USPAS Course on
Recirculated and Energy Recovered Linacs

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Injector I: Parameters, Emission, and Beam Dynamics I

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Introduction

With a short-lived (≤ μs) electron life-cycle in R&ERLs, the emphasis for delivering the beam of appropriate quality for a given application shifts almost entirely on the injector.

One way to define an injector: a part of the R&ERL up to (and including) the merge with the returning high-energy beam.

The injector determines many of the key properties of the beam (can be degraded downstream, but never improved!)

- beam current & timing structure
- horizontal & longitudinal emittances
- polarized & magnetized beam

Interface of different disciplines

Since R&ERLs can be applied to a host of different applications, each with its own parameter set from the injector, no single injector design or approach can be adequate.

<u>Thermal</u> Emission <u>Photo</u>emission <u>Field</u> Emission

The injector art is at the interface of various disciplines & techniques:

- solid state & material science (cathode physics)
- lasers (photoemission)
- single species plasma (space charge, Debye length, plasma frequency)
- high field guns (part of the injector where beam is born)

+ 'usual' accelerator physics

Parameter space

R&ERL applications

- Light sources
  - Spontaneous
  - Free electron lasers
- Nuclear physics (CEBAF)
- Electron cooling of ions
- Electron-ion collider

Beam parameters vary from sub-pC to several nC charge / bunch, less than mA to Ampere average current, may have 'special' requirements: polarization and magnetized beam

FELs

$\lambda = \lambda_p\,(1+K^2/2)\,/\,2\gamma^2$

typical $\lambda_p \geq 3$ cm

Pierce parameter $$\rho = \left[\frac{K^2[JJ]r_e n_e \lambda_p^2}{32\pi\gamma^3}\right]^{1/3}$$

$\varepsilon_{x,y} = \lambda/4\pi$ $\Delta E/E = 1/4N_p$ I_{peak}

N_p, Gain

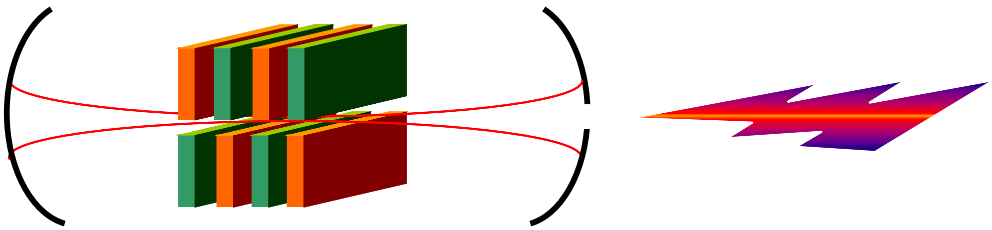
Oscillator
HGHG
SASE

$\varepsilon_{x,y} = \varepsilon_n\,/\,\gamma$, q, σ_z

FELs (contd.)

$\varepsilon_{x,y} = \lambda/4\pi \qquad \Delta E/E = 1/4N_p \qquad I_{peak}$

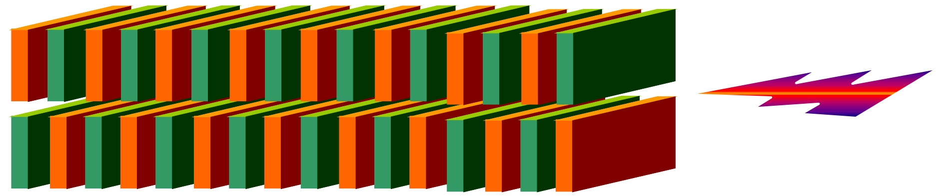
Low Gain



Longer wavelength → high reflectivity mirrors → oscillator configuration → short undulator

$\varepsilon_n \sim 10\ \mu\text{m},\ E \leq 100\ \text{MeV}$
$q \sim 0.1\ \text{nC}\ \text{ for } \lambda \sim 1\mu\text{m}$

High Gain



Shorter wavelength (XUV and down) → high gain → long undulator → more stringent specs

$\varepsilon_n \sim 1\ \mu\text{m},\ E > \text{GeV}$
$q \sim 1\ \text{nC}\ \text{ for } \lambda \sim 10\ \text{nm}$

For high power FELs $I \sim 0.1\text{-}1\ \text{A}$

Spontaneous emission ERL light source

3rd GLS storage rings, workhorse of X-ray science, are spontaneous emission sources (photons produced = $N_e \times$ single electron radiation). Various figures of merit can be used to characterize the source, e.g.

