Polarization

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Outline

- **Types of polarization**
- **Jones' matrices**
- **Birefringence**
- **Polarizing optical components**
- **Polarization in scattering**

Polarization ellipse

For light traveling along *z* direction:

$$\mathbf{k} \times \mathbf{E} = v\mathbf{B}$$

$$\mathcal{E}(z,t) = \mathrm{Re}\left\{\mathbf{A}\exp\left[j\omega\left(t - \frac{z}{c}\right)\right]\right\},$$

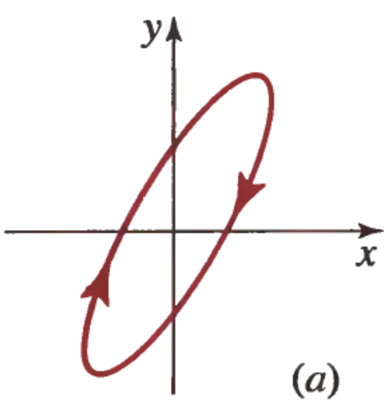
with a complete amplitude:

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}},$$

with $A_x = \mathfrak{a}_x \exp(j\varphi_x)$ and $A_y = \mathfrak{a}_y \exp(j\varphi_y)$

The electric field traces out an ellipse:

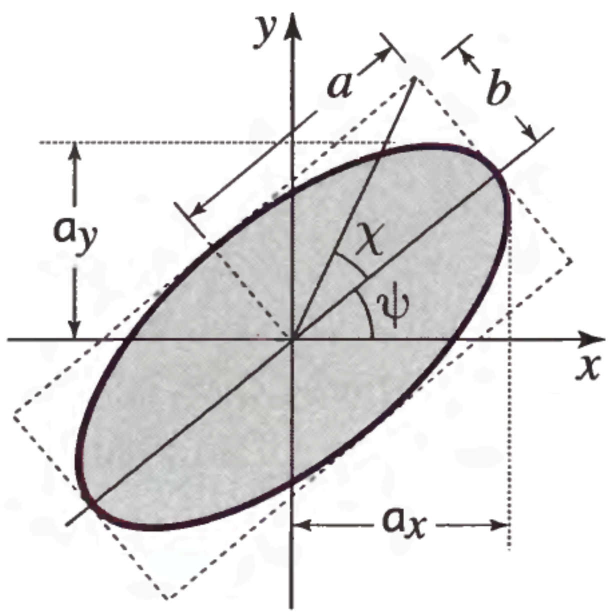
$$\mathcal{E}(z,t) = \mathcal{E}_x\hat{\mathbf{x}} + \mathcal{E}_y\hat{\mathbf{y}},$$



$$\mathcal{E}_x = \mathfrak{a}_x \cos\left[\omega\left(t - \frac{z}{c}\right) + \varphi_x\right]$$

$$\mathcal{E}_y = \mathfrak{a}_y \cos\left[\omega\left(t - \frac{z}{c}\right) + \varphi_y\right]$$

Polarization ellipse



$$\frac{\mathcal{E}_x^2}{a_x^2} + \frac{\mathcal{E}_y^2}{a_y^2} - 2\cos\varphi \frac{\mathcal{E}_x \mathcal{E}_y}{a_x a_y} = \sin^2\varphi,$$

$$\tan 2\psi = \frac{2r}{1-r^2}\cos\varphi, \qquad r = \frac{a_y}{a_x},$$

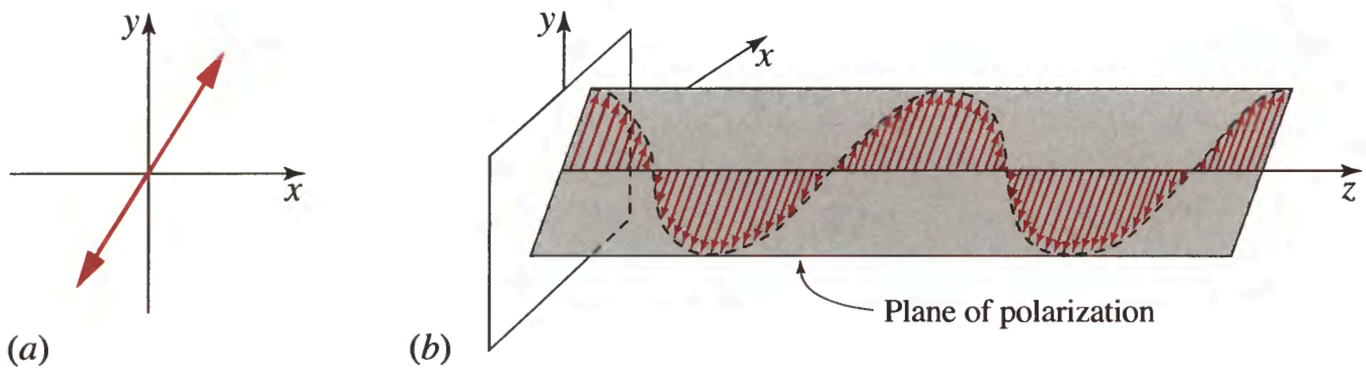
$$\sin 2\chi = \frac{2r}{1+r^2}\sin\varphi, \qquad \varphi = \varphi_y - \varphi_x$$

Polarization types:

