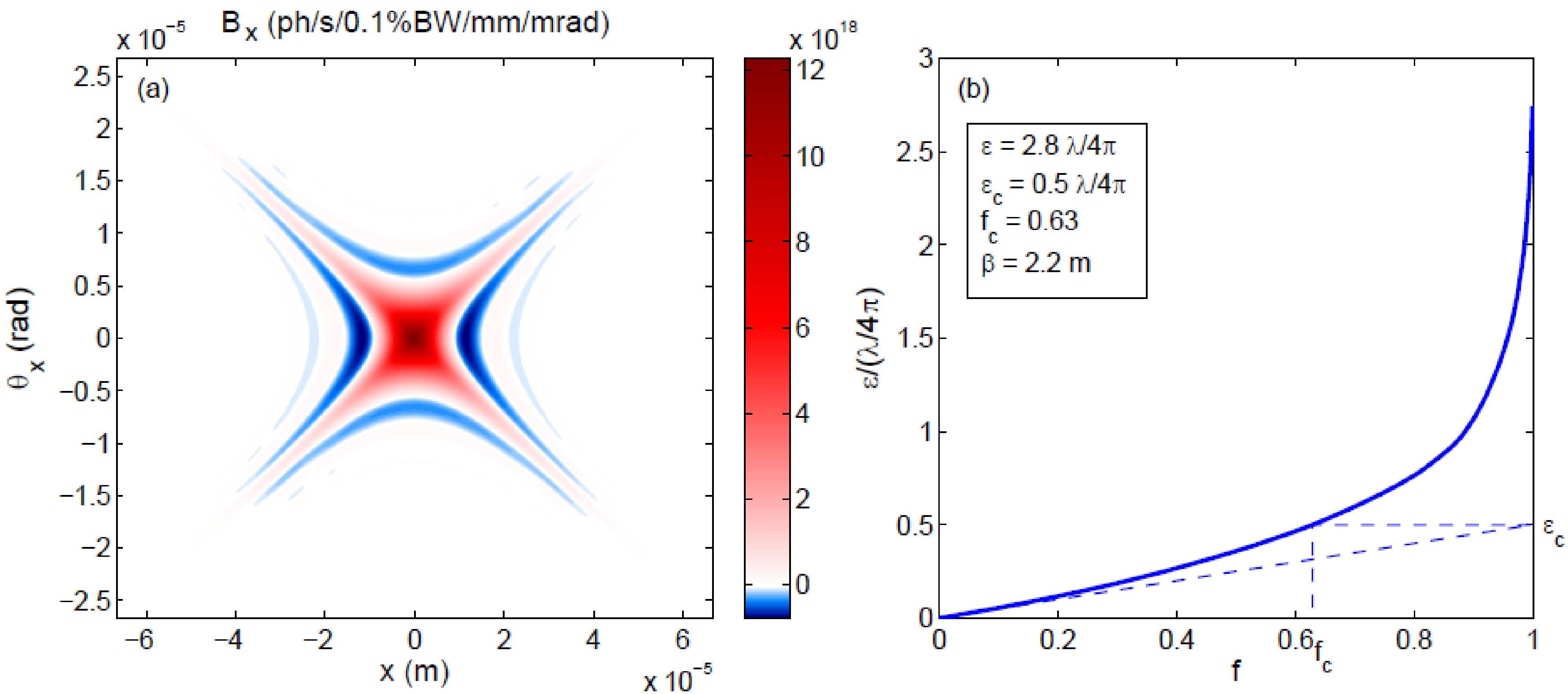
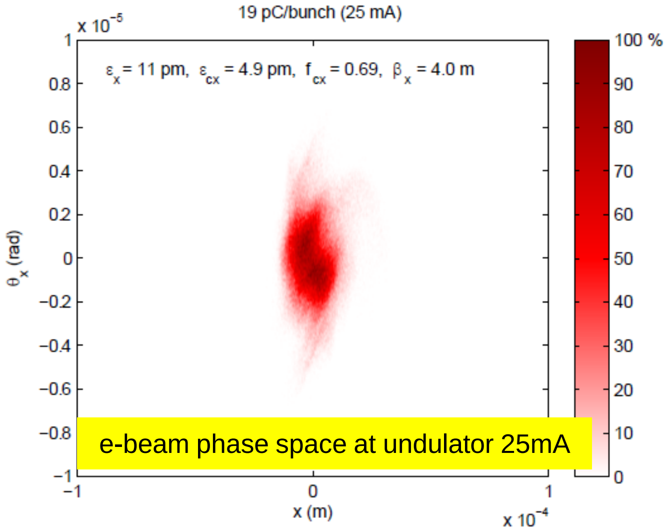