Flux [ph/s/0.1%bw]

$\propto I$

Brilliance [ph/s/0.1%/mm²/mr²]

$\propto I/(\varepsilon_x \oplus \lambda/4\pi)(\varepsilon_y \oplus \lambda/4\pi)$

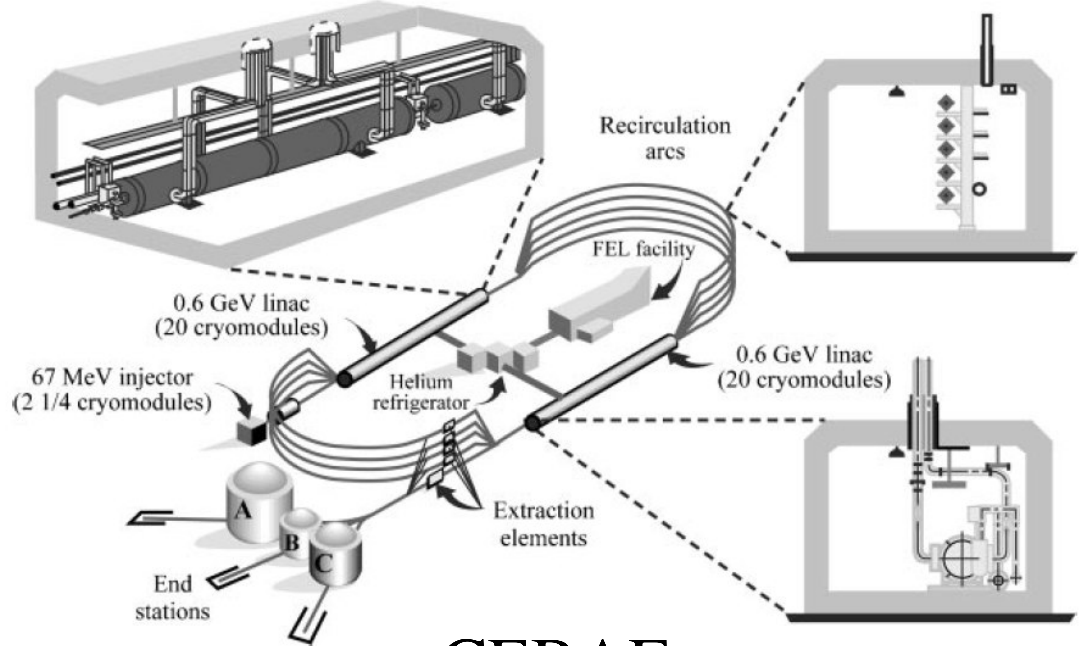
Improving flux beyond what is possible with a storage ring is unlikely, but brilliance can be improved (presently best values 10^{20}–10^{21} ph/s/0.1%/mm²/mr²).

$\varepsilon_n \sim 0.6$ μm, $q \sim 0.08$ nC, $I \sim 100$ mA, $E \sim 5$ GeV
for brilliance of $\sim 10^{22}$ ph/s/0.1%/mm²/mr²

Nuclear physics

Electrons are a probe.

E = 6 GeV
I = 200 μA
q < 0.3 pC
$\Delta E/E = 2.5 \times 10^{-5}$
Polarization > 75 %



CEBAF

Polarization is given by $$\frac{I(\uparrow) - I(\downarrow)}{I(\uparrow) + I(\downarrow)}$$

where I(↑) is the number of electrons with spin 'up' and I(↓) is the number of electrons with spin 'down' along a given axis.

