Geometric optics

Ivan Bazarov

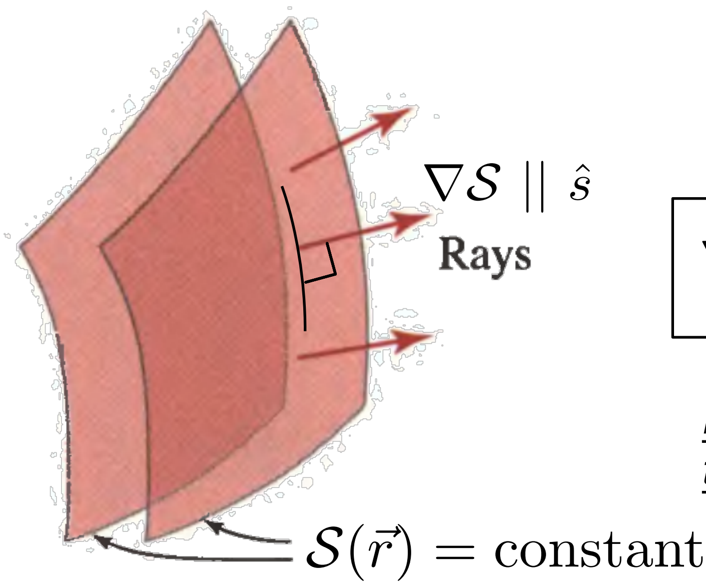
Cornell Physics Department / CLASSE

Outline

- **Eikonal equation**
- **Linear optics: ABCD matrices**
- **Non-linear optics: aberrations**
- **Depth of field, F#**
- **Diffraction limit, NA, resolving power**

Eikonal equation

Eikonal – (from gr. εἰκών) scalar 'potential' *S*, whose gradient defines the direction of rays ('field lines'). E.g. compare to: $\vec{E} = -\nabla V$

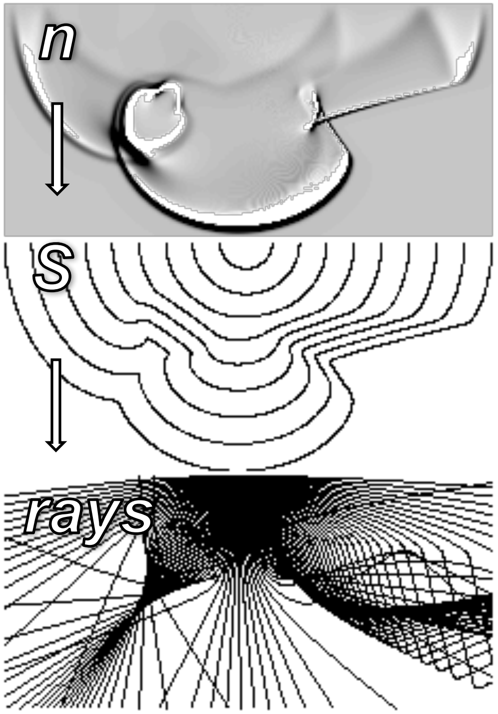


Eikonal equation

$$\boxed{\nabla\mathcal{S}(\vec{r}) = n(\vec{r})\hat{s}}$$

Mathematically equivalent to Fermat's principle!!

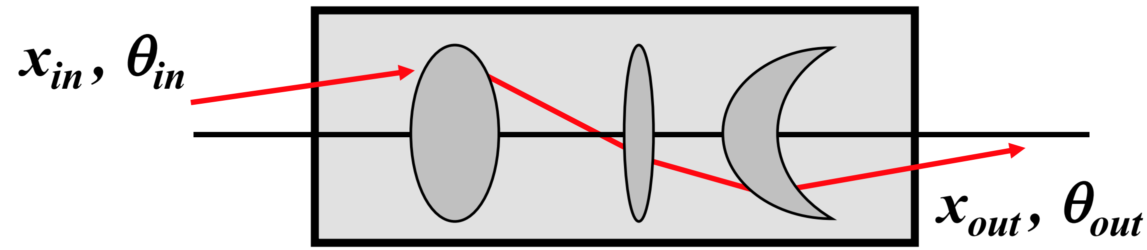
example



Difference between eikonal in points A and B gives optical path length:

$$\mathcal{S}(\vec{r}_B) - \mathcal{S}(\vec{r}_A) = \int_A^B |\nabla\mathcal{S}|\, ds = \int_A^B n\, ds = \text{optical path length}$$

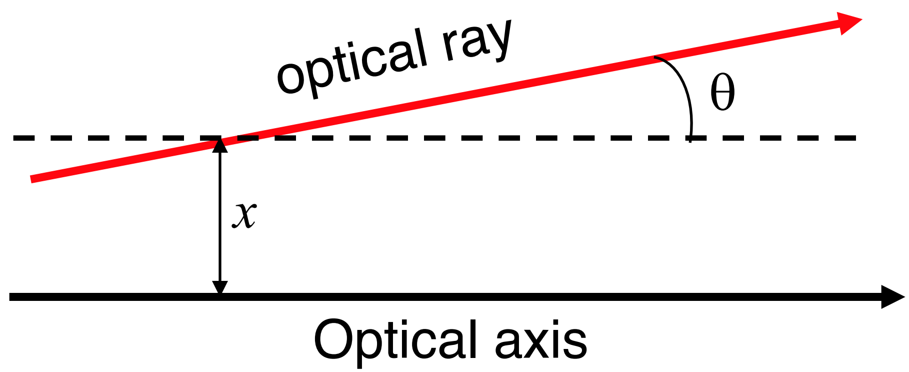
Ray vector (paraxial approximation)