- Change clipping ellipse area from ∞ to 0, record emittance vs. beam fraction contained
- Smallest M^2 ~ 3 of x-ray undulator cone (single electron), core much brighter

Exampe of accounting for realistic spreads in the electron beam



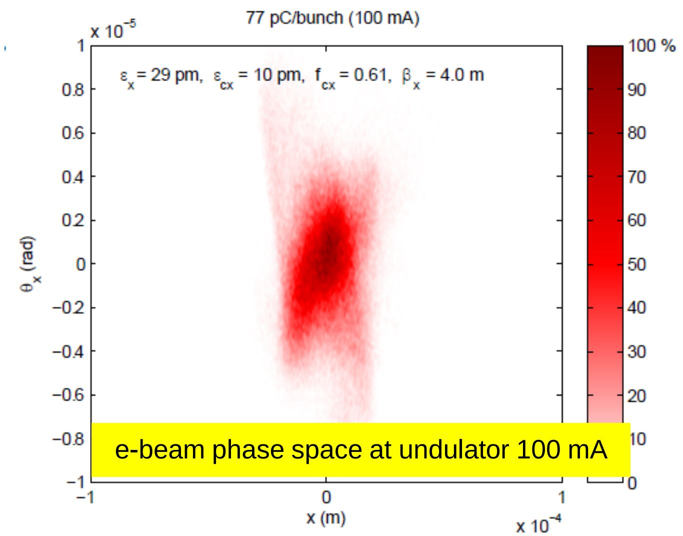
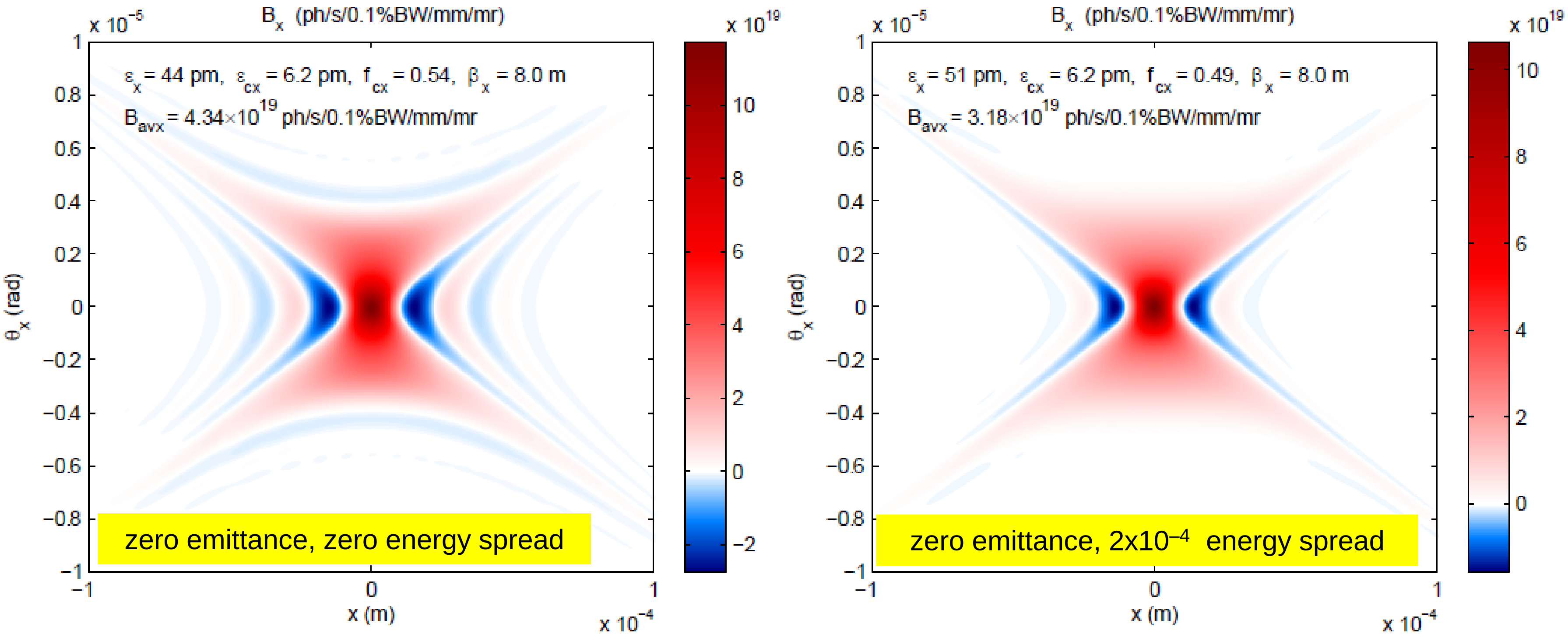


