Reflection and refraction

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Outline

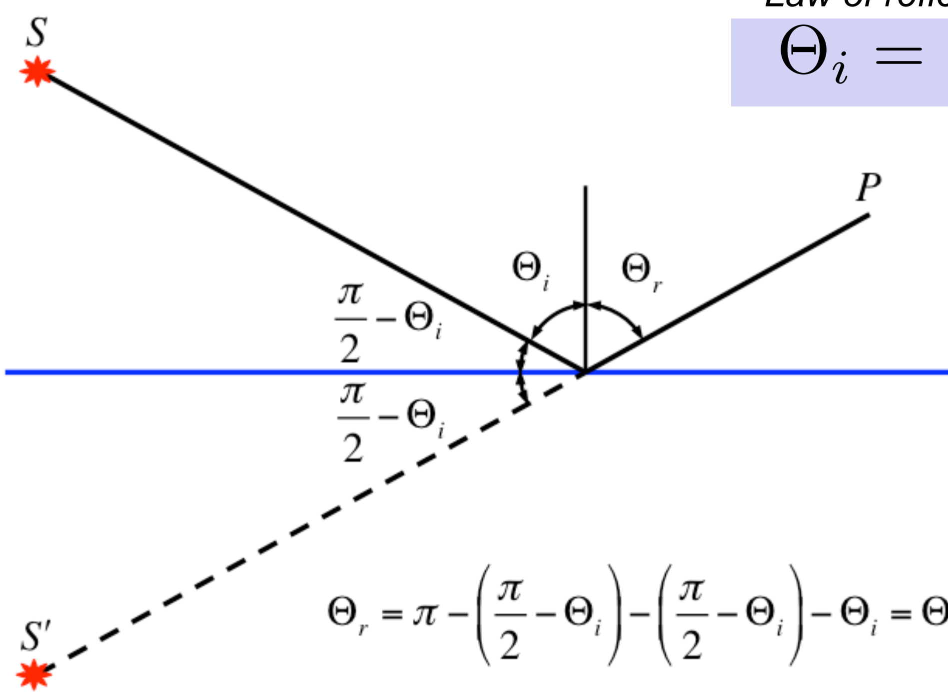
- **Reflection & refraction in geometric optics**
- **Fresnel equations**
- **Applications**

Fermat's principle: reflection

Fermat's principle: optical path is stationary (has an extremum with respect to small changes)
Optical path = path × index of refraction

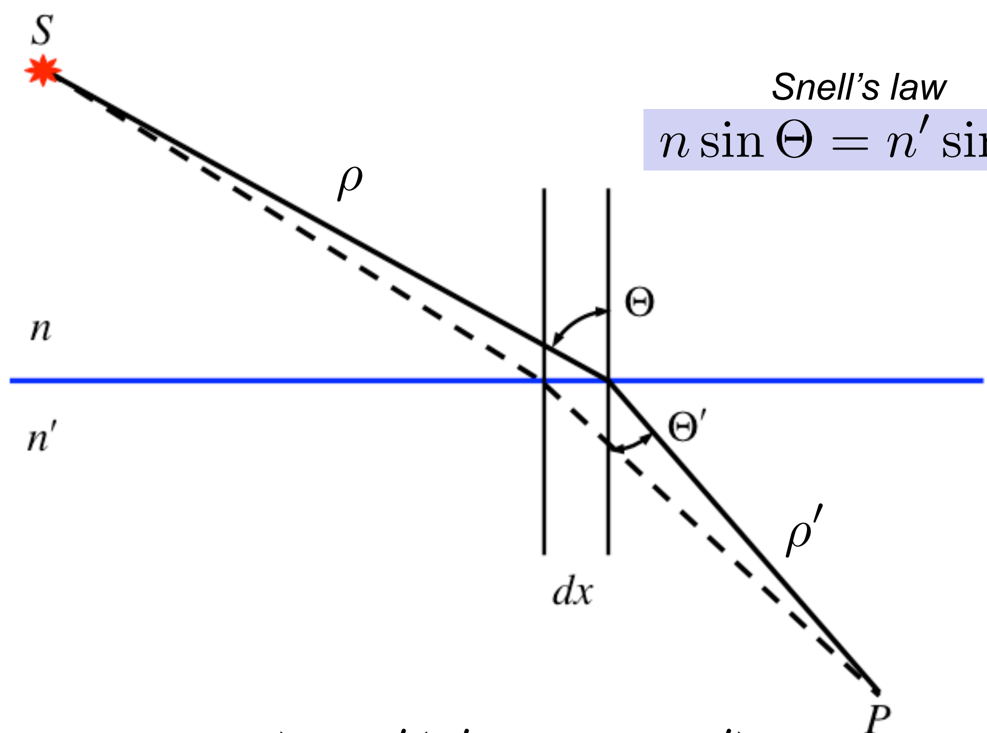
Law of reflection

$$\Theta_i = \Theta_r$$



$$\Theta_r = \pi - \left(\frac{\pi}{2} - \Theta_i\right) - \left(\frac{\pi}{2} - \Theta_i\right) - \Theta_i = \Theta_i$$

Fermat's principle: refraction



Snell's law

$$n \sin \Theta = n' \sin \Theta'$$

$$\widetilde{\mathrm{SP}} = n\rho + n'\rho'$$

$$\widetilde{\mathrm{SP}} + d\widetilde{\mathrm{SP}} = n(\rho - dx \sin \Theta) + n'(\rho' + dx \sin \Theta')$$

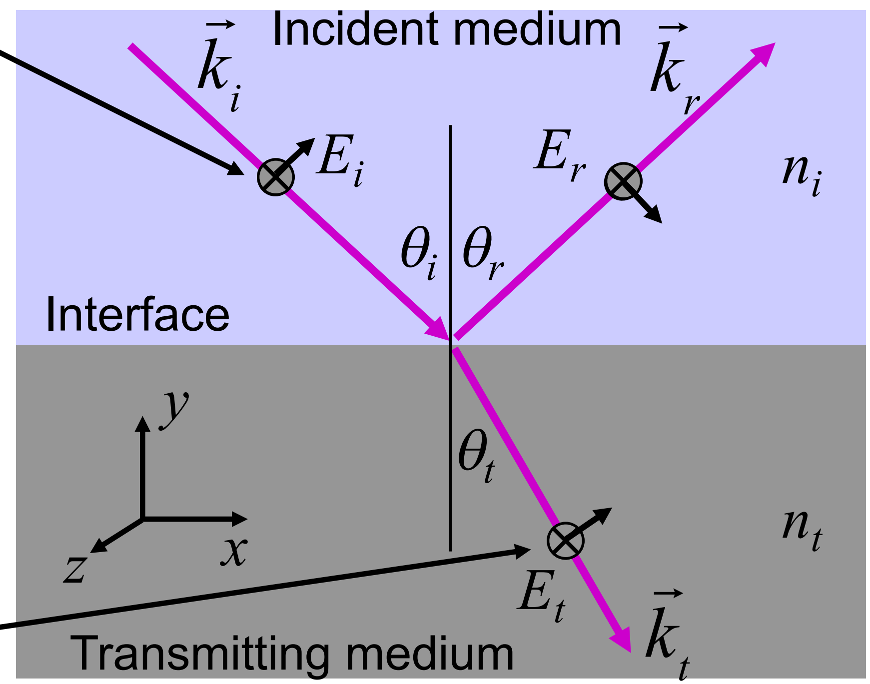
$$\frac{d\widetilde{\mathrm{SP}}}{dx} = 0 = n(-\sin \Theta) + n' \sin \Theta'$$

Look at the problem as E&M wave at the interface

- Geometric optics can provide no information about wave amplitudes
- Need E&M (vector!) theory of light

Perpendicular (“S”) polarization **“sticks out”** of or into the plane of incidence.