Electron cooling of ions

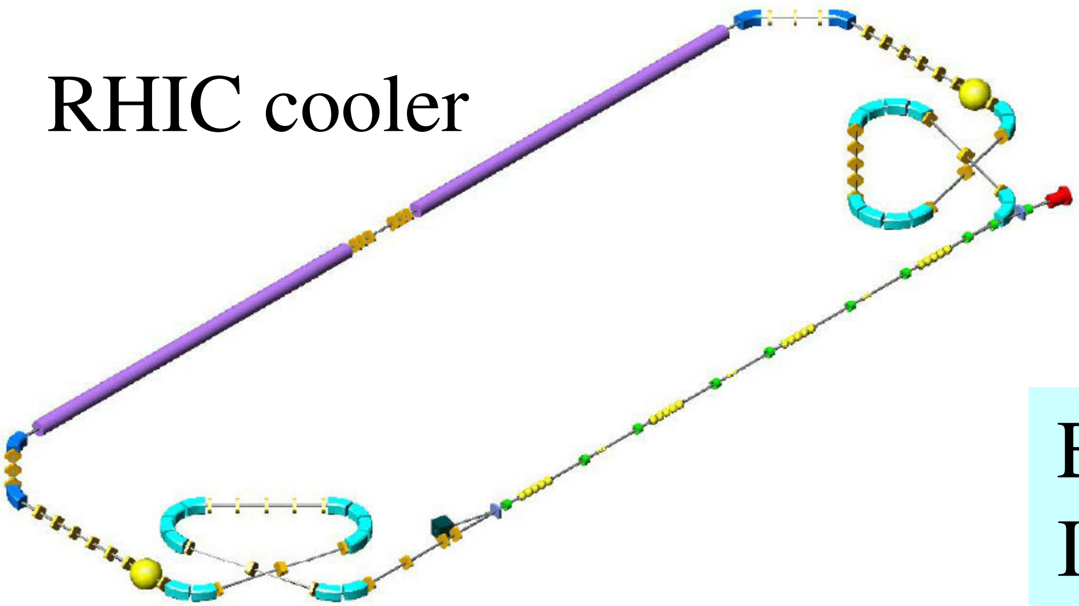
There is no radiative cooling in ion storage rings due to their large mass. One way to reduce ion's phase space volume is to co-propagate ions with a bunch of cold electrons (same $\beta = v/c$). In the rest frame, the picture resembles two-species plasma with different temperatures $T_{ion} > T_{electron}$ (interaction will heat up electrons and cool down ions).

High density of electrons is required for tolerable cooling rates.

Interaction takes place in a long focusing solenoid. To avoid enlarged 2D emittance a so-called 'magnetized' beam is needed from the source. We will discuss magnetized beam and Busch's theorem later in this lecture.

Electron cooling of ions (contd.)

RHIC cooler



$E = 55$ GeV
$I = 200$ mA
$q = 20$ nC
$\Delta E/E = 3 \times 10^{-4}$
magnetized beam

