# Standard Model + Feynman Diagrams

In the SM all forces are "quantized"

=> All interactions mediated by the exchange of field quanta

| Field → | Mediator Quanta ——→ | Field Theory |
|---|---|---|
| * E+M | Photon | Q. Electrodynamics |
| Gravity | Graviton | (unfinished) |
| Strong | gluon, (or π's in nuclei) | Q. Chromodynamics |
| * Weak | $W^\pm, Z^\circ$ bosons | Q. Flavor dynamics |

* E+M ⊕ Weak = Electroweak (successfully unified in 1970's)

=> See note on strengths.

Note: Photons (and Gravitons) have no mass, but $\pi$, $W^\pm$, $Z^\circ$ have $m \neq 0$! Massive mediator → short range! $m=0$ → Long Range! except for gluon!

Begin with Conservation Laws:

-> constrains possible interactions.

Energy, $E = \sqrt{(mc^2)^2 + (pc)^2}$ => Depends on reference frame

However $E^2 - (pc)^2 = (mc^2)^2$ invariant!

=> Can form similar compinations w/ x and t, $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial t}$, to make a relativistically invariant theory.

Turns out our theories conserve:

-> energy, momentum, angular momentun,

-> electric charge

-> conserve color: the "charge" of the strong force.

-> lepton number, quark number

However, the weak interaction often messes things up:

Lepton flavor conservation: conserved in Electroweak, but not in $\nu$ oscillations (strong force not pertinent)

Quark Flavor: conserved in strong and E+M, but not in weak.

Let's look at some examples:

1) $\Sigma^+ + n \rightarrow \Sigma^0 + p$ ⇒ Quark Flavor conserved ( $n \rightarrow p$ okay; isospin)
(uus) (udd) (uds) (uud) ⇒ Quarks moving around ⇒ ALLOWED, STRONG

2) $\Xi^- \rightarrow \Lambda^0 + \pi^-$ ⇒ $(S=-2) \rightarrow (S=-1)$ ⇒ Lepton flavor violated
dss uds $d\bar{u}$ ⇒ Quark # conserved ⇒ ALLOWED (if) WEAK

3) $B^+ \rightarrow K^0 + \pi^+$ ⇒ S and B not conserved ⇒ Quark # conserved
$u\bar{b}$ $d\bar{s}$ $u\bar{d}$ ⇒ ALLOWED (if) WEAK

4) $\pi^- \rightarrow e^- + \pi^0$ ⊗ DISALLOWED, violates lepton number cons.

5) $\eta \rightarrow \gamma + \gamma$ ⇒ Quark # and flavor conserved. ALLOWED.
$\frac{1}{\sqrt{6}}(\bar{u}u - d\bar{d} - 2s\bar{s})$ If we see a $\gamma$, force is E+M

Conservation laws discovered experimentally
⇒ How to predict scattering/decay rates?

Need Quantum Field Theories:

Sketch "ethos" of QFT

Classical mechanics ⇒ $x, t, p \Rightarrow \mathcal{L} \Rightarrow$ EOM's

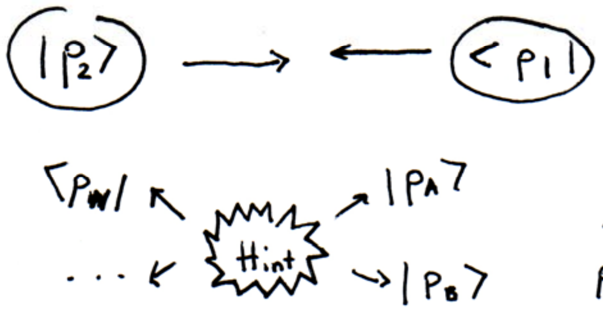
Field Theories ⇒ $\varphi(x,t), \dot{\varphi}(x,t), \vec{\nabla}(\varphi(x,t)) \Rightarrow \mathcal{L}(\varphi, \dot{\varphi}, \vec{\nabla}\varphi)$
⇒ EOM's for $\varphi$

⇒ Postulate $\mathcal{L}$'s that are Lorentz invariant
(Lorentz boost doesn't change form of $\mathcal{L}$)

⇒ Use second Quantization, write $\varphi(x,t)$ in terms of
$a^\dagger_{\vec{p}}, a_{\vec{p}}$ (creation/annihilation) at momentum $\vec{p}$

Now have a Quantum theory that is relativistic in which particles are created and destroyed! Looks like the scenario at a collider!