Plane of incidence (here the xy plane) is the plane that contains the incident and reflected k-vectors.



Parallel (“P”) polarization lies **parallel** to the plane of incidence.

B-field not shown here, but it’s always perpendicular to E-field

Fresnel equations

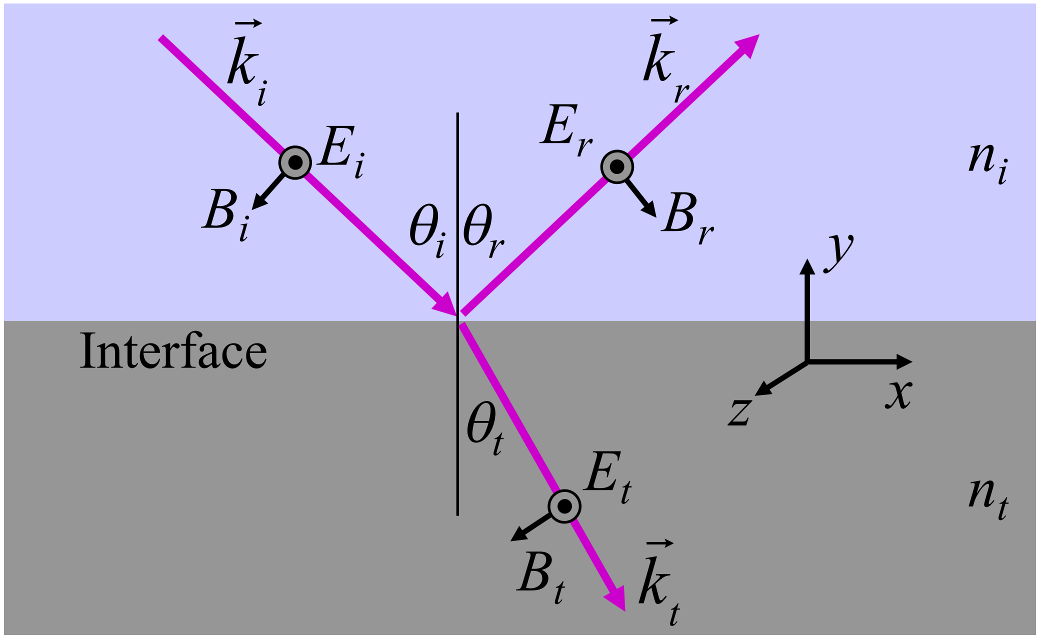
- Goal: relate amplitudes of transmitted and reflected waves
- Fresnel was the first to obtain the expressions in early 1800's.

- A complication: must consider two polarizations separately S- and P- or $\perp$ and || w.r.t. E-field orientation relative to the incidence plane

$$r_\perp = E_{0r}/E_{0i}, \quad t_\perp = E_{0t}/E_{0i} \qquad \text{for S-polarization}$$
$$r_\| = E_{0r}/E_{0i}, \quad t_\| = E_{0t}/E_{0i} \qquad \text{for P-polarization}$$

- These equations emerge from **boundary conditions** at the interface, namely a requirement for the **tangential E- and B- fields to be continuous** (assuming non-magnetic material)

S-polarization



$$E_i(x, y = 0, z, t) + E_r(x, y = 0, z, t) = E_t(x, y = 0, z, t)$$

$$-B_i(x, y=0, z, t)\cos\theta_i + B_r(x, y=0, z, t)\cos\theta_r = -B_t(x, y=0, z, t)\cos\theta_t$$

or in terms of amplitudes:

$$E_{0i} + E_{0r} = E_{0t}$$

$$-B_{0i}\cos\theta_i + B_{0r}\cos\theta_r = -B_{0t}\cos\theta_t$$

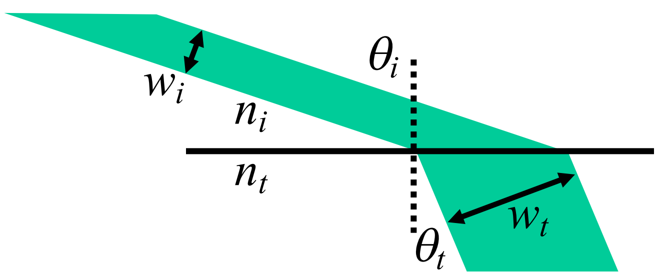
S-polarization Fresnel ratios

Using $B = nE/c$ and $\theta_r = \theta_i$ one finds

$$r_\perp = \frac{E_{0r}}{E_{0i}} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \frac{E_{0t}}{E_{0i}} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

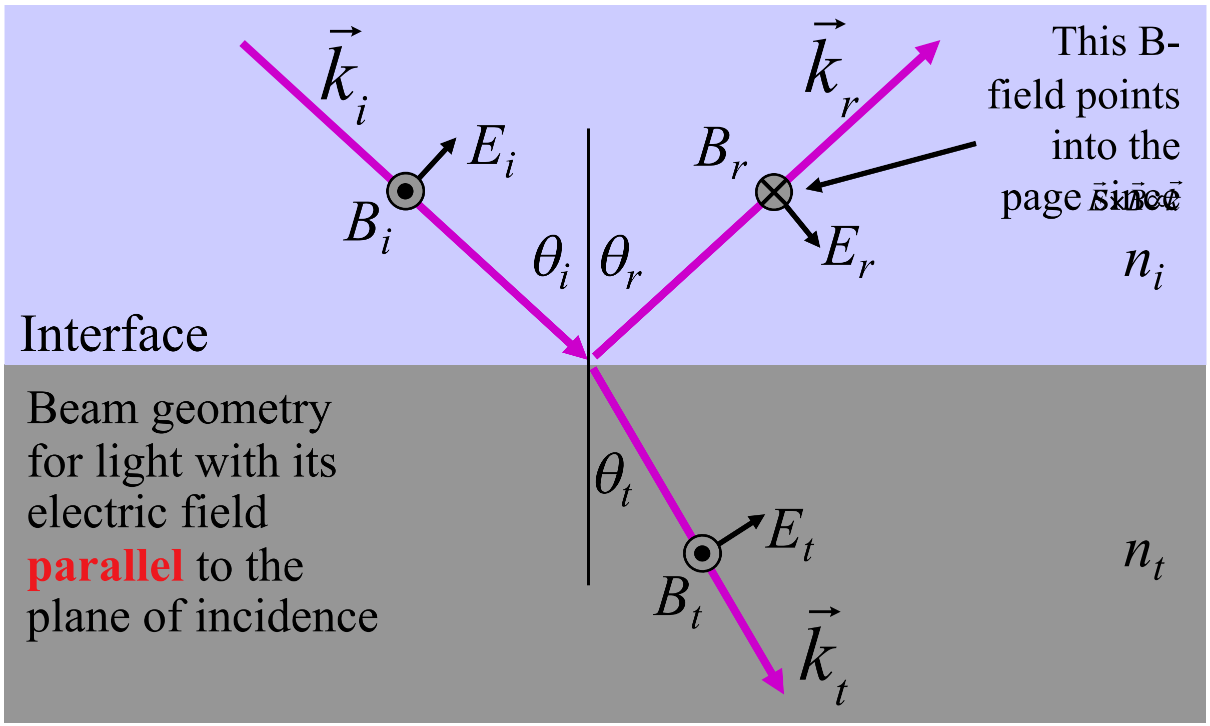
Nicer form if we define $m = \frac{w_t}{w_i} = \frac{\cos\theta_t}{\cos\theta_i}, \quad n = \frac{n_t}{n_i}$



$$r_\perp = \frac{1 - nm}{1 + nm}$$

$$t_\perp = \frac{2}{1 + nm}$$

P-polarization Fresnel ratios



From boundary conditions:

$$B_{0i} - B_{0r} = B_{0t}$$

$$E_{0i}\cos\theta_i + E_{0r}\cos\theta_r = E_{0t}\cos\theta_t$$

$$r_{\parallel} = \frac{n_i \cos\theta_t - n_t \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$t_{\parallel} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

