state : $|\chi\rangle \equiv \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$ two particles with $\frac{1}{2}$ spin

Find m_s, s ?

$\hat{S} = \hat{S}_1 + \hat{S}_2$ each only acts on particle "one" or "two" state

$|1\,2\rangle \equiv |1\rangle \otimes |2\rangle$

$\hat{S}^2|\chi\rangle = \hbar^2\, s(s+1)\,|\chi\rangle$

$\hat{S}_z|\chi\rangle = \hbar\, m_s\,|\chi\rangle$

$\hat{S}_z = \hat{S}_{1z} + \hat{S}_{2z}$;

$\hat{S}_z|\chi\rangle = \hat{S}_{1z}|\chi\rangle + \hat{S}_{2z}|\chi\rangle = \frac{\hbar}{2}\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) + \frac{\hbar}{2}\frac{1}{\sqrt{2}}\left(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$

$= 0$, $\Rightarrow$ $\boxed{m_s = 0}$

$\hat{S}^2 = \hat{S}_1^2 + \hat{S}_2^2 + 2\underbrace{\hat{S}_1\cdot\hat{S}_2}_{\hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} + \hat{S}_{1z}\hat{S}_{2z}}$

$S_{1x} = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $S_{1y} = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$

z spin up $\equiv |\uparrow\rangle$

x-proj $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}0\\ 1\end{pmatrix}$, $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$

y-proj $\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}0\\ i\end{pmatrix}$, $\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = -\begin{pmatrix}i\\ 0\end{pmatrix}$

$\hat{S}_1^2|\chi\rangle = \frac{3}{4}\hbar^2|\chi\rangle$, $\hat{S}_2^2|\chi\rangle = \frac{3}{4}\hbar^2|\chi\rangle$

$\hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} + \hat{S}_{z1}\hat{S}_{z2}|\chi\rangle = \frac{\hbar^2/4}{\sqrt{2}}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

$= \frac{1}{4}\hbar^2|\chi\rangle$

$\Rightarrow$ $\hat{S}^2|\chi\rangle = \hbar^2\left(\frac{3}{4} + \frac{3}{4} + 2\times\frac{1}{4}\right)|\chi\rangle = \overset{s(s+1)}{2}\hbar^2|\chi\rangle$; $\Rightarrow$ $\boxed{S = 1}$