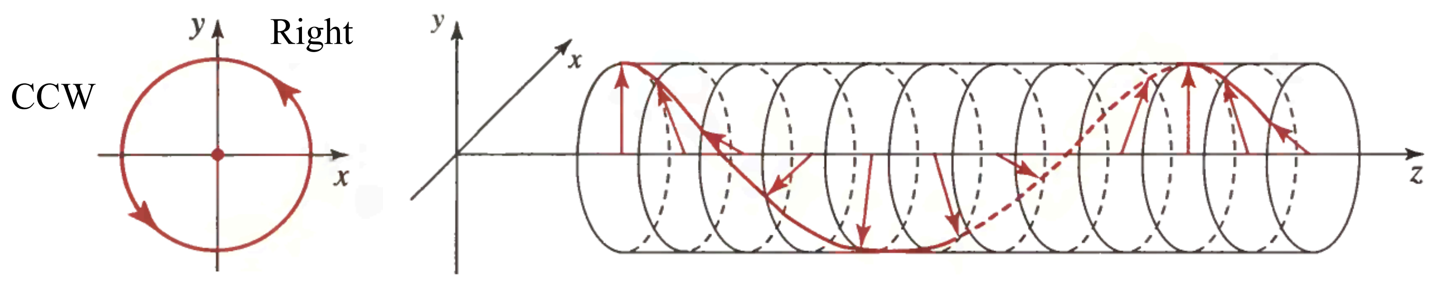
- linearly polarized light
- circularly polarized light
- unpolarized light (non-laser)

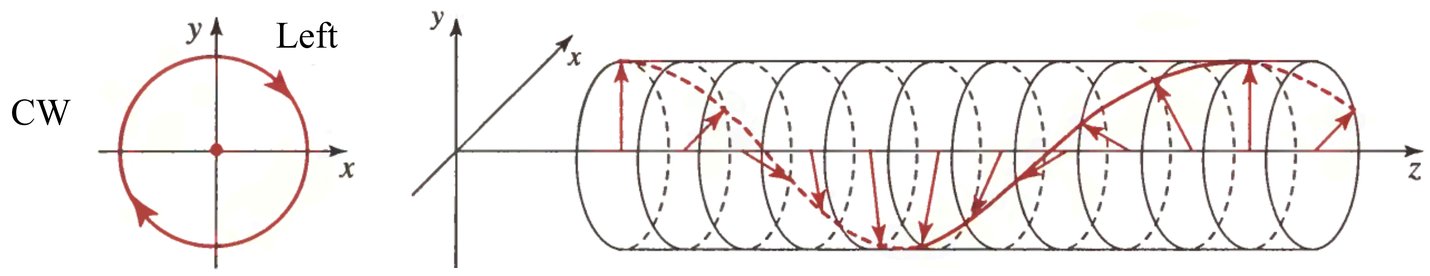
Linear & circular polarizations

linearly polarized light



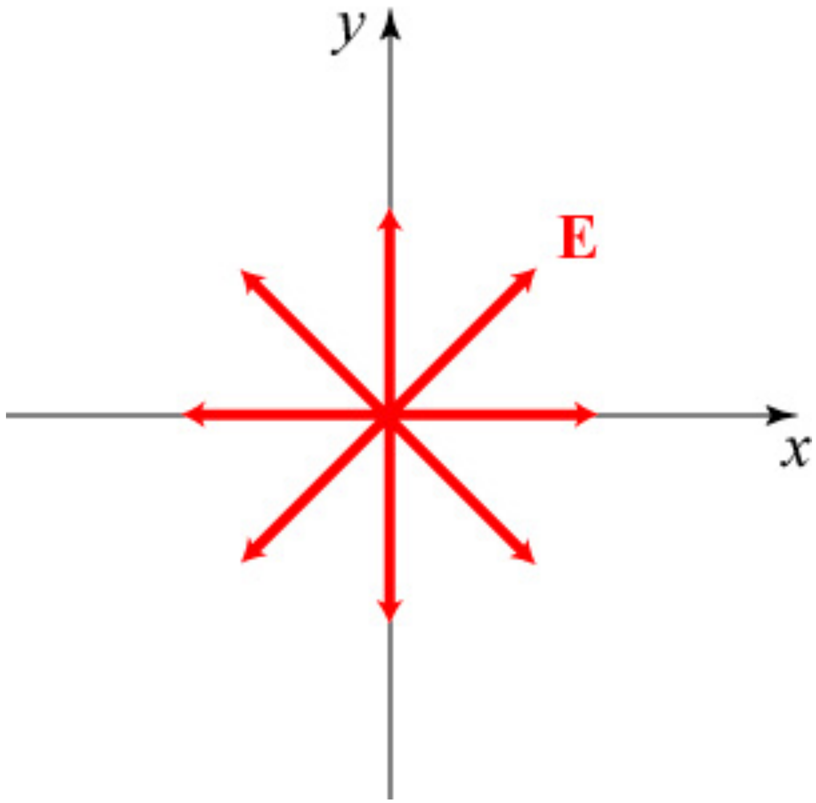
circularly polarized light





Unpolarized light?

Unpolarized light *means random (time-changing) polarization direction, e.g. excited atoms in a solid (a light bulb) emit randomly polarized light packets.*

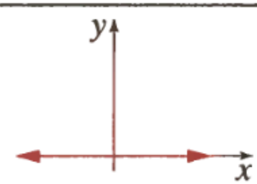
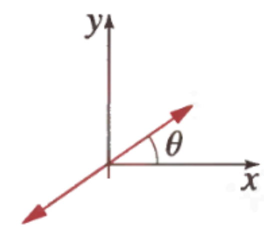
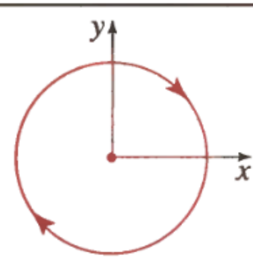
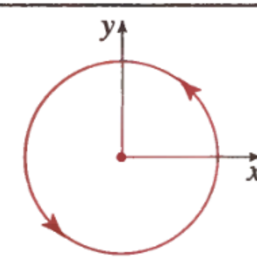


Jones vector

We can represent any monochromatic wave polarization as a ***Jones' vector***:

$$\mathbf{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

For normalized intensity $|A_x|^2 + |A_y|^2 = 1$:

LP in x direction	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$		LP at angle θ	$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$	
RCP	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$		LCP	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$	

E.g. *orthogonal polarizations* whenever $\mathbf{J_1'} \cdot \mathbf{J_2} = 0$

Exercises

Ex1: Check that HLP and VLP are orthogonal as well as RCP and LCP.