Or more compactly

$$r_{\parallel} = \frac{m-n}{m+n}$$

$$t_{\parallel} = \frac{2}{m+n}$$

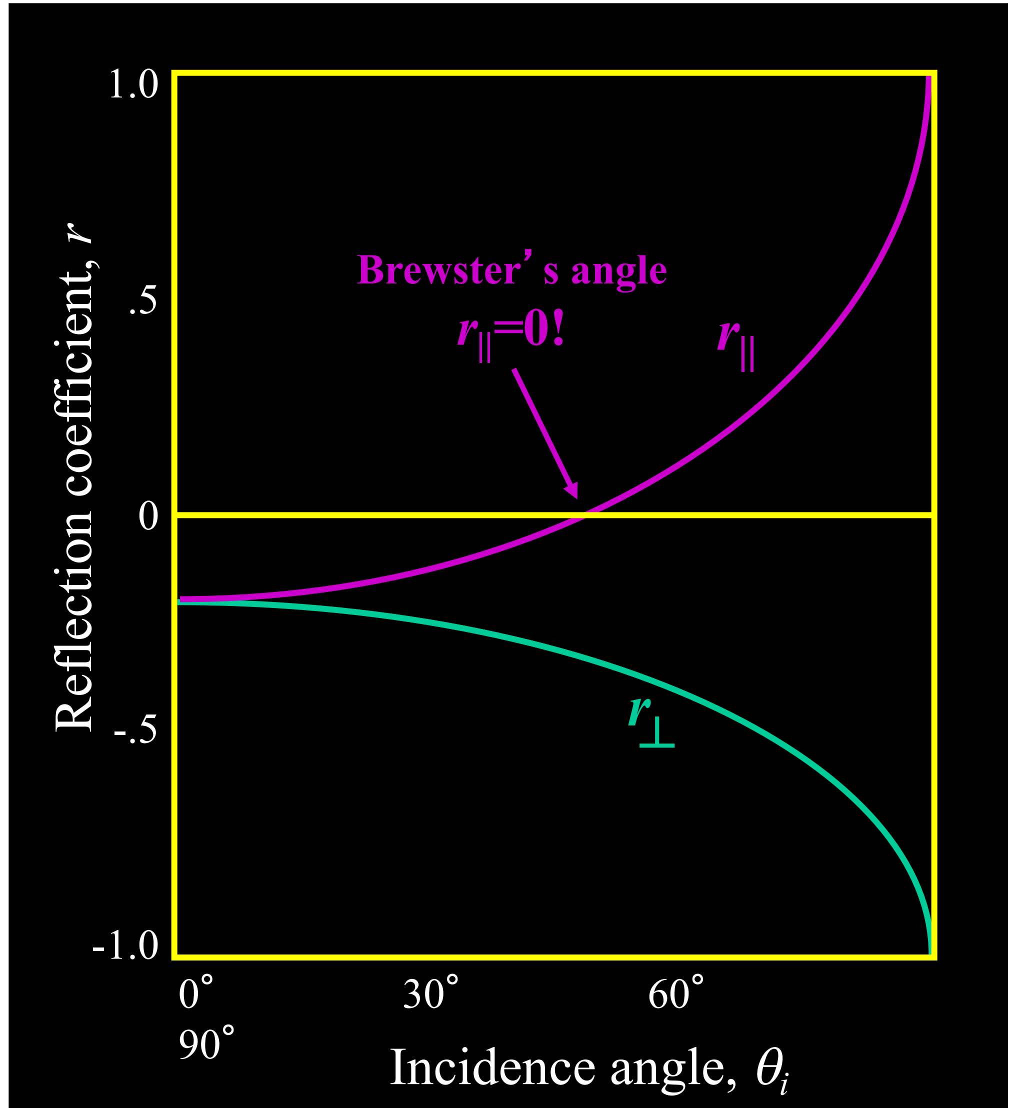
Reflection from air-to-glass interface

$$n_{air} \approx 1 \;<\; n_{glass} \approx 1.5$$

Note that:

Total reflection at $\theta = 90^\circ$ for both polarizations

Zero reflection for parallel polarization at **Brewster's angle** (56.3° for these values of n_i and n_t).



Reflection from glass-to-air interface

$n_{glass} \approx 1.5 \; > \; n_{air} \approx 1$

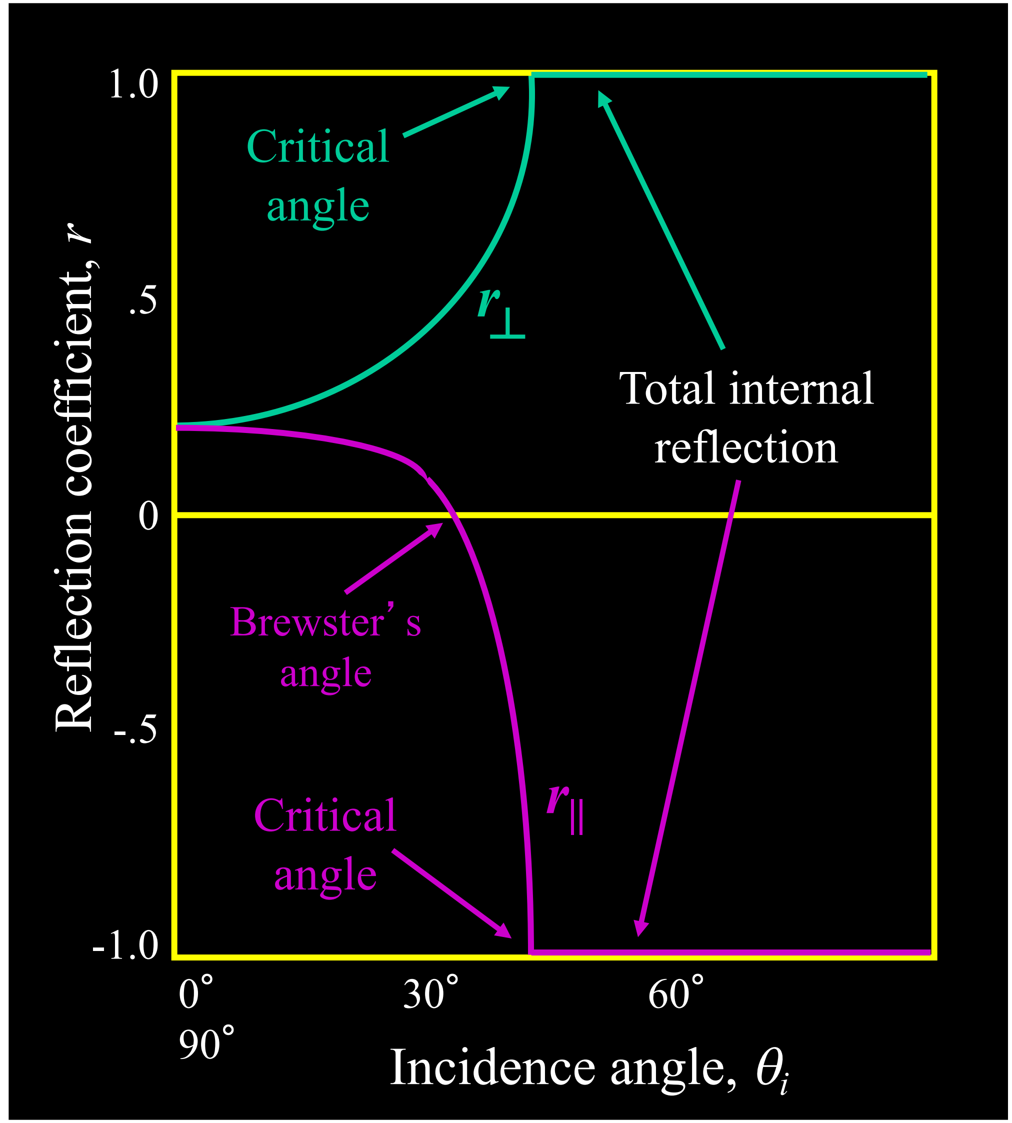
Note that:

