P3360 / AEP 3630

Lecture 8

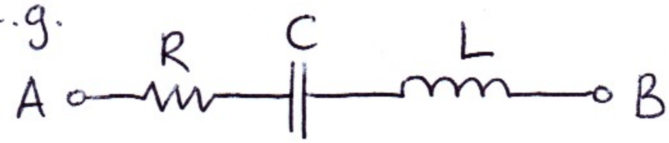
Combining R, L, C

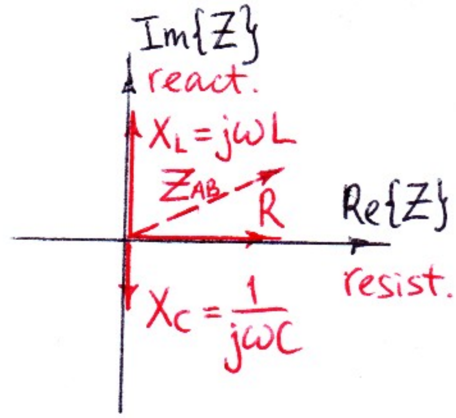
series: $Z_T = Z_1 + Z_2 + \ldots$ (add impedances)

parallel: $Y_T = Y_1 + Y_2 + \ldots$ (add admittances)

Similar to combining R's, but now add & manipulate complex numbers

E.g.



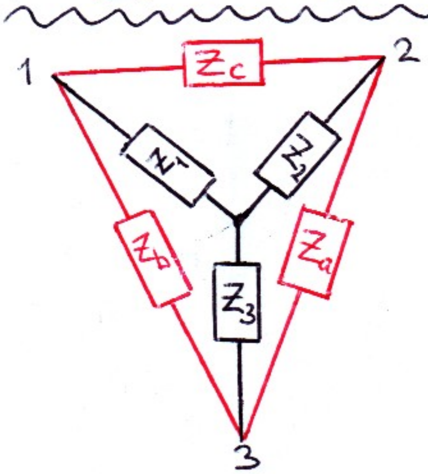


Q: is current through R, L, C different?
A: no, it is the same (KCL)

$$Z_{AB} = R + \frac{1}{j\omega C} + j\omega L \quad (\in \text{complex})$$

$$V_{AB}(\omega) = Z_{AB} \cdot I_{AB}(\omega)$$

Y – Δ transformation



same as before, but now complex

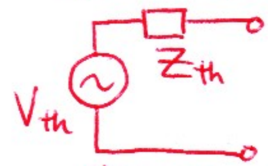
$$\Delta \to Y: \quad Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}, \ldots$$

$$Y \to \Delta: \quad Y_a = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}, \ldots$$

AC circuit analysis

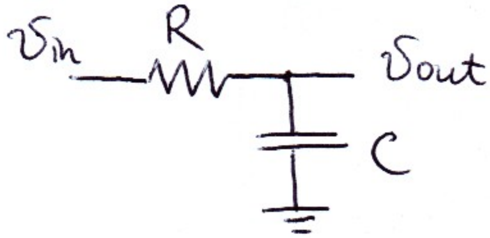
Q: which apply?

* superposition YES (if linear)
* Thevenin & Norton YES (if one freq.)
* KCL & KVL YES (always)

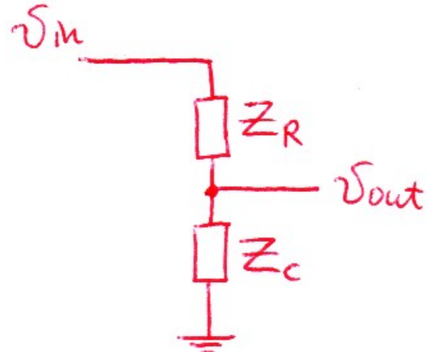


Thev. equivalent

Lo-pass filter (revisited)



$\Rightarrow$



Q: looks familiar?
A: Yes, volt. divider!