Q: What does it mean?

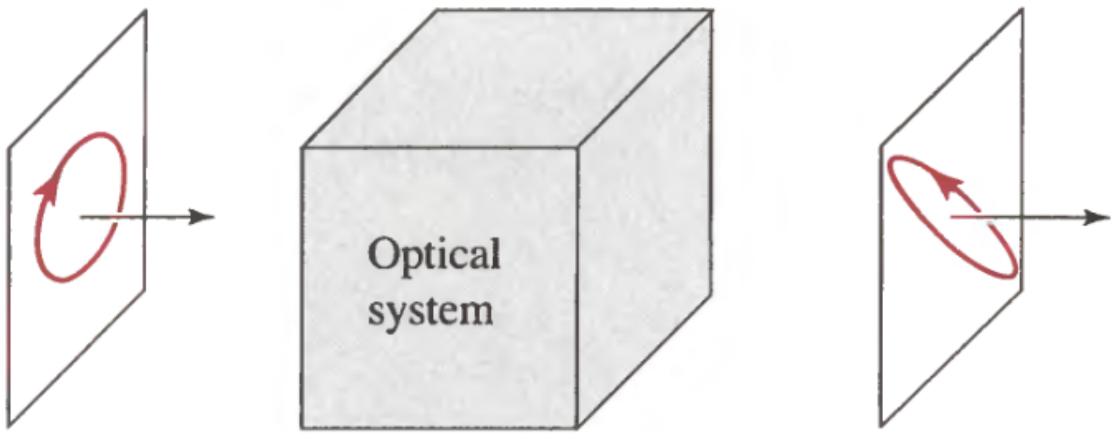
A: can use either as a basis to represent arbitrary polarization!

Ex2: How to obtain the light intensity from its Jones vector (if in vacuum)?

$$\mathbf{J}' \cdot \mathbf{J} = [A_x^* \quad A_y^*] \begin{bmatrix} A_x \\ A_y \end{bmatrix} = |A_x|^2 + |A_y|^2$$

Jones matrix (don't work with unpolarized light!)

$$\mathbf{J}_2 = \mathbf{T}\mathbf{J}_1$$



$$A_{2x} = T_{11}A_{1x} + T_{12}A_{1y}$$
$$A_{2y} = T_{21}A_{1x} + T_{22}A_{1y},$$

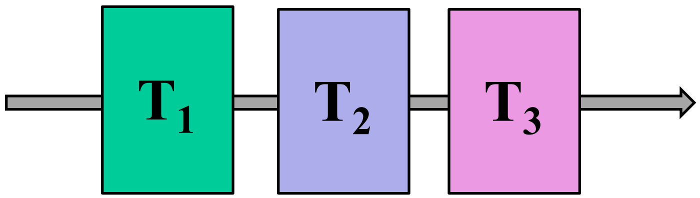
$$\begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix}$$

Normal modes:

$$\mathbf{T}\mathbf{J} = \mu\mathbf{J}$$

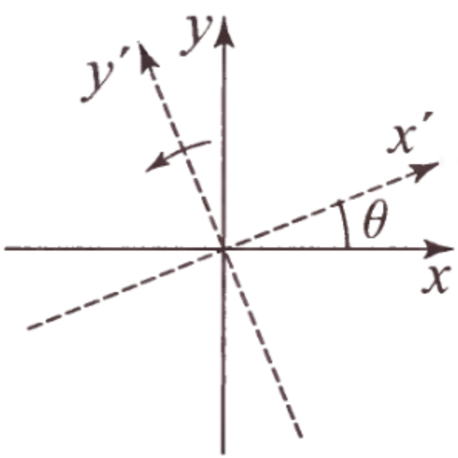
Combining polarization devices & tilt

Simply multiply the matrices in the *reverse order*:



$$\mathbf{T} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1$$

If polarization device is rotated, use: $\mathbf{T}' = \mathbf{R}(\theta)\,\mathbf{T}\,\mathbf{R}(-\theta)$



$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

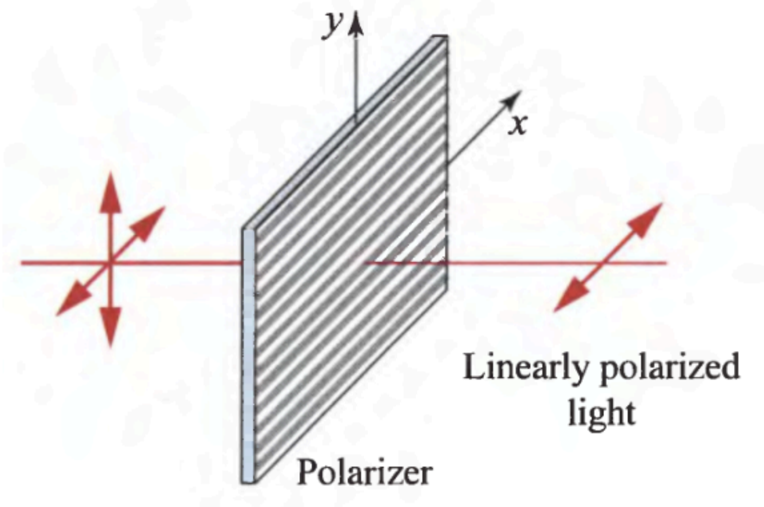
Proof: $\mathbf{J}_2' = \mathbf{R}(\theta)\,\mathbf{J}_2 = \mathbf{R}(\theta)\,\mathbf{T}\,\mathbf{J}_1.$

using $\mathbf{J}_1 = \mathbf{R}(-\theta)\,\mathbf{J}_1'$, we get

$$\mathbf{J}_2' = \underbrace{\mathbf{R}(\theta)\,\mathbf{T}\,\mathbf{R}(-\theta)}_{\mathbf{T}'}\,\mathbf{J}_1'$$