Total internal reflection above the **critical angle**

$$\theta_{crit} \equiv \arcsin(n_t / n_i)$$

(The sine in Snell's Law can't be > 1!):

$$\sin(\theta_{crit}) = n_t / n_i \sin(90°)$$

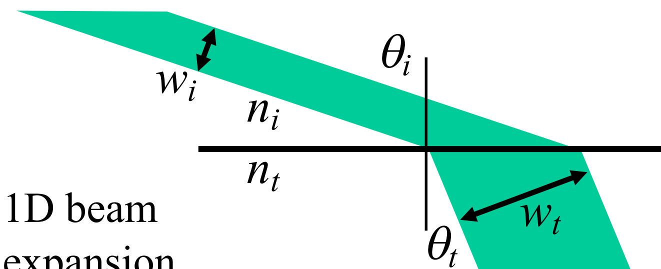


Transmittance (T)

$$I_t = \left(n_t \frac{\varepsilon_0 c_0}{2}\right)\left|E_{0t}\right|^2$$

$T \equiv$ Transmitted Power / Incident Power $= \dfrac{I_t A_t}{I_i A_i}$ $A = \text{Area}$

Compute the ratio of the beam areas:



1D beam expansion

$$\frac{A_t}{A_i} = \frac{w_t}{w_i} = \frac{\cos(\theta_t)}{\cos(\theta_i)} = m$$

The beam expands in one dimension on refraction.

$$\frac{\left|E_{0t}\right|^2}{\left|E_{0i}\right|^2} = t^2$$

$$T = \frac{I_t A_t}{I_i A_i} = \frac{\left(n_t \frac{\varepsilon_0 c_0}{2}\right)\left|E_{0t}\right|^2}{\left(n_i \frac{\varepsilon_0 c_0}{2}\right)\left|E_{0i}\right|^2}\left[\frac{w_t}{w_i}\right] = \frac{n_t \left|E_{0t}\right|^2 w_t}{n_i \left|E_{0i}\right|^2 w_i} = \frac{n_t}{n_i} t^2 \frac{\cos(\theta_t)}{\cos(\theta_i)}$$

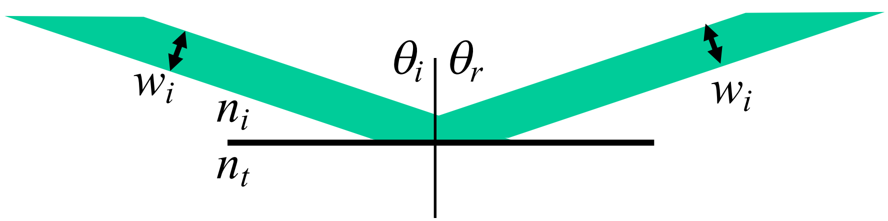
$$n = \frac{n_t}{n_i}$$

$$\Rightarrow \quad \boxed{T = \left[\frac{\left(n_t \cos\left(\theta_t\right)\right)}{\left(n_i \cos\left(\theta_i\right)\right)}\right] t^2 = n\, m\, t^2}$$

The Transmittance is also called the Transmissivity.

Reflectance (R)

R $\equiv$ Reflected Power / Incident Power $= \dfrac{I_r A_r}{I_i A_i}$ ← $A = \text{Area}$



Because the angle of incidence = the angle of reflection, the beam area doesn't change on reflection.

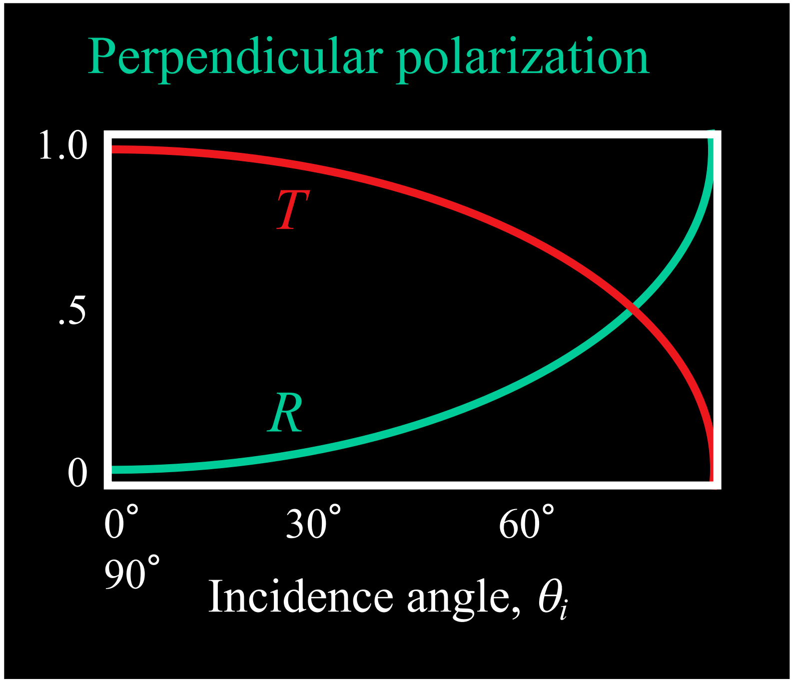
Also, $n_i = n_r$ for both incident and reflected beams.

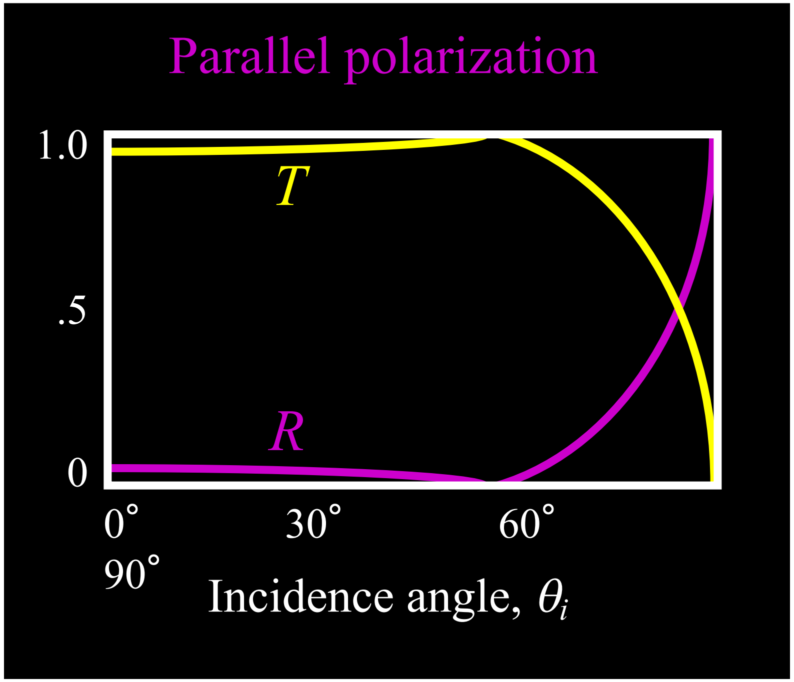
So:

$$\boxed{R = r^2}$$

The Reflectance is also called the Reflectivity.