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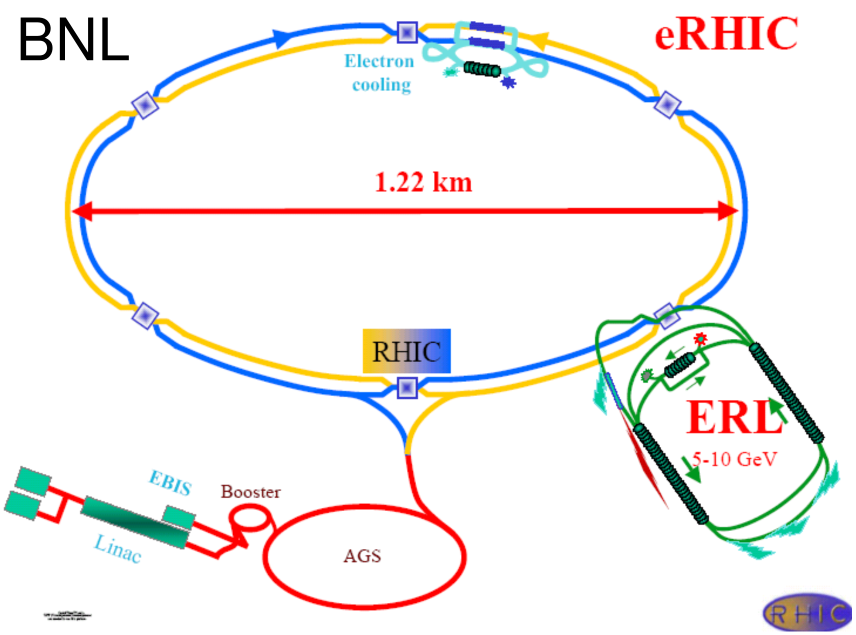
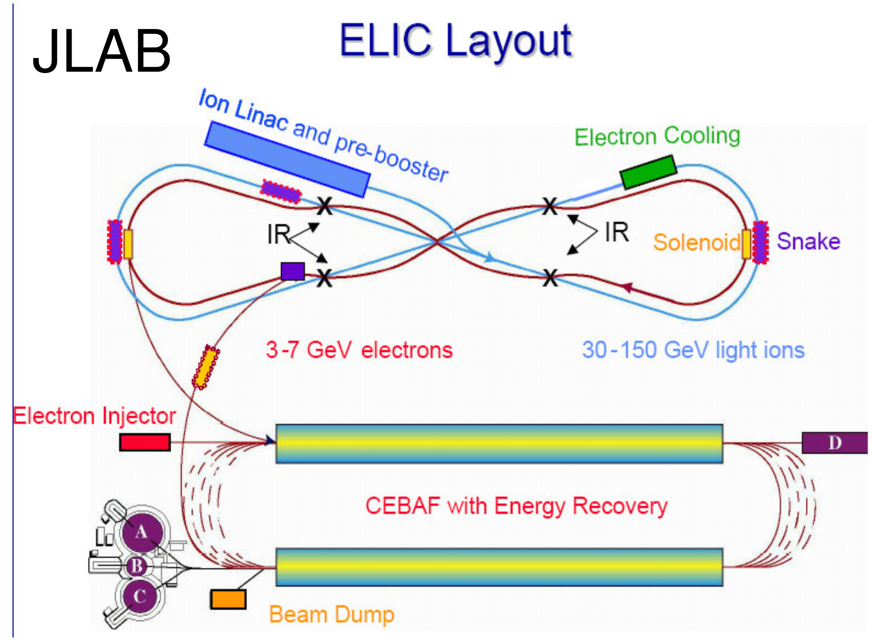
Electron-Ion collider





E = 2-10 GeV I ~ 100s mA ε_n ~ 10s mm-mrad

polarized electrons from the gun

Injector needs (bird's eye view)

The underlying feature of present and future R&ERLs is CW operation → high rep. rate, high current

FELs: need a beam close to the diffraction limit → emittance; need high peak current to lase (kA) → charge per bunch, longitudinal emittance for low final/injection energy ratio

ERL spontaneous LS: beam brightness is paramount → $I/\varepsilon_x\varepsilon_y$; medium to low charge per bunch preferred