Linear polarizer

Linear polarizer in *x*-direction



$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

A polarizer rotated by angle θ

$$\mathbf{T} = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Polarization retarders or wave plates

These devices do not affect one polarization component (*fast axis*) but add a retarding phase to the other component (*slow axis*).

If *fast axis* is along *x*-direction, then

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

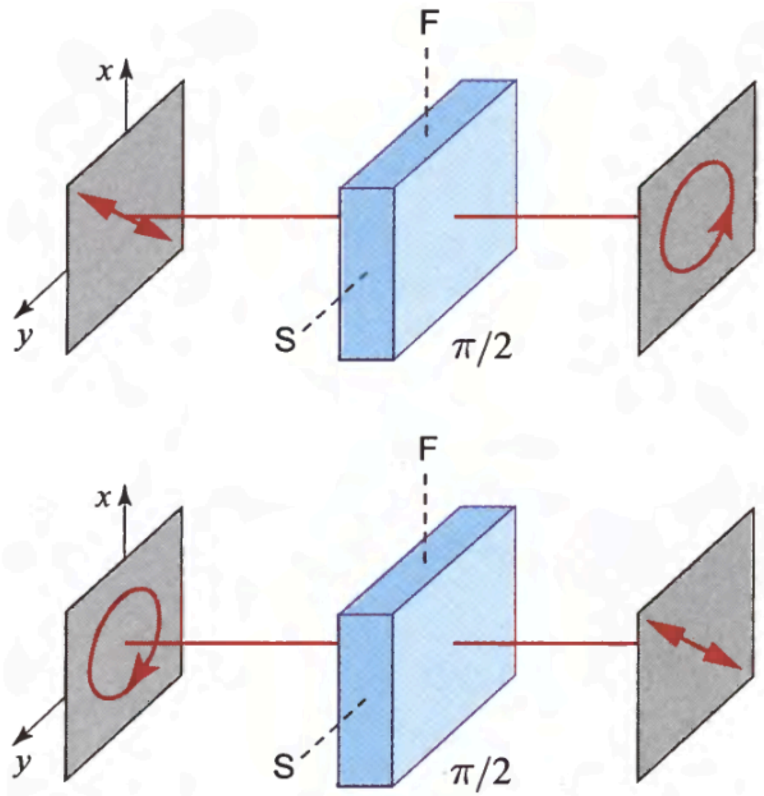
y-component gets 'retarded'
slow axis

Important cases of wave retarders:

- Quarter-wave retarder: $\Gamma = 2\pi/4 = \pi/2$
- Half-wave retarder: $\Gamma = 2\pi/2 = \pi$

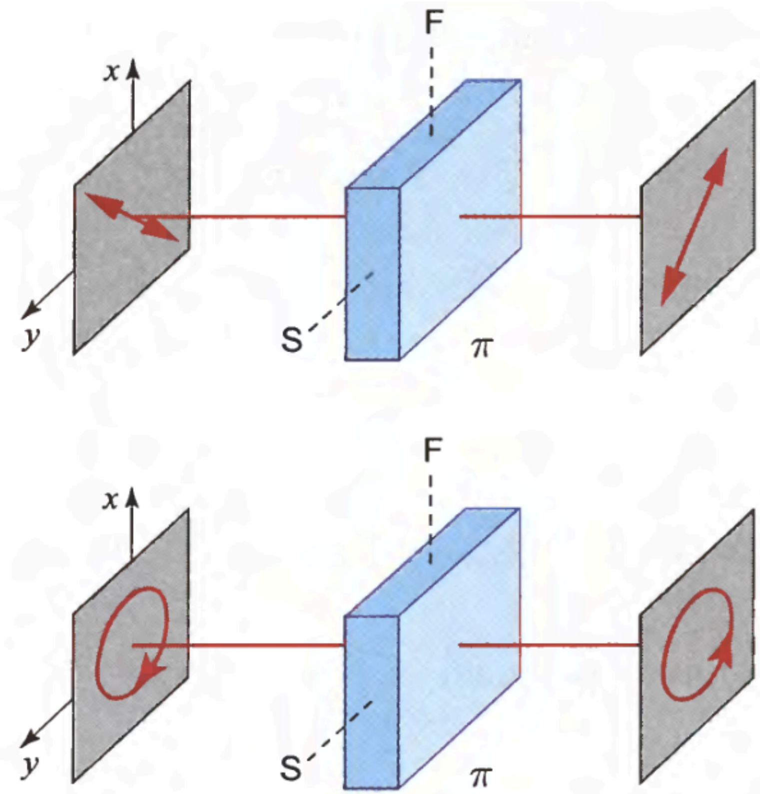
Quarter-wave plate & half-wave plate

Quarter-Wave Retarder



$$T_{\pi/2} = \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}$$

Half-Wave Retarder



$$T_{\pi} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Exercises

What is the Jones matrix of a mirror?