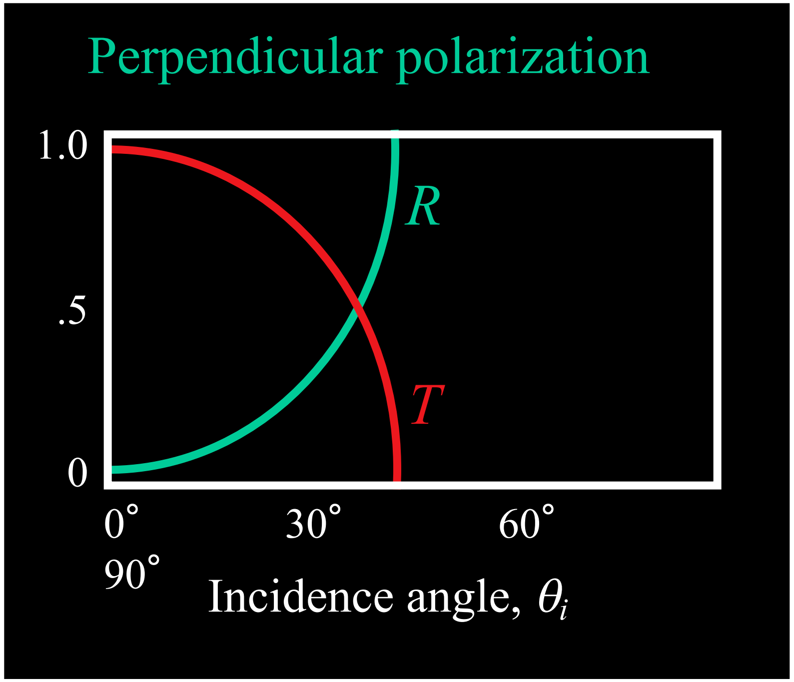
Reflectance and transmittance for an air-to-glass interface

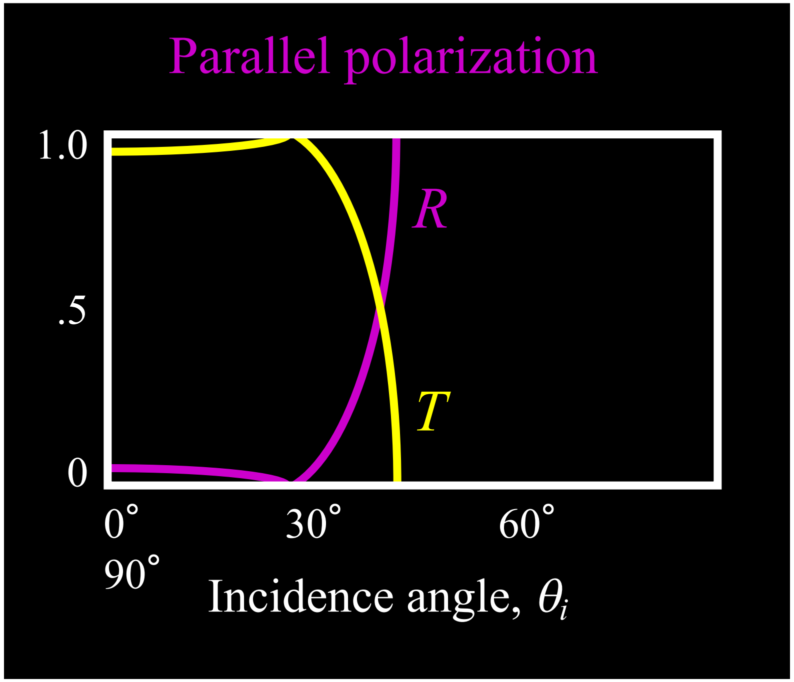




Note that $R + T = 1$

Reflectance and transmittance for a glass-to-air interface





Note that $R + T = 1$

Reflection at normal incidence

When $\theta_i = 0$,

$$R = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

and

$$T = \frac{4 n_t n_i}{\left(n_t + n_i\right)^2}$$

For an air-glass interface ($n_i = 1$ and $n_t = 1.5$),

R = 4% and T = 96%

The values are the same, whichever direction the light travels, from air to glass or from glass to air.

The 4% has big implications for photography lenses.

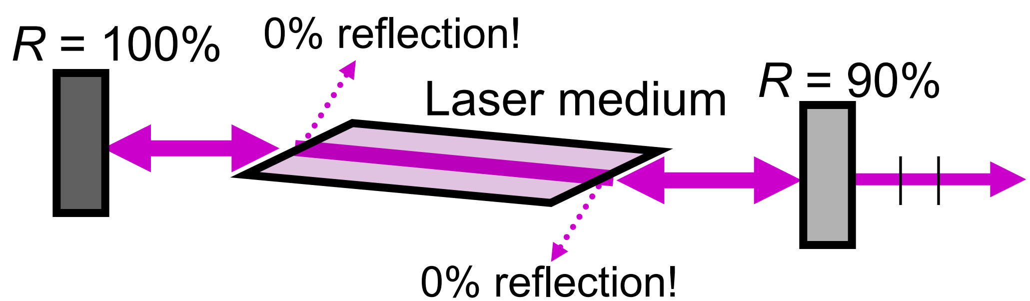
Practical implications of Fresnel equations

Windows look like mirrors at night (when you're in the brightly lit room)

One-way mirrors (used by police to interrogate bad guys) are just partial reflectors (actually, aluminum-coated).

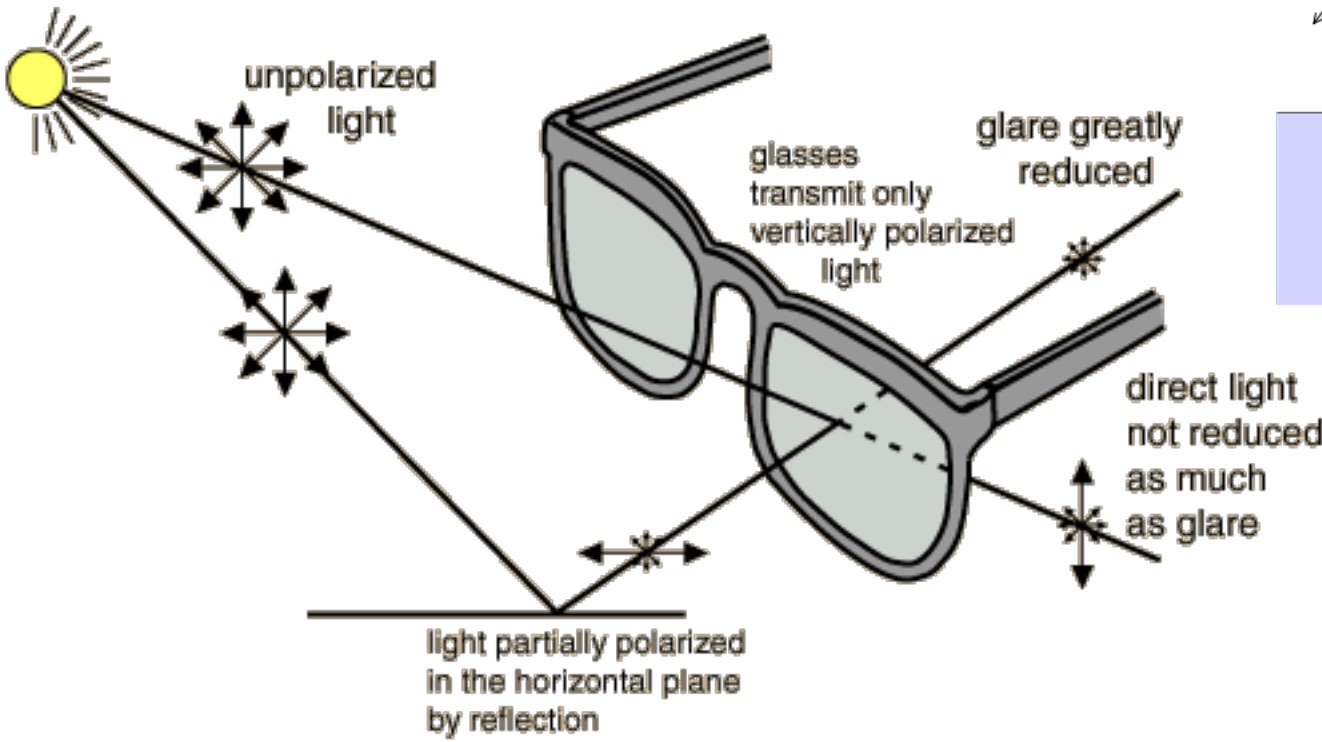
Disneyland puts ghouls next to you in the Haunted Mansion using partial reflectors (also aluminum-coated).