A light ray can be defined by two co-ordinates:

its position, x

its slope, θ



These parameters define a **ray vector**, which will change with distance and as the ray propagates through optics.

$$\begin{bmatrix} x \\ \theta \end{bmatrix}$$

Linear optics

Since the displacements and angles are assumed to be small, we can think in terms of a linear combination with partial derivatives as coefficients (leading Taylor series expansion terms…)

$$x_{out} = \frac{\partial x_{out}}{\partial x_{in}} x_{in} + \frac{\partial x_{out}}{\partial \theta_{in}} \theta_{in}$$

$$\theta_{out} = \frac{\partial \theta_{out}}{\partial x_{in}} x_{in} + \frac{\partial \theta_{out}}{\partial \theta_{in}} \theta_{in}$$

$$\frac{\partial x_{out}}{\partial x_{in}} \qquad \frac{\partial x_{out}}{\partial \theta_{in}}$$

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

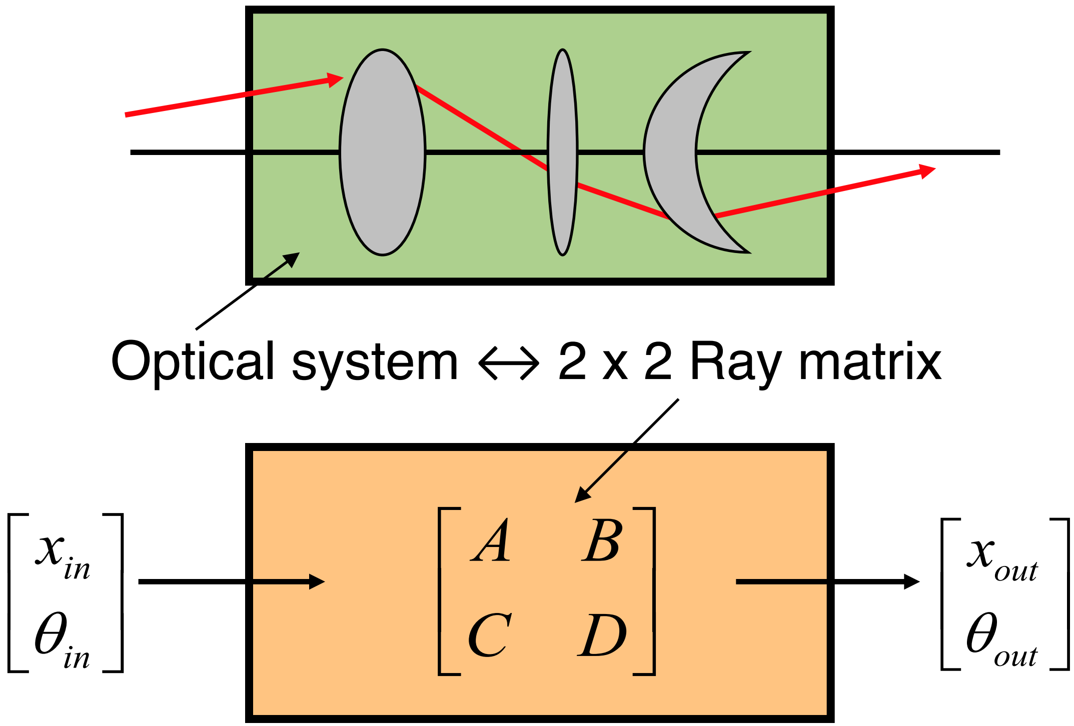
$$\frac{\partial \theta_{out}}{\partial x_{in}} \qquad \frac{\partial \theta_{out}}{\partial \theta_{in}}$$

We can write these equations in matrix form.

ABCD matrix

Thus, we can define 2 x 2 **ray matrices** for any element.

An element’s effect on a ray is found by multiplying its ray vector.



Ray matrices can describe simple and complex systems.

These matrices are often called ABCD Matrices.

Physical meaning of the matrix elements

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} \overset{\frac{\partial x_{out}}{\partial x_{in}}}{A} & \overset{\frac{\partial x_{out}}{\partial \theta_{in}}}{B} \\ \underset{\frac{\partial \theta_{out}}{\partial x_{in}}}{C} & \underset{\frac{\partial \theta_{out}}{\partial \theta_{in}}}{D} \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

A = "spatial magnification".

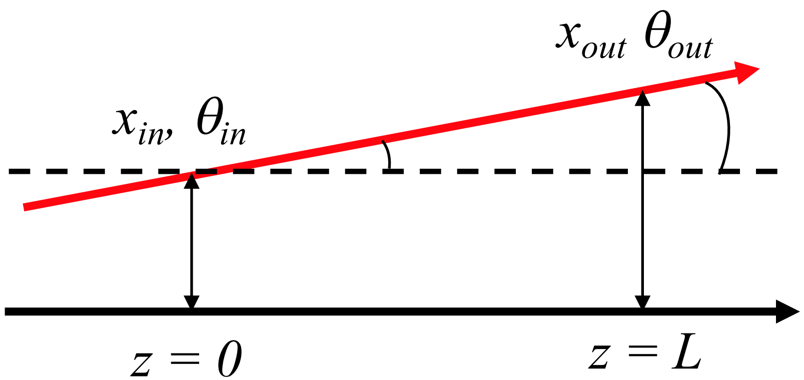
B = "length to the object". B = 0 means that rays emitted at different angles end up at the same *x* offset (condition to form an image).