What happens to HLP light upon reflection?

What happens to VLP light upon reflection?

What happens to RCP light upon reflection?

What happens to LCP light upon reflection?

Q: How is it different from the half-wave plate retarder?

Jones calculus example

Poor man's optical isolator:

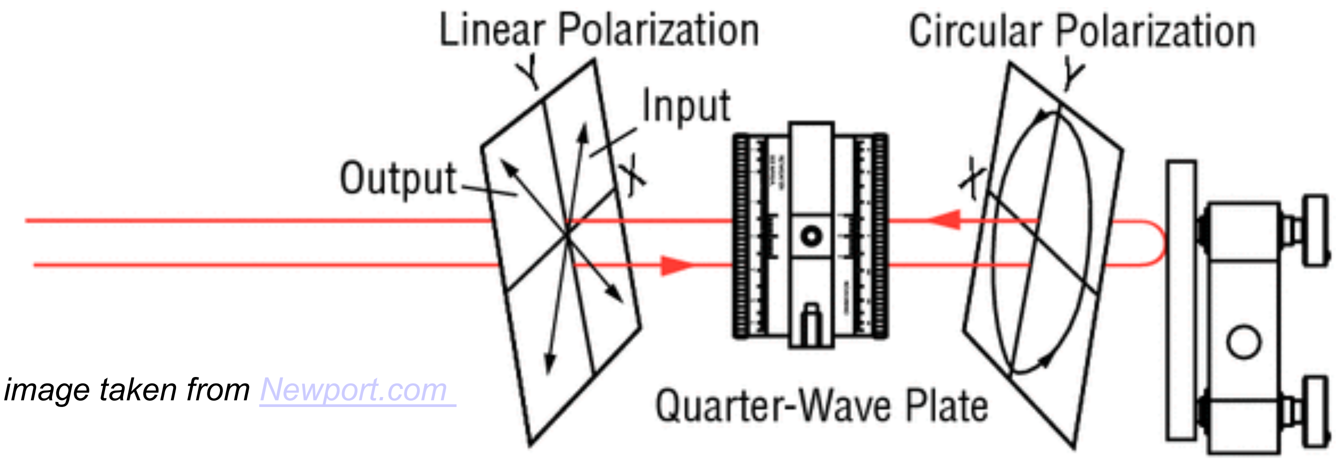


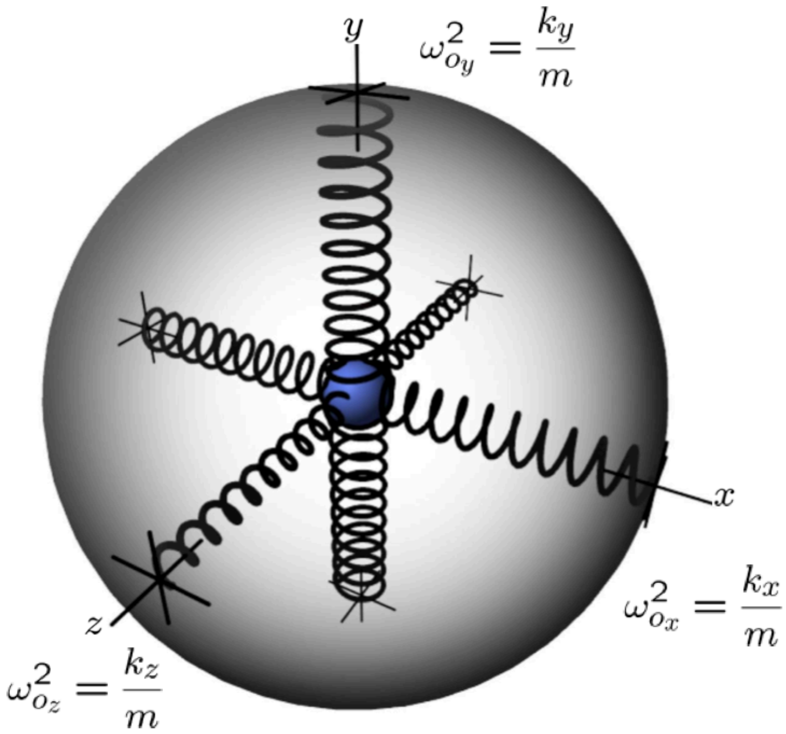
image taken from Newport.com

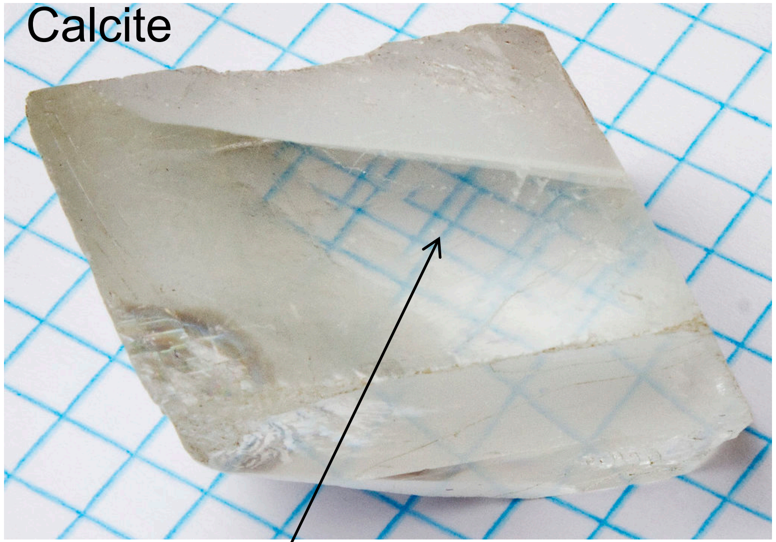
Jones matrix: $$\mathbf{T} = \mathbf{T}_{\text{pol,x}}(-45^\circ)\mathbf{T}_{\pi/2}\mathbf{T}_{\text{mirror}}\mathbf{T}_{\pi/2}\mathbf{T}_{\text{pol,x}}(45^\circ)$$

$$\mathbf{T} = \frac{1}{4}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Birefringence

Spring model of a molecule:





*different indexes of refraction depending on the polarization direction → **birefringence***

When 'spring constants' are not the same, the material is said to be optically ***anisotropic*** (otherwise, it is said to be *isotropic*).