<u>In a collider</u>:



momentum eigenstates "in"

$\lambda H_{interaction}$, where $\lambda \ll 1$
produces new "out" states

$H = H_0 + \lambda H_{int}$

Remember evolution operator $e^{i\hat{H}t/\hbar}$

Probability to produce $\underbrace{\{p_1^A, p_2^B, \dots, p_N^N\}}_{\{p_{final}\}}$ different particles w/ different momenta:

$$P \propto \left| {}_{out}\langle\{p_{final}\}| e^{i\hat{H}_{int}} |p_1 p_2\rangle_{in} \right|^2$$

expand in Taylor series:

$$P \propto \left| \langle\{p_f\}|(1 - i\lambda\hat{H}_{int} + O(\lambda^2) + \dots)|p_1 p_2\rangle \right|^2$$

Not so easy:

$a^{1\dagger}_p a^{2\dagger}_p |0\rangle = |p_1 p_2\rangle, \quad \langle\{p_f\}| a_A a_B a_C \cdots$

$\hat{H}_{int} = \hat{H}_{int}(a^\dagger_A, a^\dagger_B, a^1_p, a^2_p, a^\dagger_p, \dots$ creation/ann. ops for all fields/particles present$)$

$$\Rightarrow P \propto \left| \dots + \lambda\langle 0|(-i)\underbrace{(a_A a_B a_C \cdots)}_{\{P_f\}}\underbrace{(\cdots a_A a^\dagger_B a^\dagger_C \cdots)}_{\{H_{int}\}}(a^\dagger_1 a^\dagger_2)|0\rangle + \lambda^2\langle 0|\dots\dots|0\rangle \right|^2$$

Remember your HW $\langle 0|a a^\dagger a^\dagger a a^\dagger \cdots|0\rangle = [a,a^\dagger]+[a^\dagger,a]+\cdots$

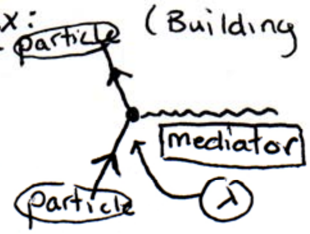
for different a's (different particles) ⇒ expressions are long and complex.

## Enter Feynman:

Keep track of commutators with diagrams ⇒ commutators involve different fields → connect them with lines.

⇒ Draw all possible diagrams ⟺ capture all commutators.

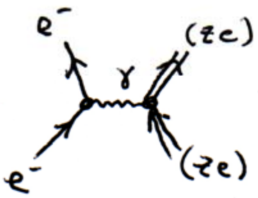
**Vertex:** (Building block of diagrams)



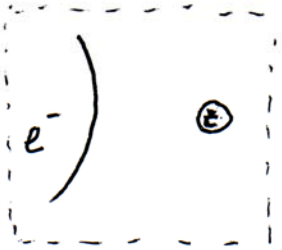
at each vertex, get an extra power of $\lambda$

⇒ more complicated diagrams contribute less and less.

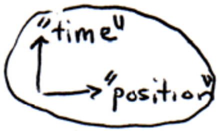
Electron scatters off nucleus:



→ "looks" a lot like →



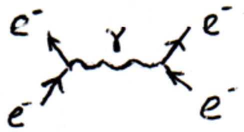
* associate



w/ diagrams!

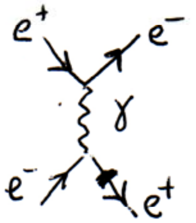
an electron scatters off another:

$e^- + e^- \to e^- + e^-$



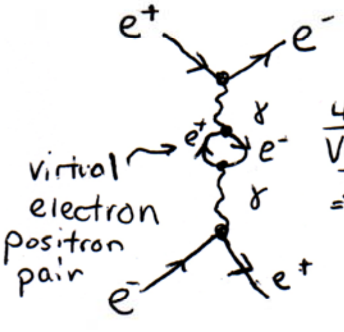
Crossing symmetry (rotate!)

$e^- + e^+ \to e^- + e^+$



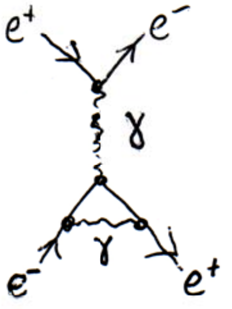
antiparticles point the wrong way.

Other diagrams for $e^+e^- \to e^+e^-$



virtual electron positron pair

4 Vertices $\Rightarrow P \propto (\lambda^4)^2$



Feynman: "Read as:

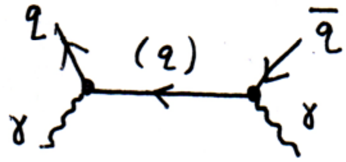
There once was an electron and a positron, they interacted like so, and out came an electron and positron".