C = "focusing power" or negative inverse focal length. C = 0 means that parallel rays stay parallel (collimating optics).

D = "angular magnification".

Ray matrix for a free space or a medium

If x_{in} and θ_{in} are the position and slope upon entering, let x_{out} and θ_{out} be the position and slope after propagating from $z = 0$ to L.



$$x_{out} = x_{in} + L\,\theta_{in}$$

$$\theta_{out} = \theta_{in}$$

Rewriting these expressions in matrix notation:

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix}$$

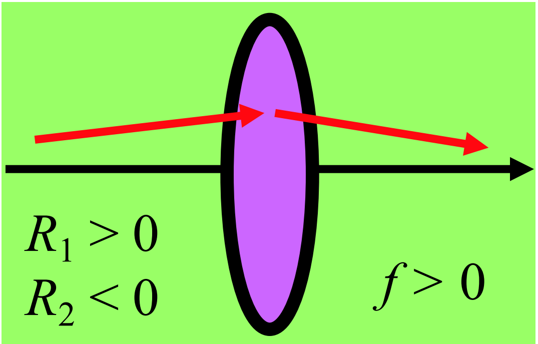
$$\boxed{O_{space} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}}$$

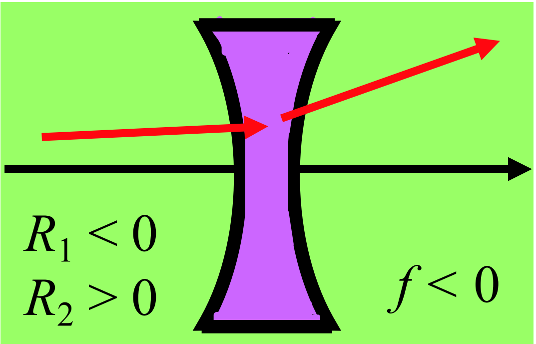
Ray matrix for a thin ideal lens

$$x_{out} = x_{in}$$
$$\theta_{out} = \theta_{in} + \frac{x_{in}}{-f}$$

$$\boxed{O_{lens} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}}$$

The quantity, f, is the **focal length** of the lens. It's the single most important parameter of a lens. It can be positive or negative.





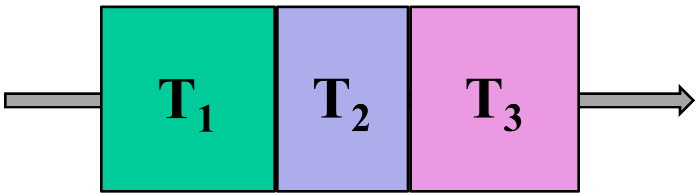
If $f > 0$, the lens deflects rays toward the axis.

If $f < 0$, the lens deflects rays away from the axis.

lens maker formula

$$1/f = (n-1)(1/R_1 - 1/R_2)$$

Combining matrices of multiple elements (must include drifts!!)



$$\mathbf{T} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1$$

As with Jones' matrices: multiply in the reverse order

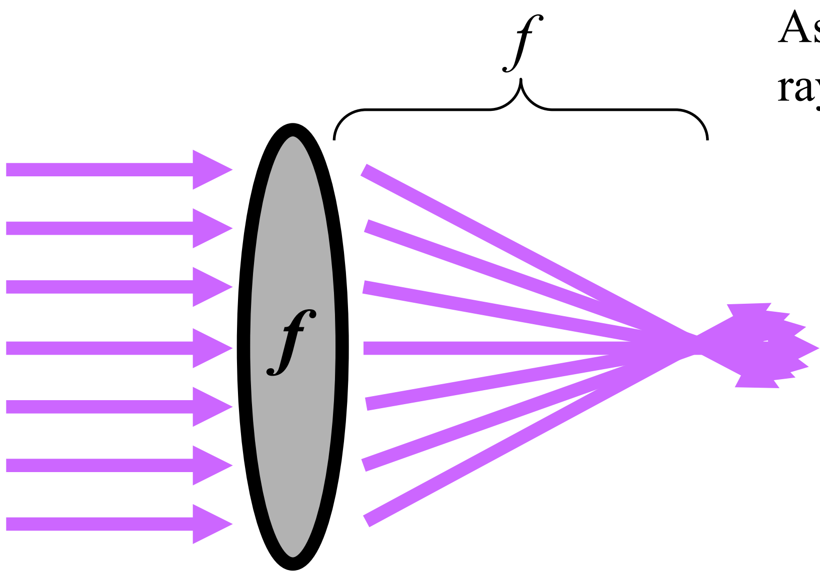
Example 1: a lens focuses parallel rays to a point one focal length away

A lens followed by propagation by one focal length:

For all rays $x_{out} = 0!$

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & f \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x_{in} \\ 0 \end{bmatrix} = \begin{bmatrix} \boxed{0} \\ -x_{in}/f \end{bmatrix}$$

Assume all input rays have $\theta_{in} = 0$



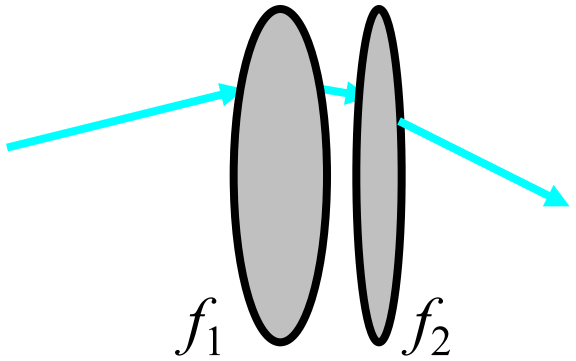
At the focal plane, all rays converge to the z axis ($x_{out} = 0$) independent of input position.