Uniaxial crystals

If the 'spring constants' $k_1 = k_2 \neq k_3$, index of refraction $n_3 = n_e$ in k_3 polarization direction (***optical* axis**) is different (***extraordinary wave***) from $n_1 = n_2 = n_o$ in k_1, k_2 directions (***ordinary waves***). Such materials are known as ***uniaxial crystals***.

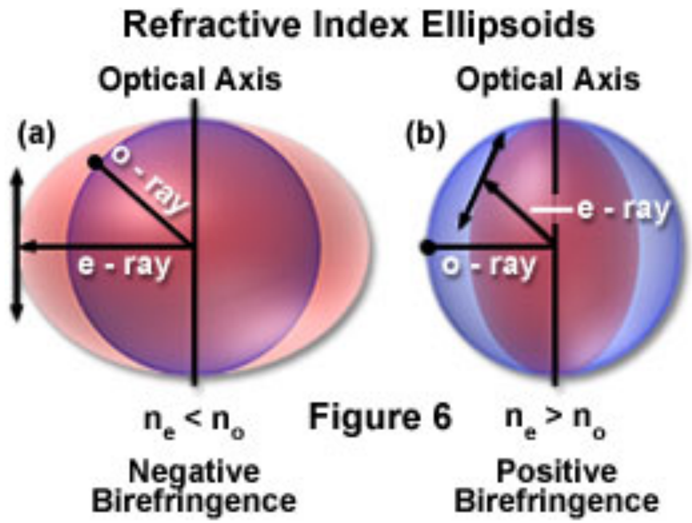


image taken from microscopyu.com

calcite unit cell

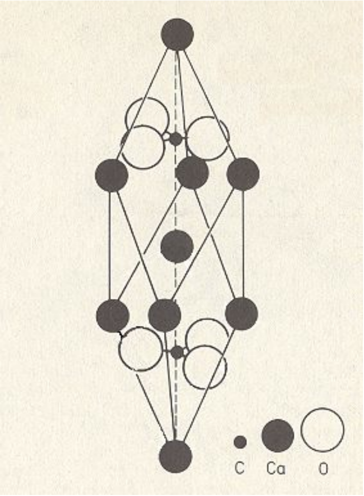


image taken from metafysica.nl

If $k_1 \neq k_2 \neq k_3$, the crystal as said to be ***biaxial***.

Retarders

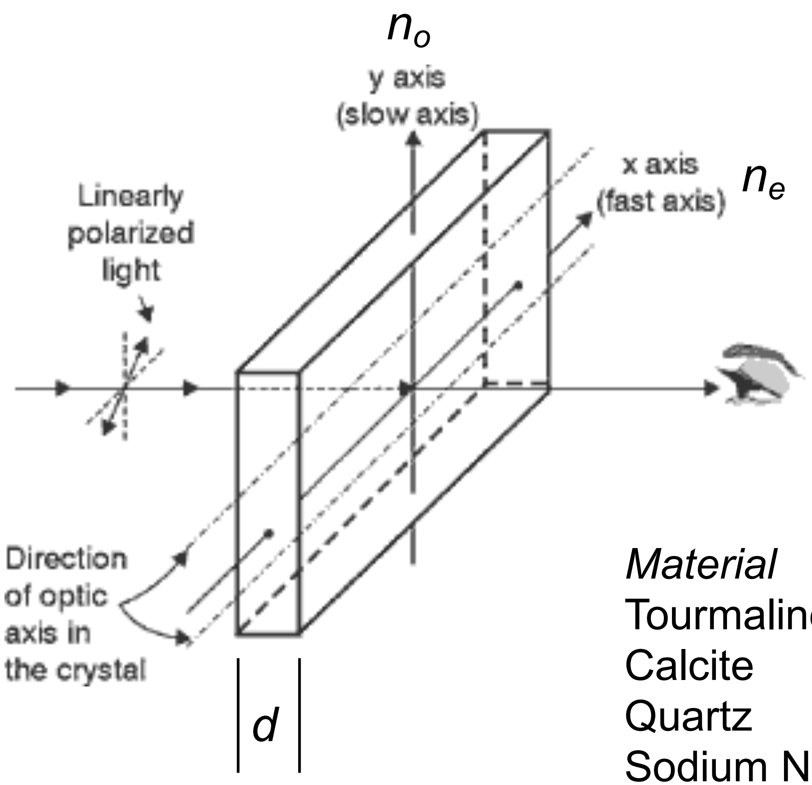


image taken from philem.me.uk

Phase retardation for *x*,*y*-polarizations

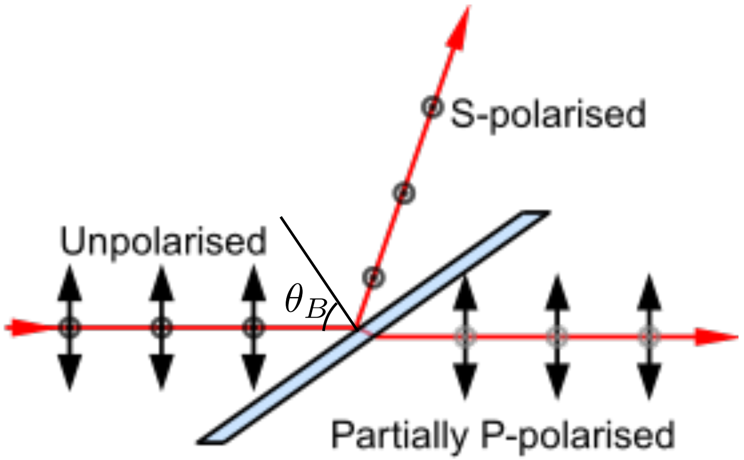
$$\Gamma = \frac{2\pi}{\lambda_0} d(n_0 - n_e)$$