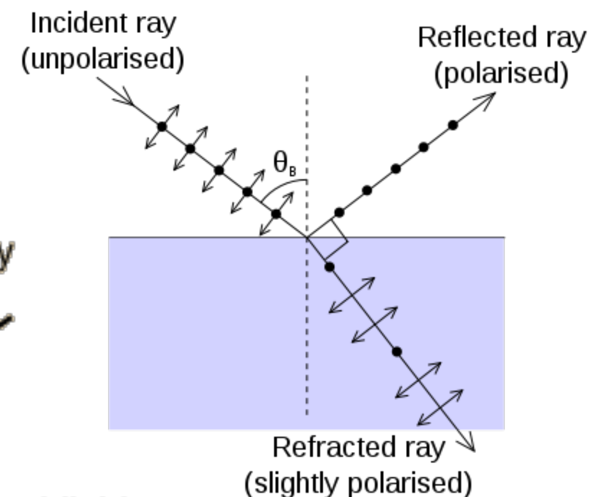
Lasers use Brewster's angle components to avoid reflective losses:



Optical fibers use total internal reflection. Hollow fibers use high-incidence-angle near-unity reflections.

Polarized sunglasses





Sunglasses are made to transmit only vertically polarized light to cut glare due to reflections from water

Photography with polarizers

(no polarizer)

(with polarizer)

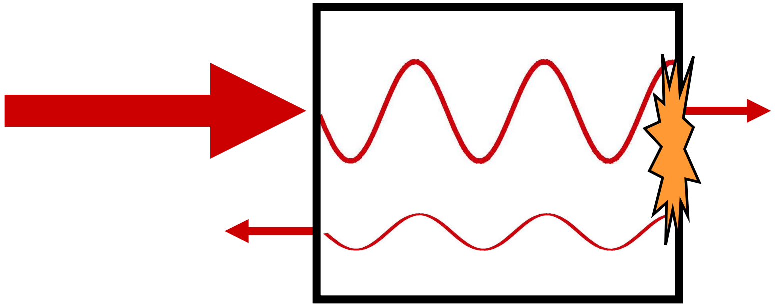
Same principle to ‘see below the surface’ or remove an unwanted reflection/glare

Phase shifts for reflections

r has a sign, i.e. when negative it means 180º phase shift

180˚ if low-index-to-high and 0 if high-index-to-low.

E.g. if you slowly turn up a laser intensity incident on a piece of glass, where does damage happen first, the front or the back?



Actually, at the back surface because of constructive interference, which gives about 44% higher intensity than at the front:

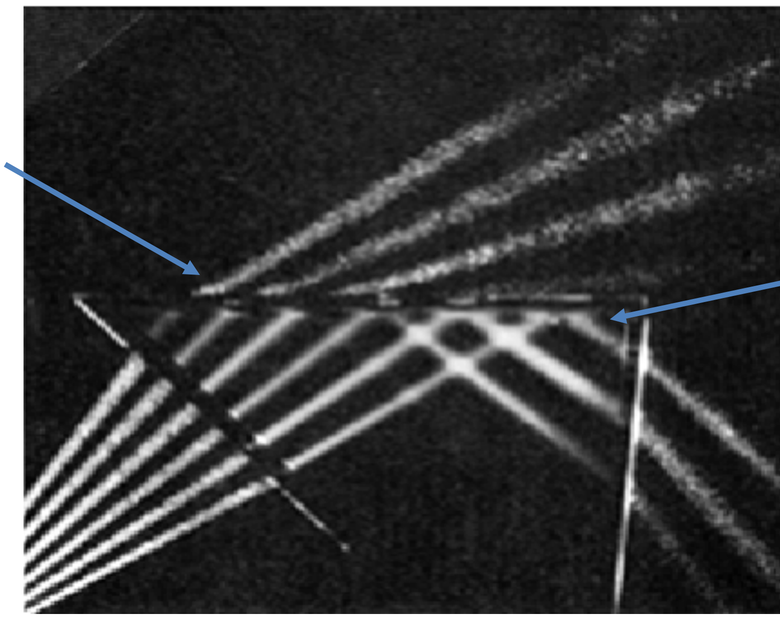
$$(1+r)^2 = (1+0.2)^2 = 1.44$$

Important application of phase shifts: anti-reflection coating

Total Internal Reflection occurs when $\sin\theta_t > 1$, and no transmitted beam can occur.

Note that the irradiance of the transmitted beam goes to zero (i.e., TIR occurs) as it grazes the surface.

Brewster's angle

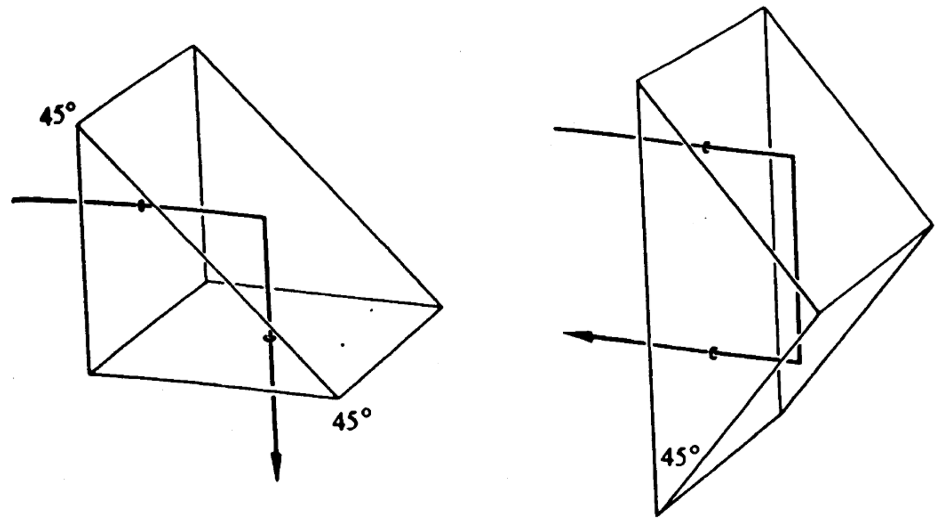


Total Internal Reflection

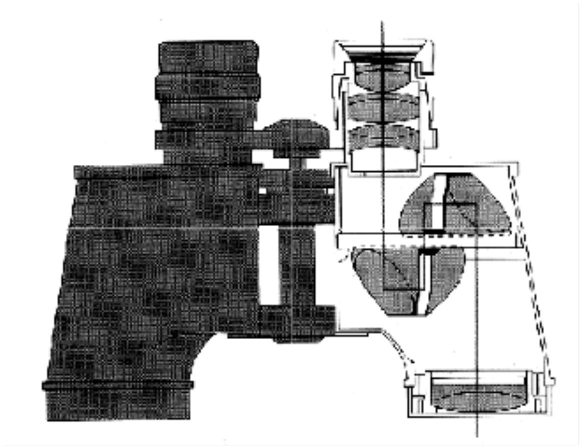
Total internal reflection is **100% efficient**, that is, all the light is reflected.