Parallel rays at a different angle focus at a different x_{out}.

Looking from right to left, rays diverging from a point are made parallel.

Example 2: two consecutive lenses

Suppose we have two lenses right next to each other (with no space in between).



$$O_{tot} = \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_1 - 1/f_2 & 1 \end{bmatrix}$$

$$1/f_{\mathrm{tot}} = 1/f_1 + 1/f_2$$

So two consecutive lenses act as one whose inverse focal length is the sum of the two.

As a result, we define a measure of **inverse** lens focal length, the **diopter**.

1 diopter = 1 m^{-1}

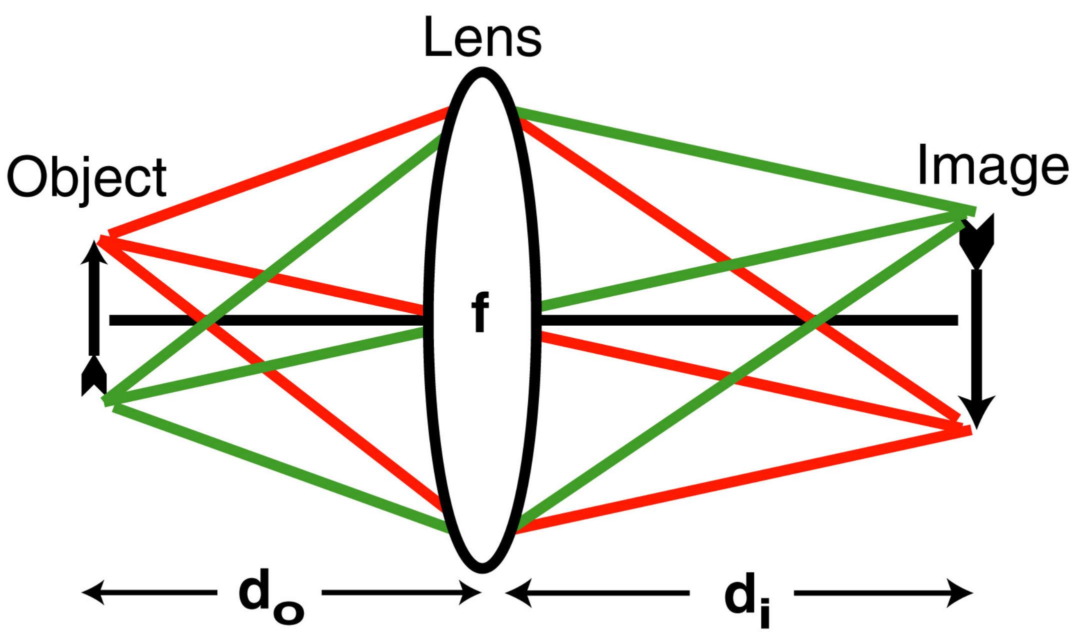
Recall: a system images an object when B = 0

When $B = 0$, all rays from a point x_{in} arrive at a point x_{out}, independent of angle.

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} = \begin{bmatrix} A\,x_{in} \\ C\,x_{in} + D\,\theta_{in} \end{bmatrix}$$

$$\boxed{x_{out} = A\,x_{in}}$$

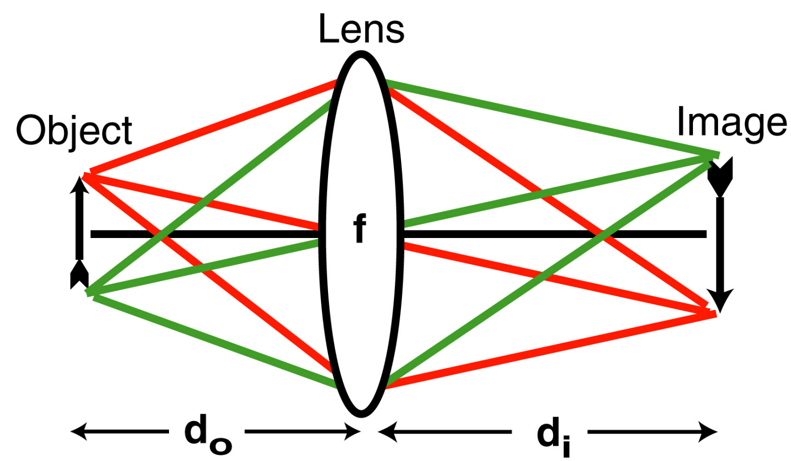
When $B = 0$, A is the **magnification**.



Example 3: the Lens Law

From the object to the image, we have:

1) A distance d_o
2) A lens of focal length f
3) A distance d_i



$$O = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_o \\ -1/f & 1-d_o/f \end{bmatrix}$$

$$= \begin{bmatrix} 1-d_i/f & \boxed{d_o + d_i - d_o d_i / f} \\ -1/f & 1-d_o/f \end{bmatrix}$$

$$B = d_o + d_i - d_o d_i / f = d_o d_i \left[1/d_o + 1/d_i - 1/f \right] = 0 \text{ if}$$

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

This is the Lens Law.