For E+M with e's, $\lambda = \underset{\substack{\downarrow \\ \text{F.S.C.}}}{\sqrt{\alpha}}$

quarks have $|q| = \frac{1}{3}e$, $\frac{2}{3}e \rightarrow \lambda = \sqrt{\frac{1}{3}\alpha}, \sqrt{\frac{2}{3}\alpha}$

Pair production: (E+M)

$\gamma + \gamma \rightarrow q + \bar{q}$

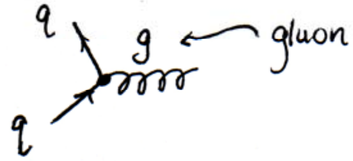


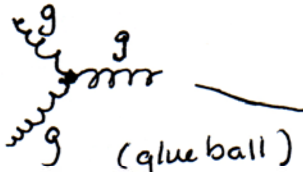
q here is virtual particle

⇒ we can't connect two γ's together (superposition principle)

## Strong interactions

fundamental vertex





At each vertex:

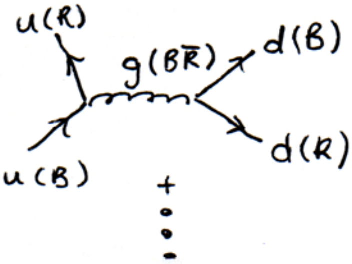
→ factor of $\sqrt{\alpha_s}$ ⇒ "alpha-strong", dependent on E.

→ Color changes, flavor unchanged

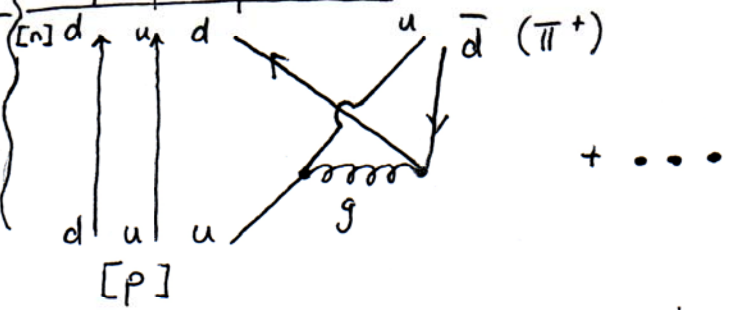
→ gluon "charged" (has color charge), unlike photon,

→ can interact with each other.

Examples: $u + d \rightarrow u + d$



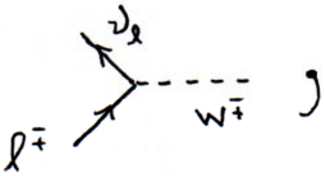
$p + p \rightarrow p + n + \pi^+$

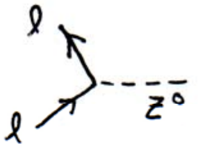


But we also know that $\alpha_s$ is not small ⇒ Perturbation theory not valid!?

Well, sort of; can play tricks to do these sums, but these calculations are very hard.

## Weak interactions





at vertex

⇒ get factor of $\alpha_w \sim \frac{1}{30}$; perturbation works

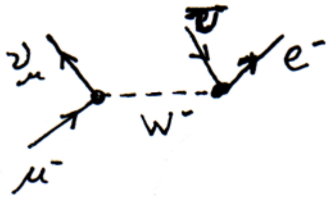
→ if $\alpha_w > \alpha_{EM}$, why is it "weak"?

Mediator Bosons are massive!

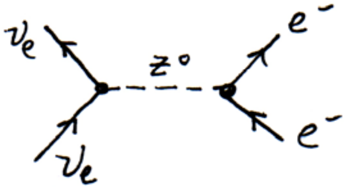
⇒ Short Ranged.

# Interactions and Crossing Symmetry

$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$

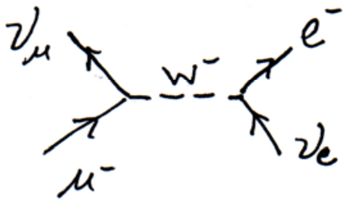


$e^- + \nu_e \rightarrow e^- + \nu_e$



↓

$\mu^- + \nu_e \rightarrow e^- + \nu_\mu$



A note on force strengths: (Normalize E+M force to be 1 in all situations)

| | Gravity | E+M | Weak | strong (quark-gluon) |
|---|---|---|---|---|
| 2 u @ 0.03fm | $10^{-41}$ | 1 | close! unified? 0.8 | 25 |
| 2 u @ 0.001fm | $10^{-41}$ | 1 | $10^{-4}$ | 60 |