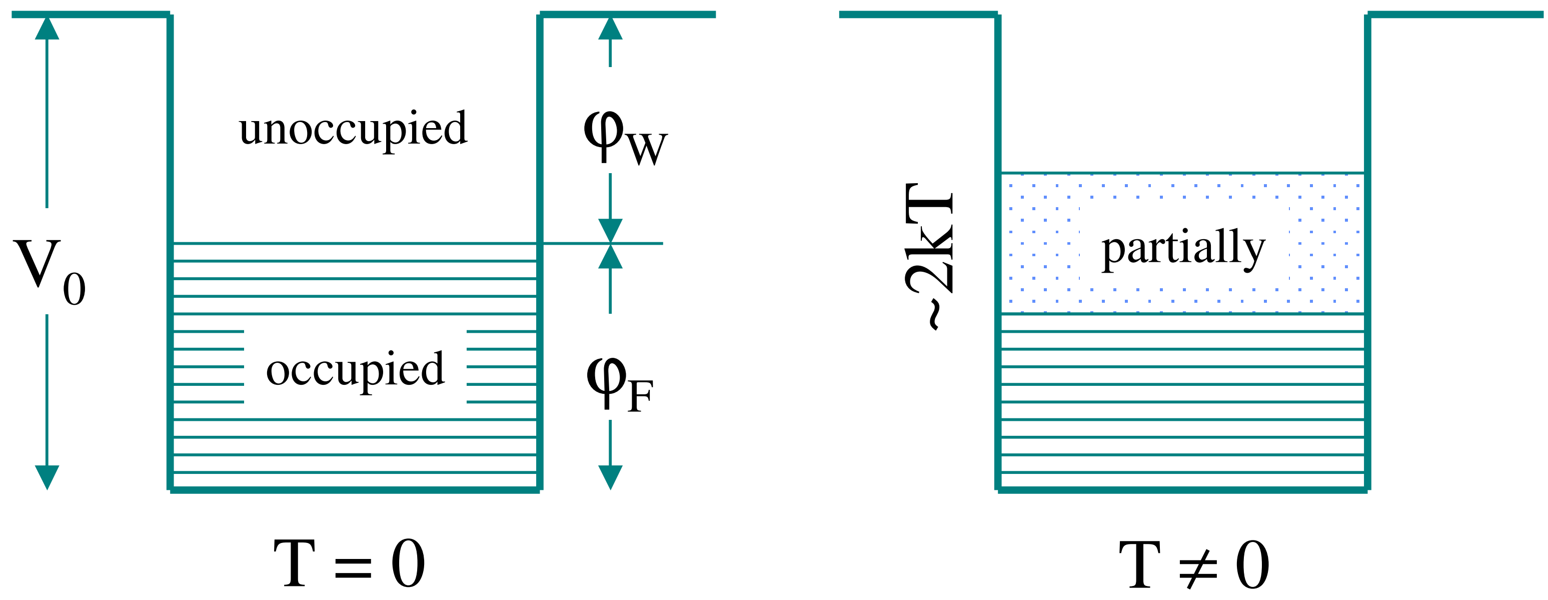
Electron cooling: high bunch charge (~10 nC) and current (0.1-1 Ampere); magnetized beam

Electron-ion collider: polarized source with high average current

Cathode figures of merit

- available current density
- energy distribution of emitted electrons $\varepsilon_{n,th} = \sigma_{\perp}\sqrt{\frac{E_{th}}{mc^2}}$
- lifetime (resistance to contamination)
- response time

Emission: thermal in metals



Richardson-Dushman equation (1923)

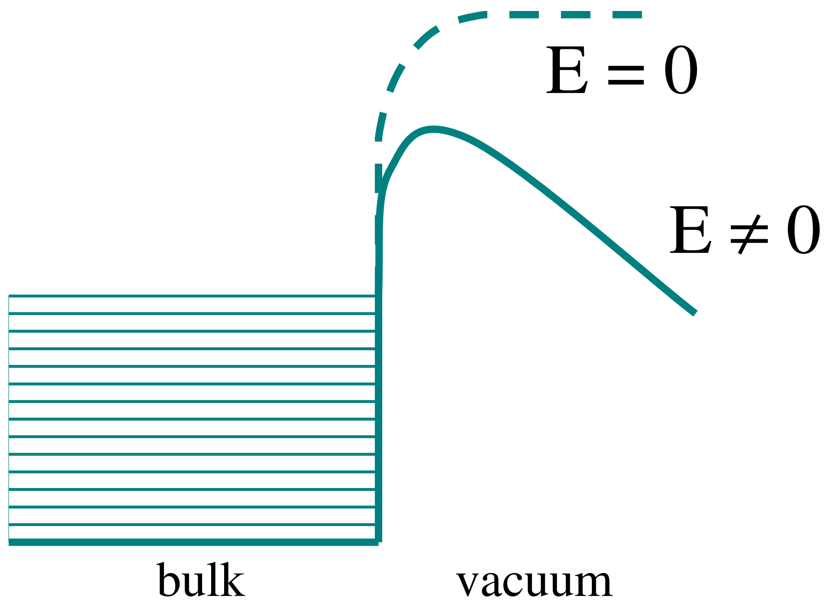
current density $J = AT^2 \exp\left[-\frac{e\varphi_W}{kT}\right]$