Transfer fcn (a.k.a. complex gain)

$$G(\omega) \equiv \frac{v_{out}(\omega)}{v_{in}(\omega)}$$

L.P.: $$G(\omega) = \frac{v_{in} \cdot Z_c}{Z_c + Z_R}\frac{1}{v_{in}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

Q: $C = \frac{A}{B}$, magn & phase?

$$|C| = \frac{|A|}{|B|}, \quad \varphi[C] = \varphi[A] - \varphi[B]$$

magn. $$|G(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

phase $$\varphi[G] = \varphi[1] - \varphi[1 + j\omega RC]$$
$$= -\tan^{-1}(\omega RC)$$

can be measured by network analyzer

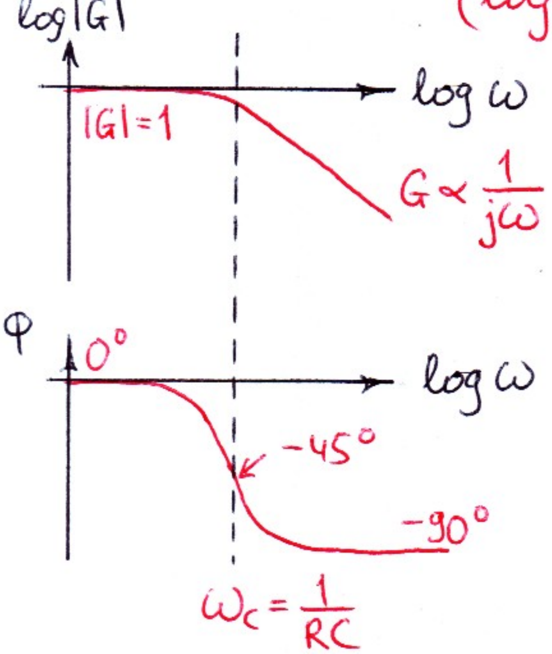
VNA – vector NA
both magn & ph

SNA – scalar NA
only magn.

Bode plots

$$G(\omega) = |G(\omega)|\, e^{j\varphi[G(\omega)]}$$

Bode plots = plot $|G|$ and φ versus ω
(log-log) (semi-log)

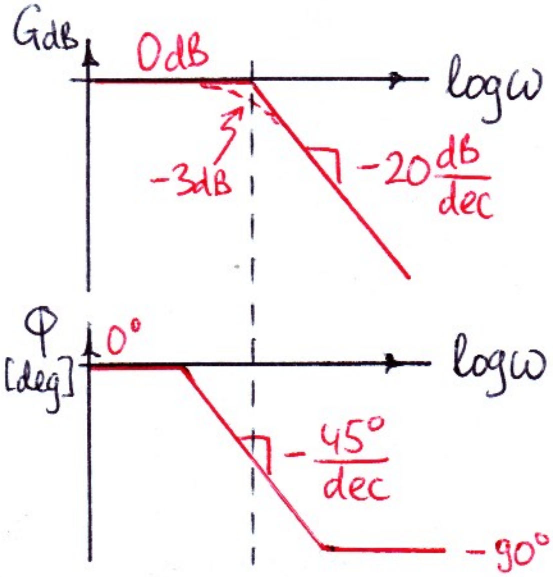


$\omega_c \equiv \dfrac{1}{RC}$ "corner" freq

$$\begin{cases} \omega \ll \omega_c, & G \approx 1 \\ \omega \gg \omega_c, & G \approx \dfrac{\omega_c}{j\omega} \end{cases}$$

Q: phase? $\begin{cases} \omega \ll \omega_c & 0^\circ \\ \omega \gg \omega_c & -90^\circ \end{cases}$

Usually draw uncorrected Bode plots.
use dB for $|G|$ and deg. for φ



To make Bode plots:
mentally sweep through all freq.