Material	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1. 5534
Sodium Nitrate	1.5854	1. 3369
Ice	1.309	1.313
Rutile (TiO_2)	2.616	2.903

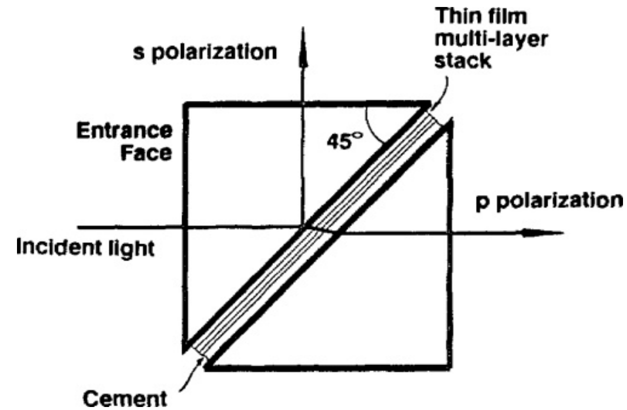
Example: quarter-wave plate of calcite must have thickness $d_{\pi/2}$ = 600 nm/4/($n_o - n_e$) = 0.87 μm, and $d_{\pi/2}$ = 38 μm for ice.

Polarizing beam splitters

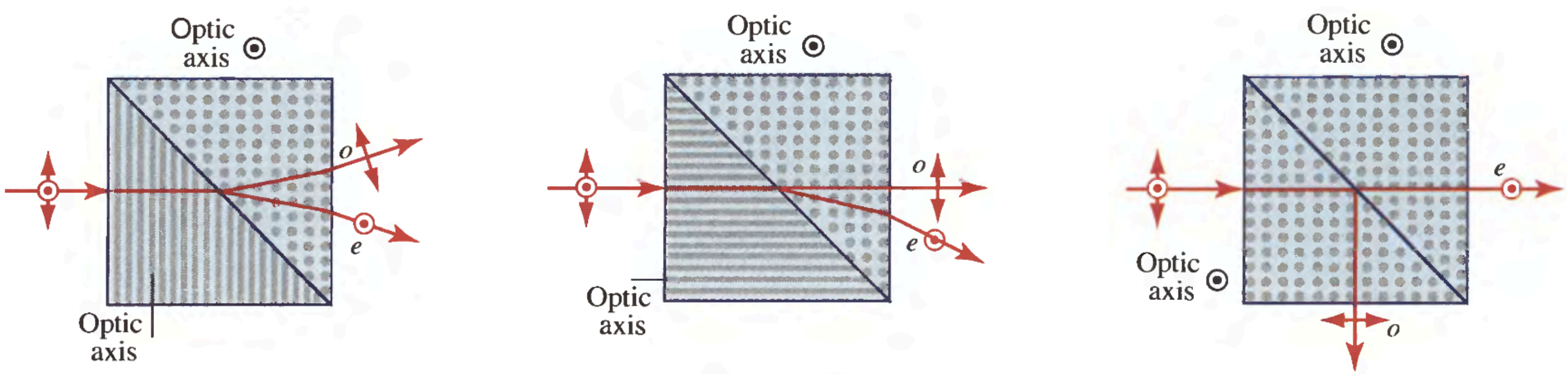
Simplest kind – use Brewster's angle: $\theta_B = \tan^{-1}(n_2/n_1)$



MacNeille polarizing beamsplitter cube



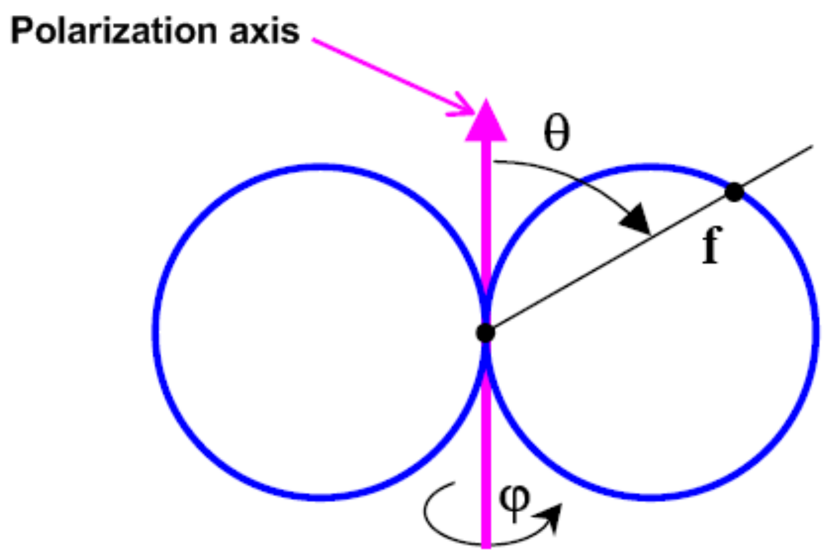
Other types rely on splitting beam into *ordinary* and *extraordinary* waves:



(*a*) Wollaston prism (*b*) Rochon prism (*c*) Glan–Thompson prism

Rayleigh scattering

Recall that a driven electric dipole emits radiation (fully polarized!)