$A = 120\ \mathrm{A/(cm^2K^2)}$

$\varphi_W = 4.5$ V (tungsten)

$\varphi_W = 2.0$ V (dispenser)

Emission: Schottky correction



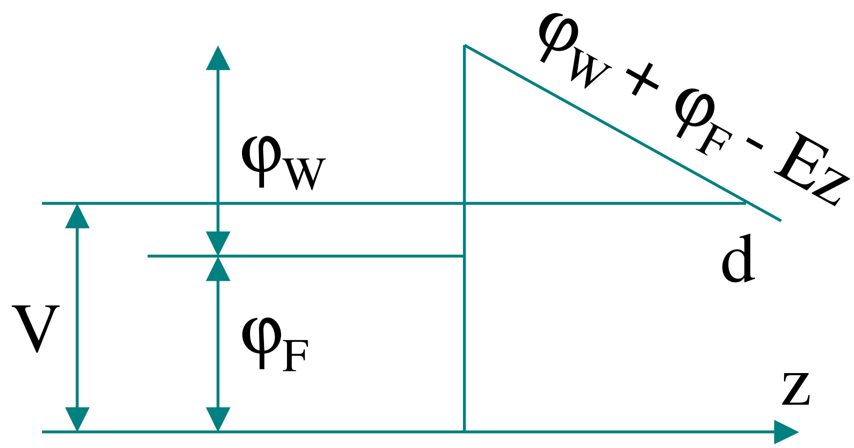
$$J = AT^2 \exp\left[-\frac{e\varphi_W^*}{kT}\right]$$

External field lowers work function $\varphi_W^* \rightarrow \varphi_W - \Delta\varphi$

Schottky correction $\Delta\varphi = \sqrt{\frac{eE}{4\pi\varepsilon_0}}$ $\quad \Delta\varphi[\mathrm{V}] = 0.038\sqrt{E[\mathrm{MV/m}]}$

Emission: Fowler-Nordheim tunneling

Schottky correction does not include probability of escaping from a finite width potential barrier that follows from quantum mechanics. In 1928 Fowler and Nordheim have published a paper where they derived current density from this effect.



$$J_z = \int_0^\infty e P v_z n(v) dv$$

$$P \propto \exp\left[-\frac{4}{3}\sqrt{\frac{2m}{\hbar^2}}\frac{(e\varphi_W + e\varphi_F - W_z)^{3/2}}{eE} \right]$$

(A/cm²)

$$J_z = 6.2\times 10^{-6}\,\frac{\sqrt{\varphi_F/\varphi_W}}{\varphi_F+\varphi_W}\,E^2 \exp\left[-6.8\times 10^7\,\frac{\varphi_W^{3/2}}{E} \right]$$

(V) (V/cm)

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Emission: field enhancement

Interestingly, F-N dependence of field emission with E is the same as the functional R-D dependence of thermal emission with T.

In reality, field emission fits F-N dependence if E $\rightarrow$ Eβ. β is known as an enhancement factor that translates macroscopic field E to a local value (local geometry dependant).

In summary,

$$J_{thermal} = AT^2 \exp\left[-\frac{e\varphi_W}{kT}\right]$$

$$J_{Schottky} = AT^2 \exp\left[-\frac{e(\varphi_W - B\sqrt{\beta E})}{kT}\right]$$

$$J_{field} = C\beta^2 E^2 \exp\left[-\frac{D}{\beta E}\right]$$

Child-Langmuir limit

So far we have discussed current density available from a cathode.