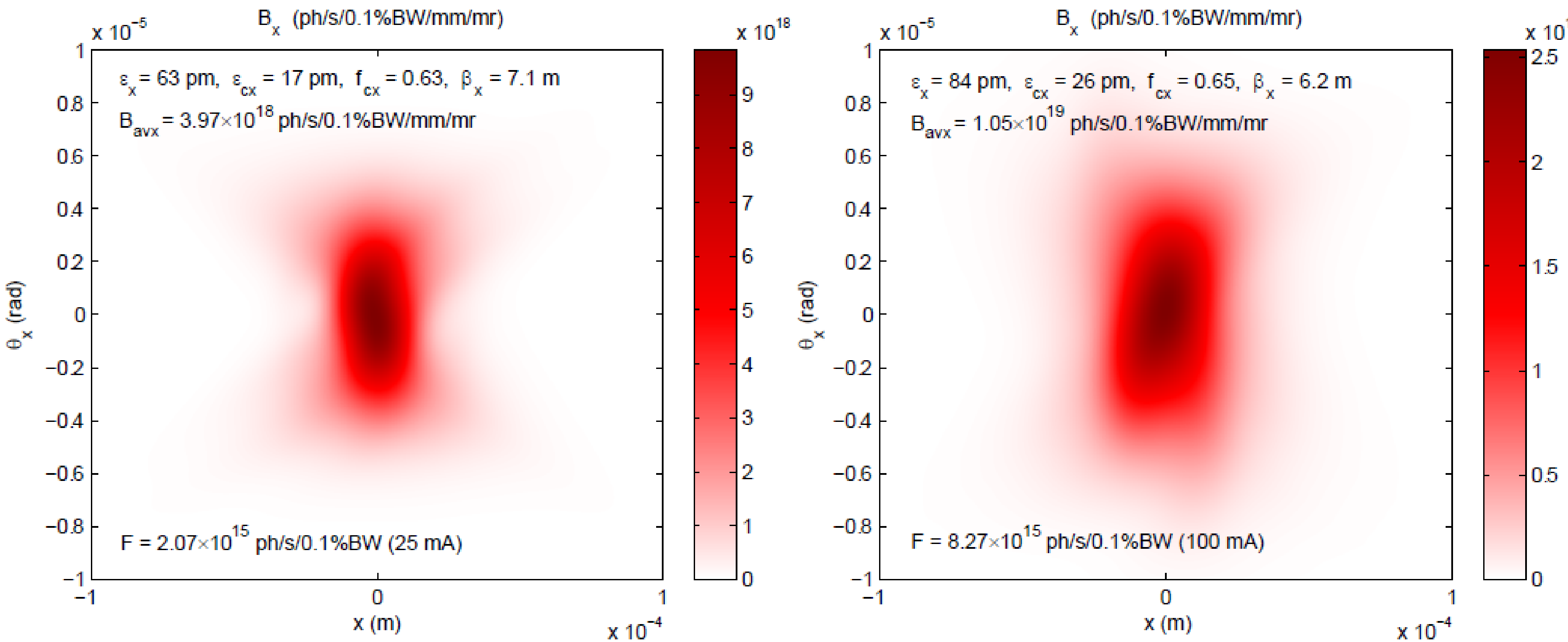
TABLE II. Parameters used in computing the radiation phase space.