Applications of total internal reflection

Beam steerers

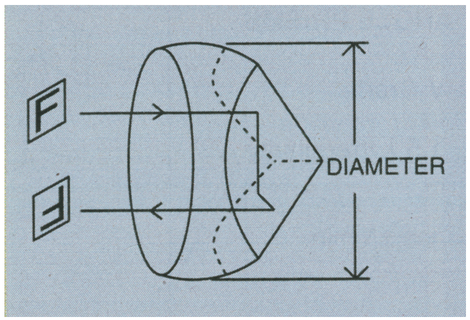


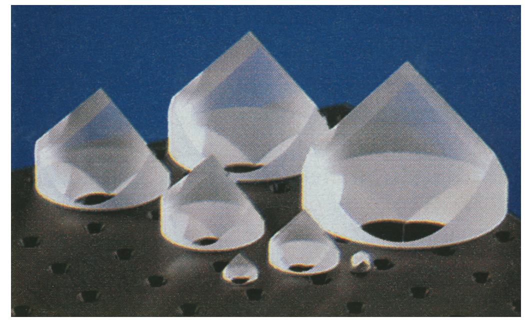
Beam steerers used to compress the path inside binoculars



Back-reflectors: corner cubes and cat's eyes

Corner cubes involve three reflections and also displace the return beam in space. Even better, they always yield a parallel return beam:

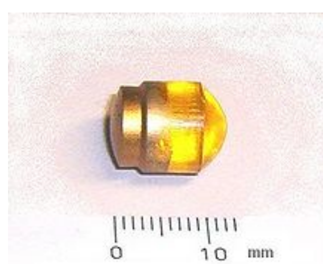


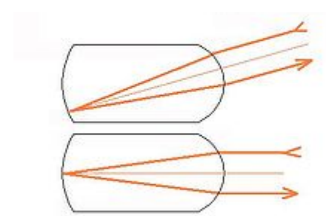


If the beam propagates in the z direction, it emerges in the -z direction, with each point in the beam (x,y) reflected to the (-x,-y) position.

Hollow corner cubes avoid propagation through glass and don't use TIR.

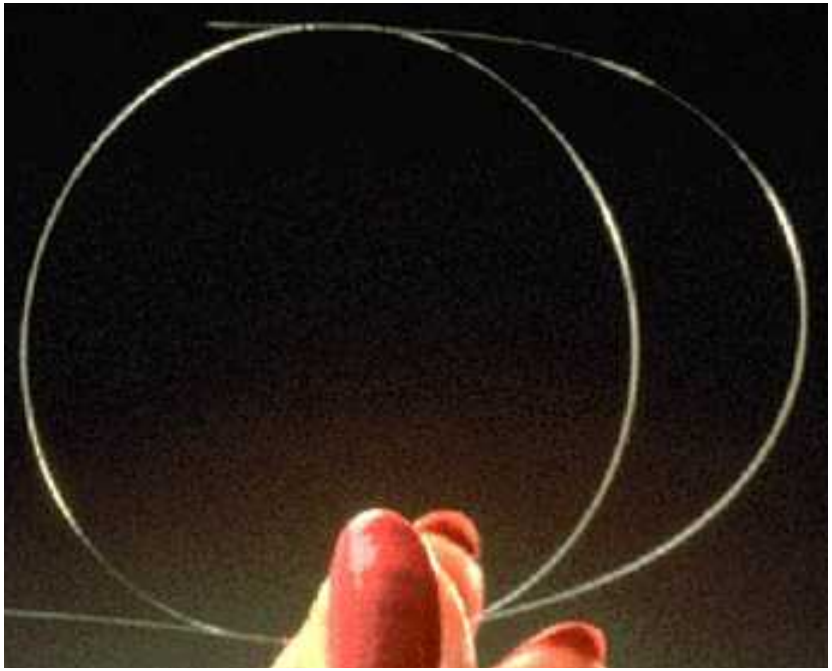
Cat's eye reflectors use a spherical geometry to achieve the same goal.





Optical fibers

Optical fibers use TIR to transmit light long distances.



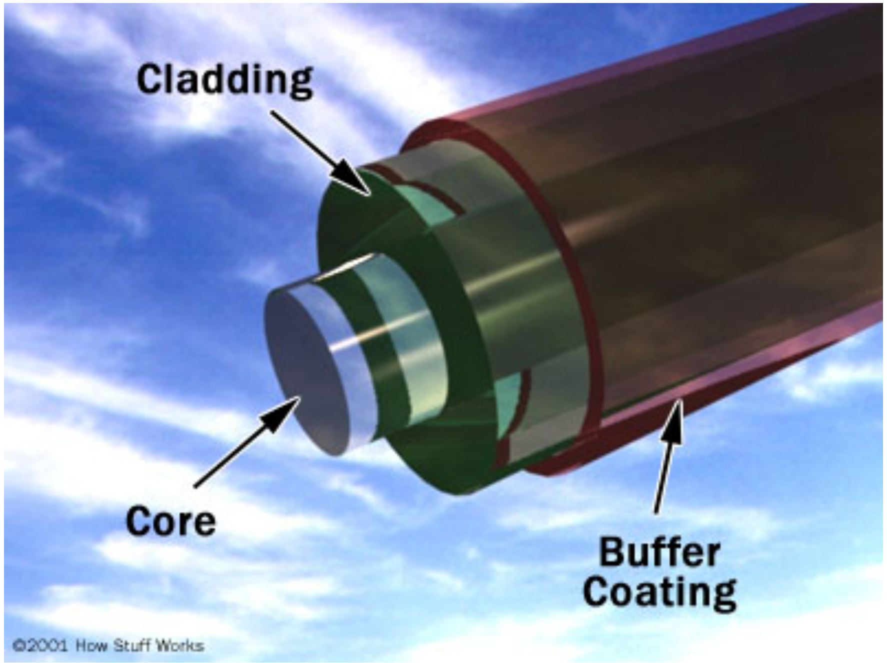
They play an ever-increasing role in our lives!