Image magnification

If the imaging condition,

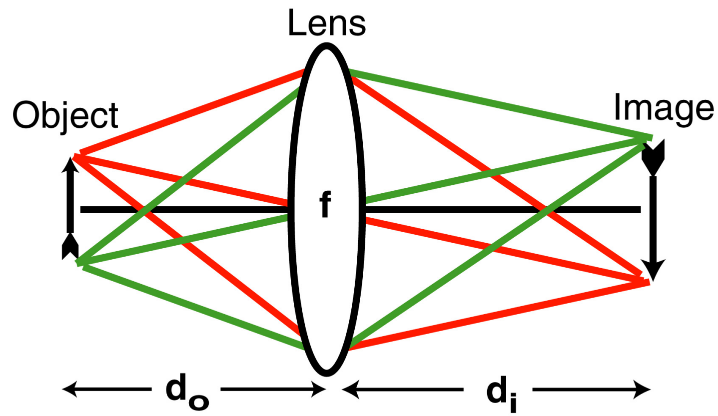
$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

is satisfied, then:

$$O=\begin{bmatrix} 1-d_i/f & 0 \\ -1/f & 1-d_o/f \end{bmatrix}$$

So:

$$\boxed{O=\begin{bmatrix} M & 0 \\ -1/f & 1/M \end{bmatrix}}$$



$$A=1-d_i/f=1-d_i\left[\frac{1}{d_o}+\frac{1}{d_i}\right]$$

$$\Rightarrow \qquad \boxed{M=-\frac{d_i}{d_o}}$$

$$D=1-d_o/f=1-d_o\left[\frac{1}{d_o}+\frac{1}{d_i}\right]$$

$$=-\frac{d_o}{d_i}=1/M$$

Linear vs. nonlinear optics

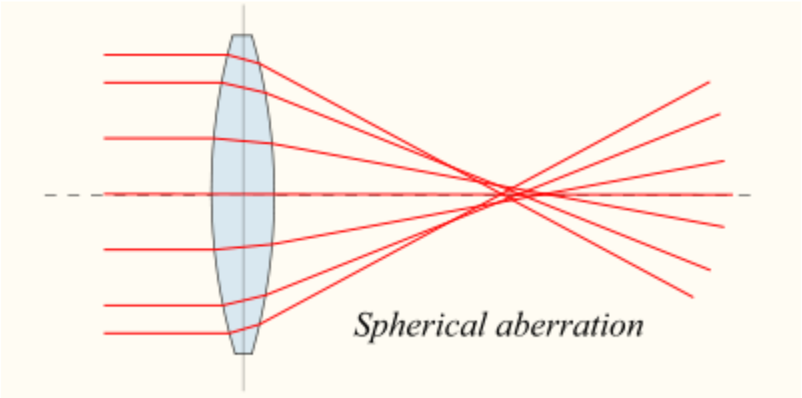
- With linear optics, we only kept the single leading term(s) from the Taylor expansion.
- If we include higher-order terms, we get ***nonlinear optics***: e.g. x_{out} depends on not only x_{in} and θ_{in}, but also x_{in}^2, $x_{in}\theta_{in}$, θ_{in}^2, x_{in}^3, etc.
- Linear optics matrices have det M = 1 (phase space area or divergence × size conserving optics, a.k.a. Liouville's theorem – more at the next lecture).
- Nonlinear optics is no longer phase space area conserving, beam experiences distortions known as ***aberrations***.
- Nonlinear optics design is highly non-trivial, nowadays uses computer ray tracing: e.g. OSLO or ZeMAX software packages.

Aberrations

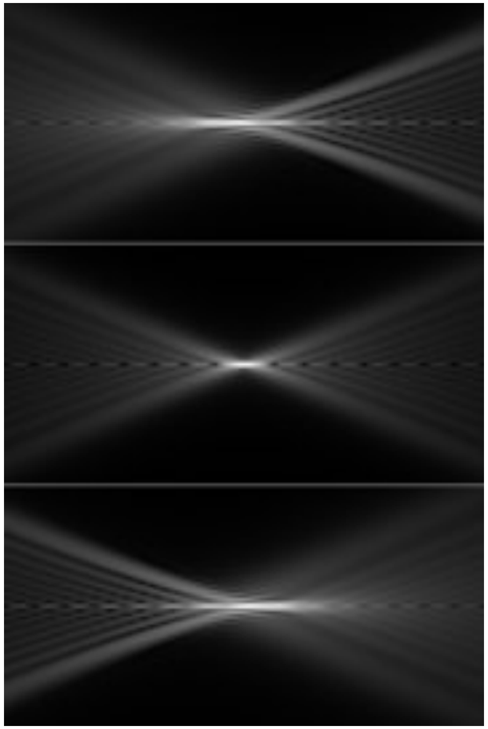
- Main types can be classified into ***geometric*** and ***chromatic aberrations***
- ***Chromatic*** means color-dependent (e.g. index of refraction dependence on the wavelength)
- ***Geometric*** come from the higher orders of the Taylor expansion. Examples are:
  - spherical aberration (all spherical surfaces have it but a parabollic reflector doesn’t)
  - coma (off-axis artifact, even aspheric lenses have this)
  - astigmatism (focal length is different for different planes, e.g. asymmetry in lenses)

Spherical aberration

- Rays at different radii focus at different points
- Makes for a mushy focus, with a halo
- Positive spherical lenses have positive S.A., where exterior rays focus closer to lens
- Negative lenses have negative S.A. “Overcorrecting” a positive lens (going too far in making asphere) results in neg. S.A.