$G(\omega_c) = \dfrac{1}{1+j}$, =>

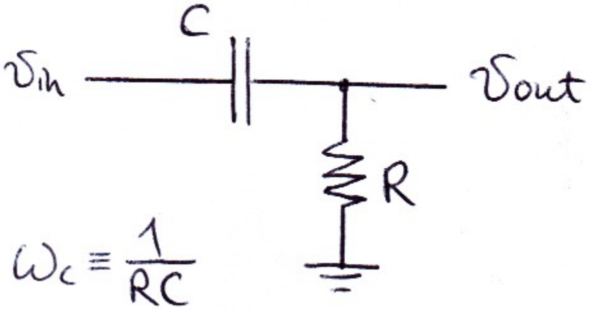
$|G(\omega_c)| = \dfrac{1}{\sqrt{2}}$ or -3dB

$\varphi[G(\omega_c)] = -45^\circ$

decade: $\dfrac{\omega_2}{\omega_1} = 10$ octave: $\dfrac{\omega_2}{\omega_1} = 2$

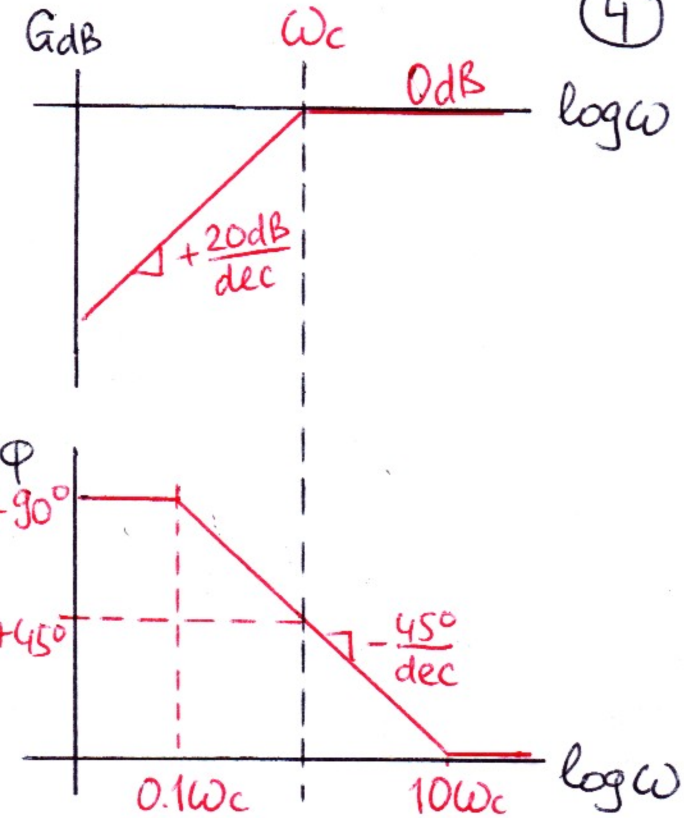
20 dB/dec = 6dB/oct

Hi-pass filter



$\omega_c \equiv \frac{1}{RC}$

$$G(\omega) = \frac{Z_R}{Z_R + Z_c} = \frac{j\omega/\omega_c}{1 + j\omega/\omega_c}$$



Note: Bode plots ⇒ always two

$$G(\omega) = |G(\omega)|\, e^{j\varphi[G(\omega)]}$$

$$\ln G(\omega) = \underbrace{\ln|G(\omega)|}_{\text{real}} + j\underbrace{\varphi[G(\omega)]}_{\text{imag}}$$

- magnitude & phase are not independent
- e.g. change in magn. requires change in phase (a.k.a. Kramers–Kronig relation)

Standard (zero-pole) form of G

$$G(\omega) = \frac{N(\omega)}{D(\omega)} = K(j\omega)^n \frac{(1+j\omega/\omega_{z_1})(1+j\omega/\omega_{z_2})\ldots}{(1+j\omega/\omega_{p_1})(1+j\omega/\omega_{p_2})\ldots}$$

$N(\omega \to j\omega_z) \to 0$, $|G| \to 0$ called *zero*

$D(\omega \to j\omega_p) \to 0$, $|G| \to \infty$ called *pole*