Design of an optical fiber

Core: Thin glass center of the fiber that carries the light

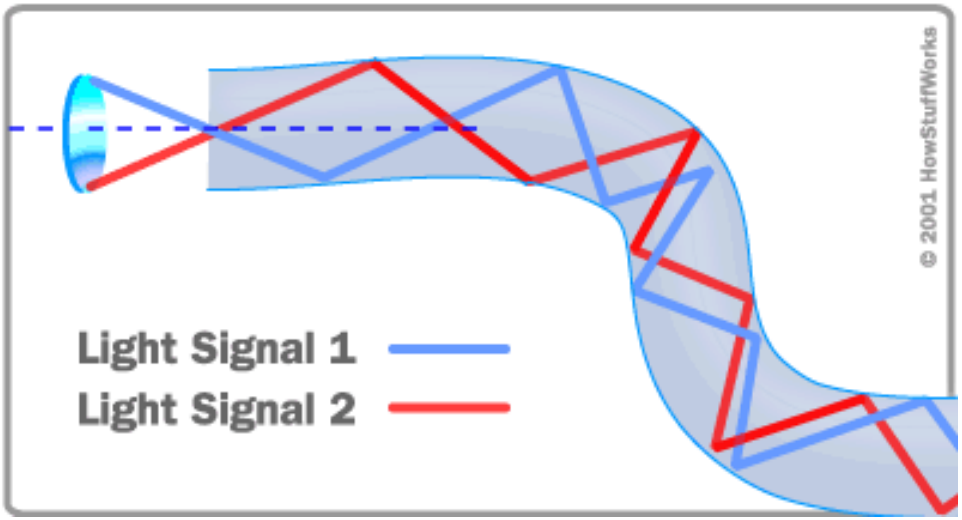
Cladding: Surrounds the core and reflects the light back into the core

Buffer coating: Plastic protective coating



$$n_{core} > n_{cladding}$$

Propagation of light in optical fiber



Light travels through the core bouncing from the reflective walls. The walls absorb very little light from the core allowing the light wave to travel large distances.

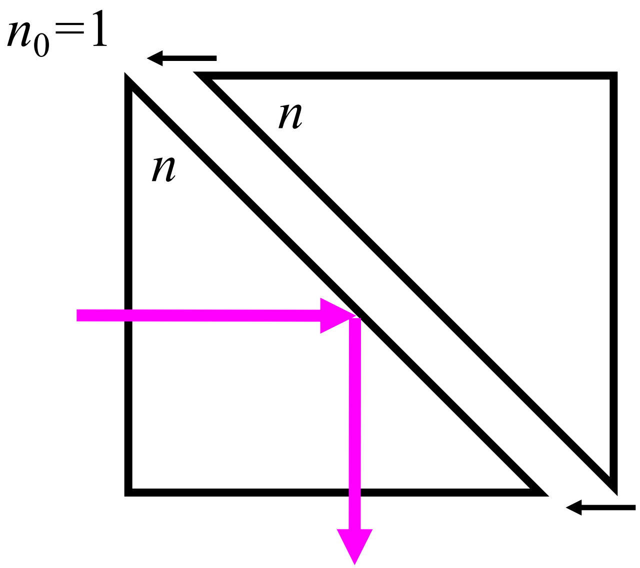
Some signal degradation occurs due to imperfectly constructed glass used in the cable. The best optical fibers show very little light loss -- less than 10%/km at 1,550 nm.

Maximum light loss occurs at the points of maximum curvature.

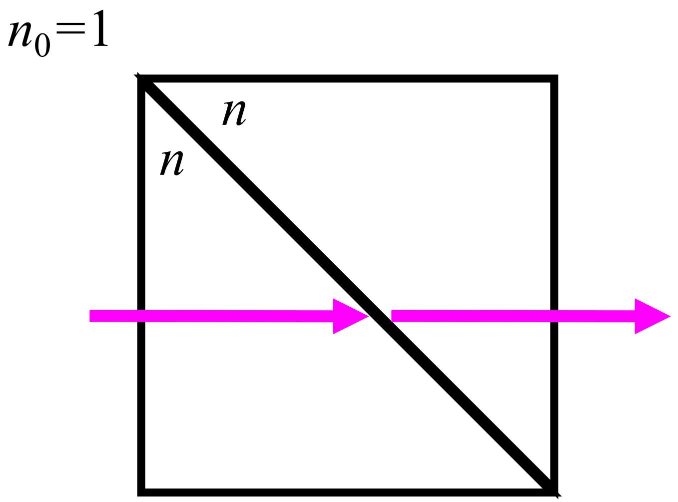
Frustrated total internal reflectance

By placing another surface in contact with a totally internally reflecting one, total internal reflection can be frustrated.

Total internal reflection



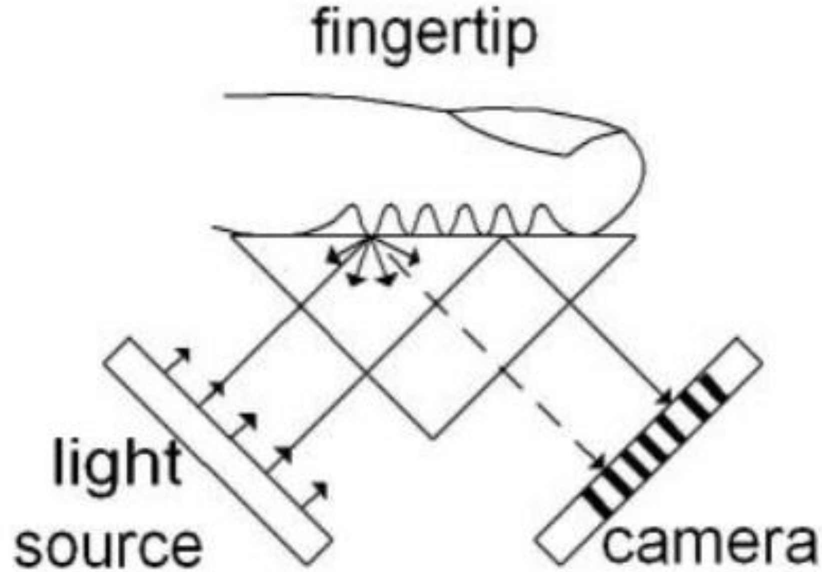
Frustrated total internal reflection

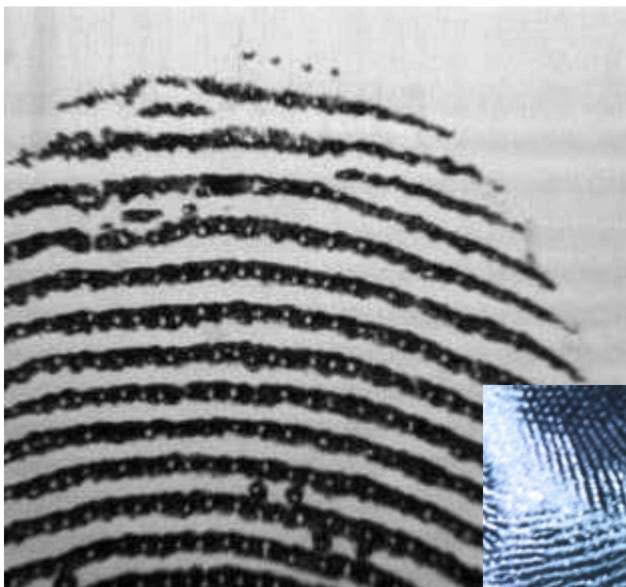


How close do the prisms have to be before TIR is frustrated?

This effect provides evidence for **evanescent fields**—fields that leak through the TIR surface–and is the basis for a variety of spectroscopic techniques.

FTIR for fingerprinting





See TIR from a fingerprint valley and FTIR from a ridge.

Links/References

http://www.teknik.uu.se/ftf/education/ftf2/Optics_FresnelsEqns.pdf

www.physics.rutgers.edu/ugrad/389/FresnelsEqns.ppt

http://optics.hanyang.ac.kr/~shsong/23-Fresnel%20equations.pdf

Some figures taken from the lecture notes by Dr. Agladze and Wikipedia