lens side

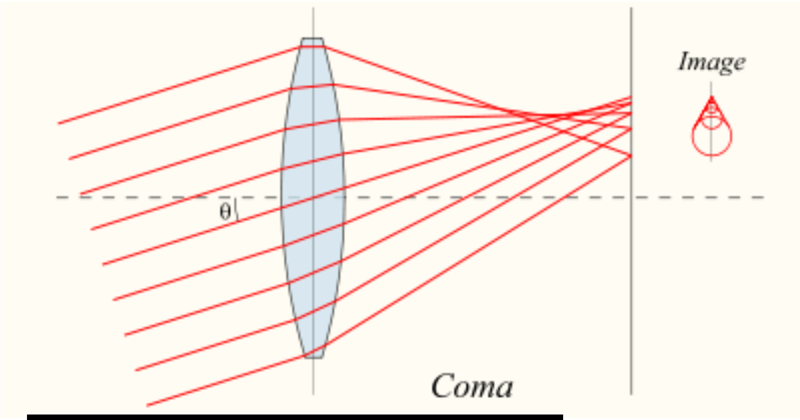


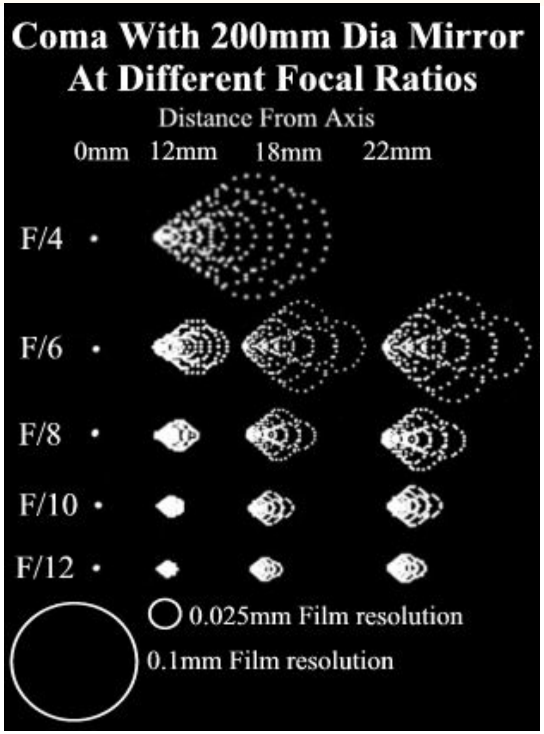
neg. S.A.

zero S.A.

pos. S.A.

Coma

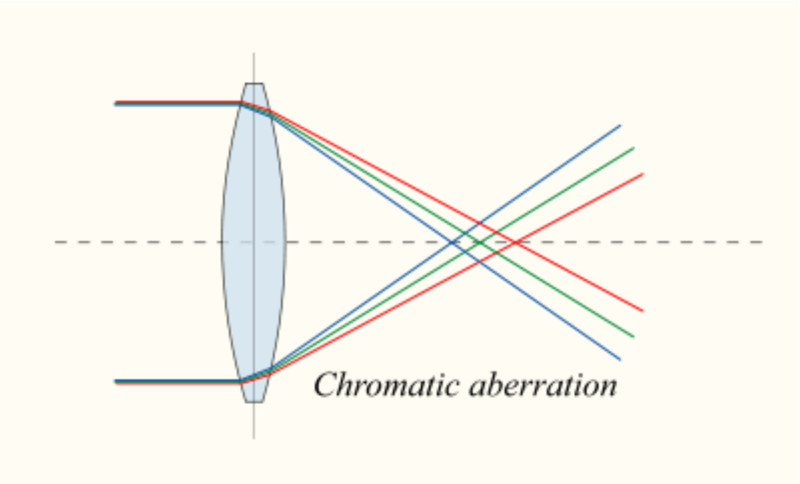


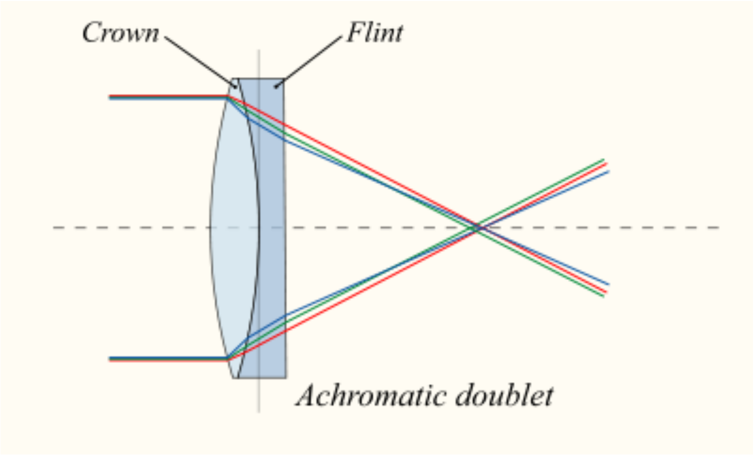


- Off-axis rays meet at different places depending on ray height
- Leads to asymmetric image, looking something like a comet (with nucleus and flared tail)
  - thus the name coma
- As with *all aberrations*, gets worse with larger apertures
- Exists in parabolic reflectors, even if no spherical aberration

Chromatic aberration

- Glass has slightly different refractive index as a function of wavelength
  - so not all colors will come to focus at the same place
  - leads to colored blur
  - why a prism works
- Fixed by pairing glasses with different *dispersions* ($dn/d\lambda$)
  - typically a positive lens of one flavor paired with a negative lens of the other
  - can get cancellation of this aberration
  - also helps spherical aberration to have multiple surfaces (more design freedom)