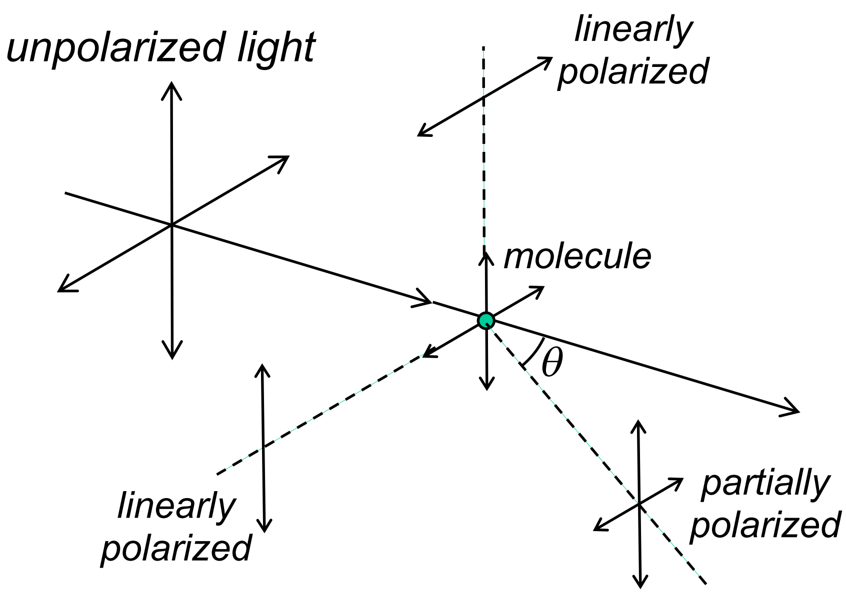
$f(\theta,\varphi) = A \cdot \sin(\theta)$

Electric field of light drives little dipoles (bound electrons) → light is re-radiated or scattered (Rayleigh scattering).

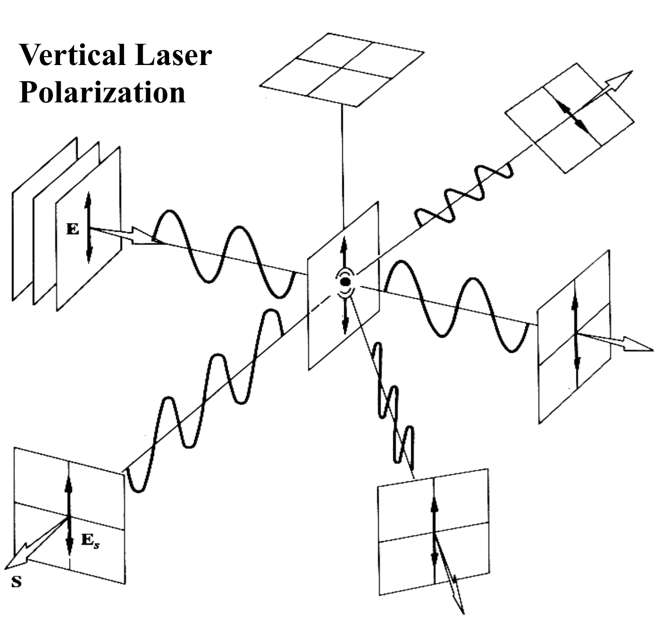
Its most salient feature is that $I_{scat.} \propto 1/\lambda^4$ (i.e. the blue light gets scattered much more than red, the reason behind blue skies)

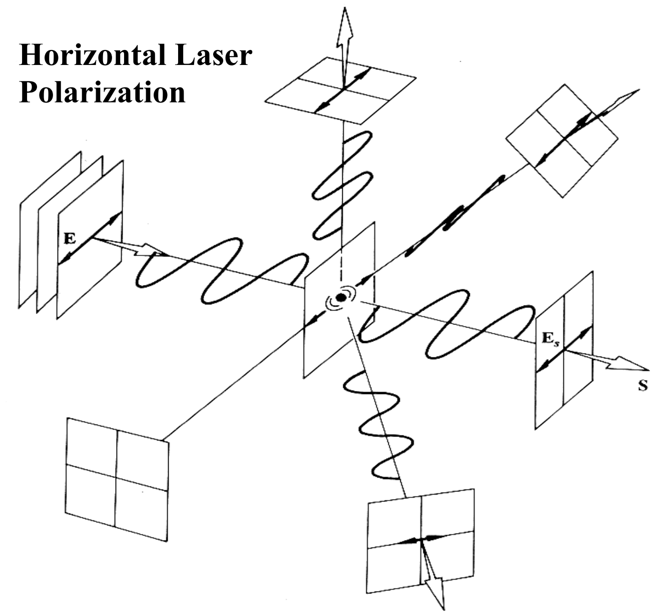
Rayleigh scattering polarization

Scattered light is linearly polarized when viewed at 90º from the scatterers; & partially polarized at other angles.



Rayleigh scattering from laser





Other scattering regimes:

- Rayleigh scattering is when scatterer size $\ll$ wavelength
- When scattering particles ~ wavelength, it's called ***Mie*** scattering, which is a more general theory. ***Tyndall*** effect refers to being able to see the laser path in a colloidal solution.

Links/References

Most figures taken from Saleh & Teich

Some figures from Wikipedia

http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-007-electromagnetic-energy-from-motors-to-lasers-spring-2011/lecture-notes/MIT6_007S11_lec25.pdf