Number of periods, $N_u = 1250$
Undulator period, $\lambda_u = 2\,\mathrm{cm}$
Harmonic number, $n = 1$
Resonant photon energy, $\hbar\omega = 8\,\mathrm{keV}$
Detuning radiation frequency, $\Delta\omega = -0.75\omega/N_u$
Beam energy, $E = 5\,\mathrm{GeV}$
Electron energy spread, $\sigma_{\delta_e} = 2 \times 10^{-4}$
Electron emittance, $\epsilon_x = 11, 29\,\mathrm{pm}$
Average current, $I = 25, 100\,\mathrm{mA}$
β-function, $\beta_x = 4\,\mathrm{m}$

Accounting for energy spread (phase space of x-rays)



And finite emittance…
(phase space of x-rays)



Back to the comparison

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- **TODAY: Cornell ERL photoinjector project has already achieved beam brightness that at 5 GeV would be equivalent to 100mA 0.5nm-rad × 0.005nm-rad storage ring Gaussian beam**
- **TOMORROW: both technologies (SR and ERL) can reach diffraction limited emittances at 100mA**
- **SR can easily do several 100's mA (x-ray optics heat load??), ERLs not likely (less appealing for several reason)**
- **ERL is better suited for very long undulators (small energy spread) and Free-Electron-Laser upgrades (using its CW linac)**

Simultaneous short pulses and generic ERL running

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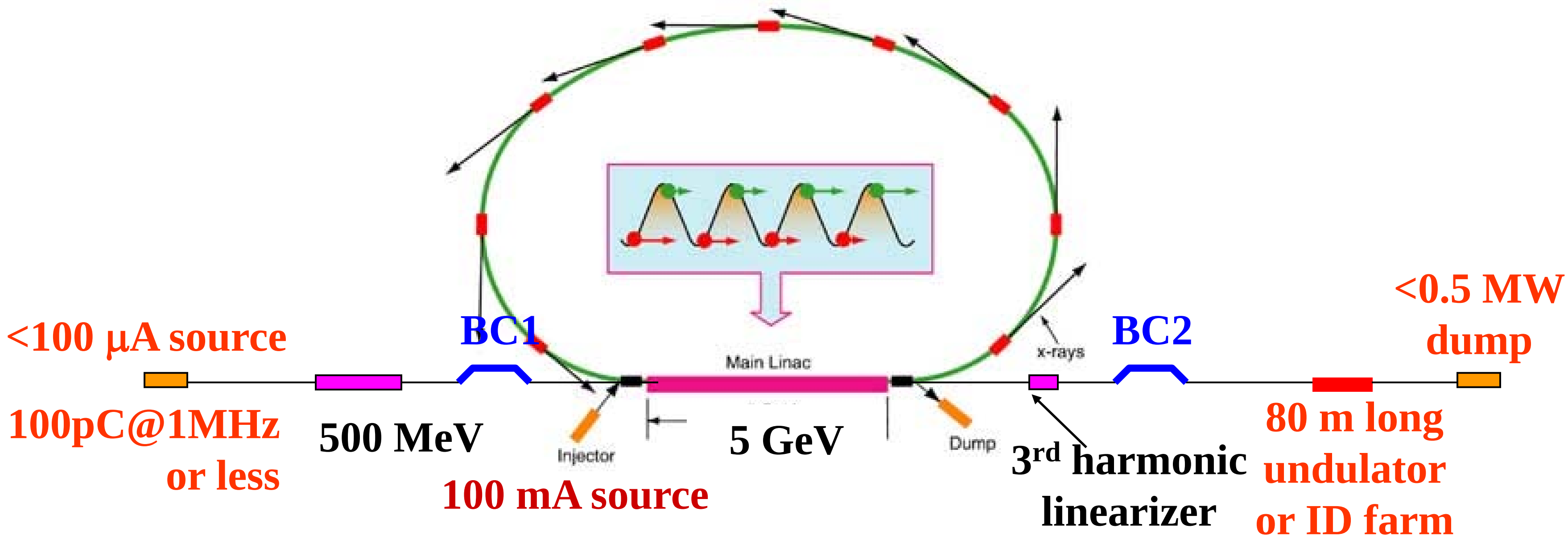

- Initial analysis to meet XFELO specs shows it's doable using non-energy recovered beamline
- Simultaneous operation of the two sources (100mA and 100μA appears feasible)

Conclusions

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- **Few people do correct brightness calculations (there are a lot fewer Gaussians than one might be imagining); proper procedure discussed (more in arXiV 1112.4047)**

- **Both technologies can deliver super-bright x-rays with a CW SRF linac of ERL having an edge for FEL techniques**

- **Can a future source be made more affordable?? Cost of ~billion should be a hard cutoff in my opinion (including beamlines)**