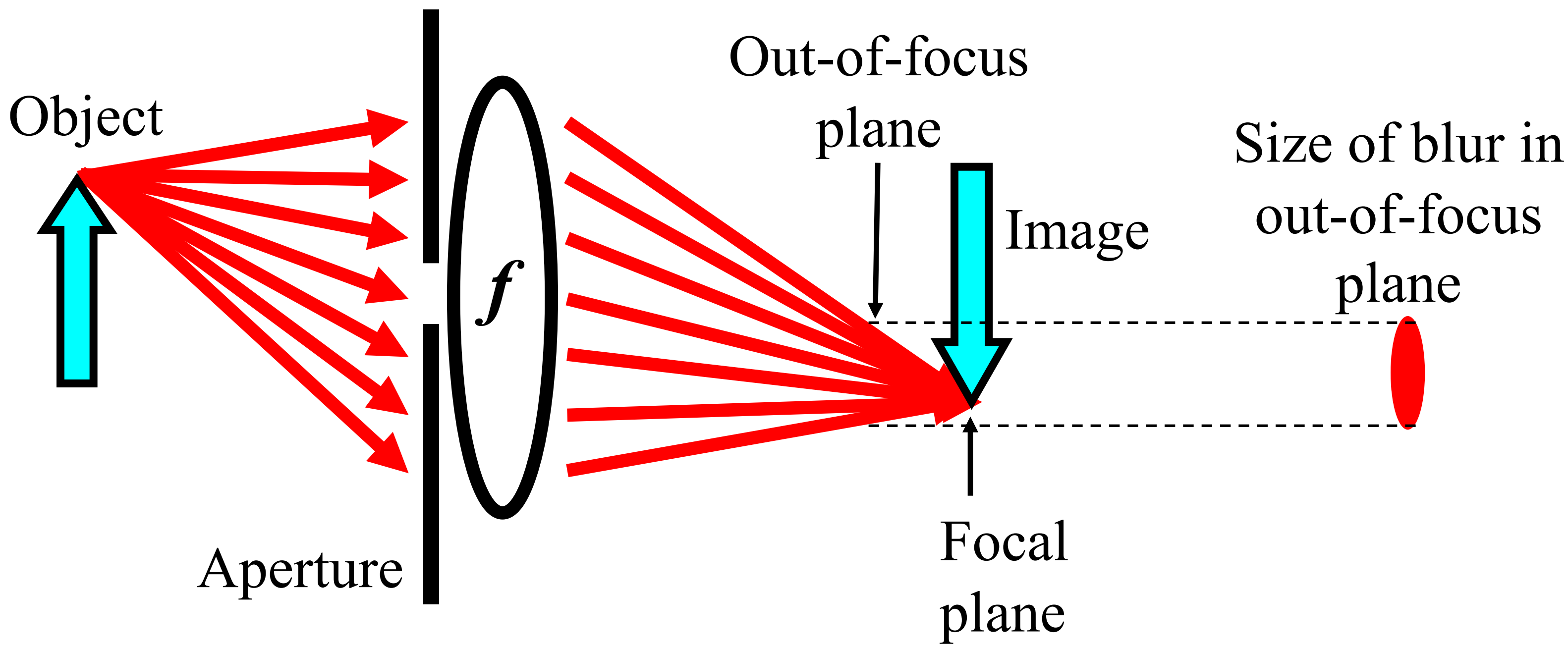




Depth of field

Only one plane is imaged (i.e., is in focus) at a time. But we'd like objects near this plane to at least be almost in focus. The range of distances in acceptable focus is called the **depth of field.**

It depends on how much of the lens is used, that is, the **aperture**.

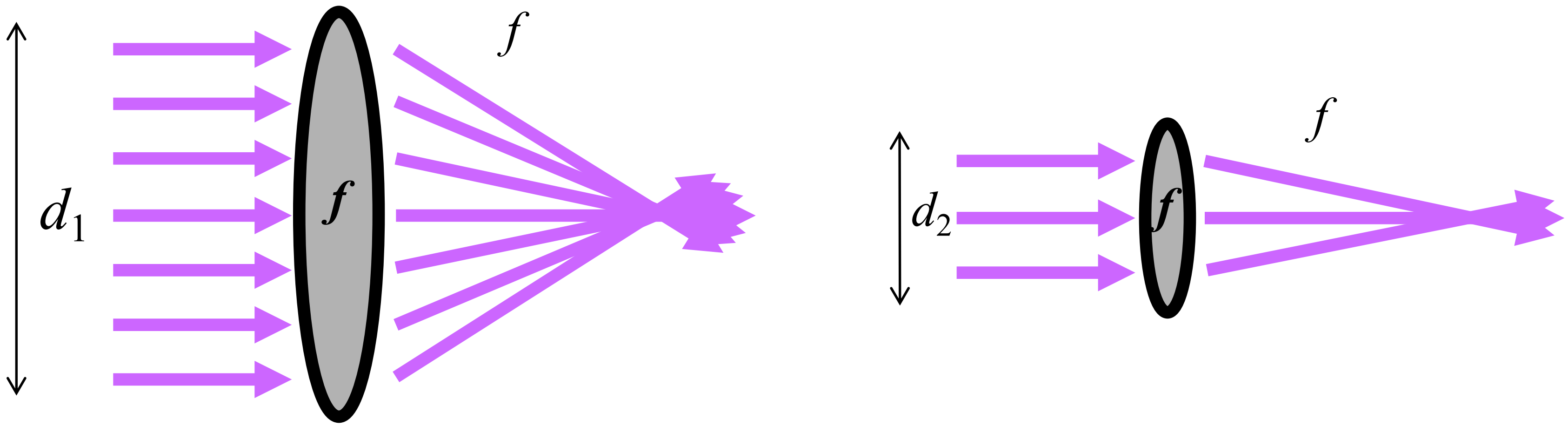


The smaller the aperture, the more the depth of field.

F-number

The F-number, "$f\,/\,\#$", of a lens is the ratio of its focal length over its diameter.

$$\boxed{f/\# = f/d}$$



$f/\# = f/d_1$ $\qquad$ $f/\# = f/d_2$

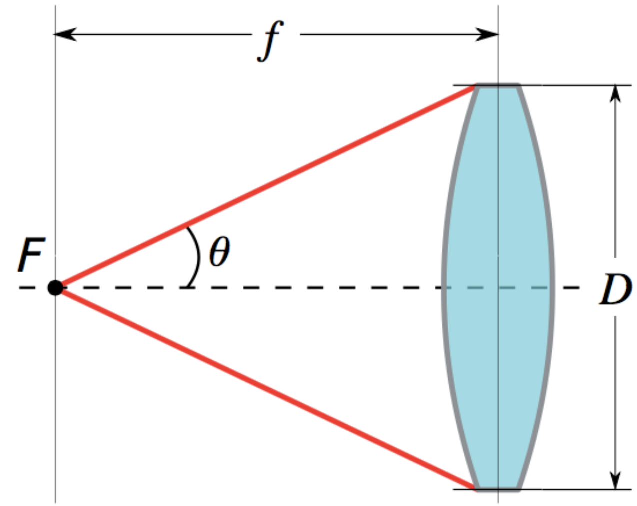
Large f-number lenses collect more light but are harder to engineer.

Numerical aperture/diffraction limited resolution

A related quantity is ***numerical aperture***

$$\mathrm{NA} = n \sin\theta$$

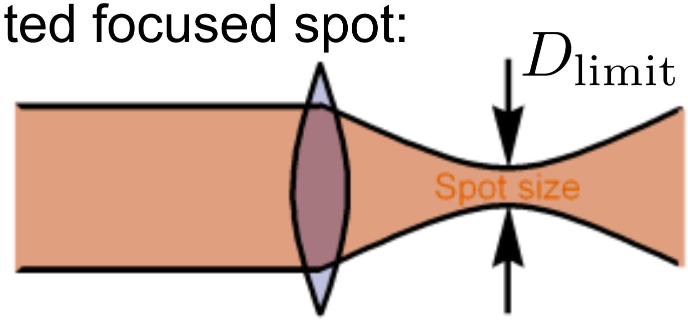
for medium, i.e. 1 for air



For small angles: $\mathrm{NA} = \dfrac{1}{2(f/\#)}$

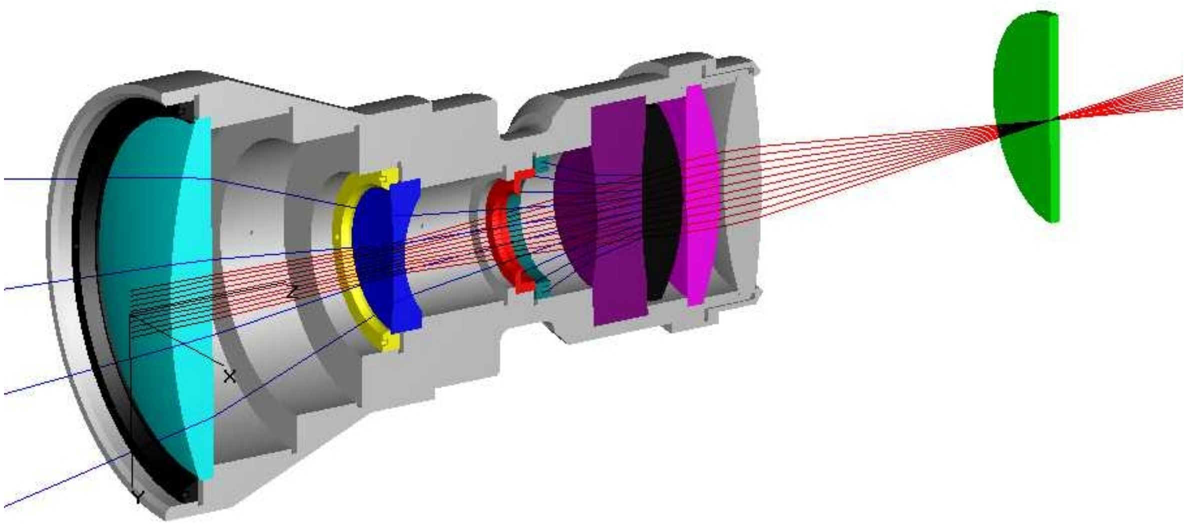
Most significant application – diffraction-limited focused spot:

$$D_{\mathrm{limit}} = \frac{2}{\pi}\frac{\lambda}{\mathrm{NA}}$$



Ray tracing software: OSLO

OSLO (Optics Software for Layout and Optimization) is a powerful optical design software. Does ray tracing (all types of aberrations) but can also include diffraction effects.



A free version (limited to 10 optical surfaces) is downloadable from

http://www.lambdares.com/buy/educators-and-students

Links/References

www.gabrielse.us/physics/optics/powerpoints/Geometric%20Optics.ppt

http://physics.ucsd.edu/~tmurphy/phys121/lectures/05_optics.ppt