Child-Langmuir law specifies maximum current density for a space-charge limited, nonrelativistic, 1-D beam *regardless* of available current density from the cathode. The law has a limited applicability to R&ERLs guns (applies to continuous flow, ~100 kV DC guns), but provides an interesting insight (and a home problem).

$$J = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}}{d^2} \qquad J[\mathrm{A/cm^2}] = 2.33E^{3/2}[\mathrm{MV/m}]/\sqrt{d[\mathrm{cm}]}$$

E is field in a planar diode in the absence of the beam: E = V/d

Emission: photoeffect

Photoemission offers several advantages over thermal (field) emission:

- Higher current density
- Bunched beam is generated through a laser with appropriate time structure. No need for chopping / extensive bunching as in the case of thermal emission (shortest pulse available through a grid pulser is ~ ns, while L-band RF needs bunches ~10 ps long – readily available from photocathodes).
- Colder beam (lower thermal emittance) is possible
- Polarized electrons from special photocathodes

Photocurrent

$$I[\text{mA}] = \frac{\lambda[\text{nm}]}{124} \times P_{laser}[\text{W}] \times \eta[\%] \quad \leftarrow \quad \frac{N_e}{N_{ph}}$$

For *metals* such as copper work function is 4.5 eV → UV light is required (frequency multiplication in nonlinear medium crystals)

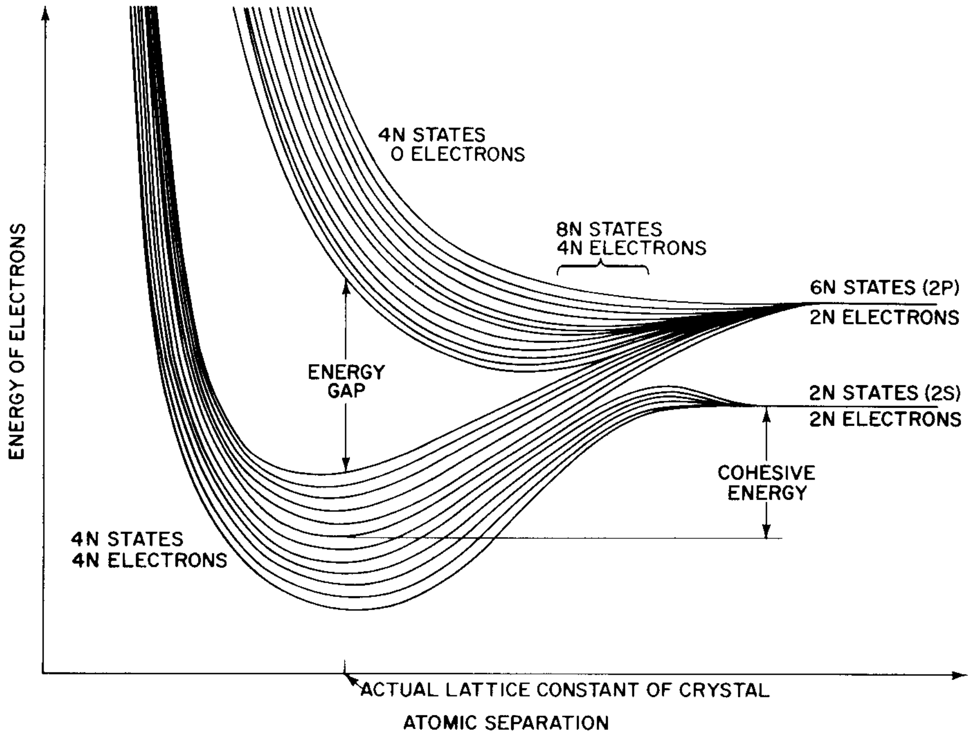
Furthermore, η is low (< 10^{-3}) due to electron-electron scattering in conduction band (1/2 energy lost per collision on average)

Metals have very fast response time (< ps)

Thermal emittance numbers vary, values quoted for thermal energy E_{th} = 0.2-0.7 eV

In short, metals (Cu, Mg, etc.) are suitable for pulsed applications, but is a poor choice for a high average current gun

Photoemission: semiconductors



Energy banding of allowed levels in diamond as a function of spacing between atoms

Gap and availability of electrons in conduction band determines whether material is

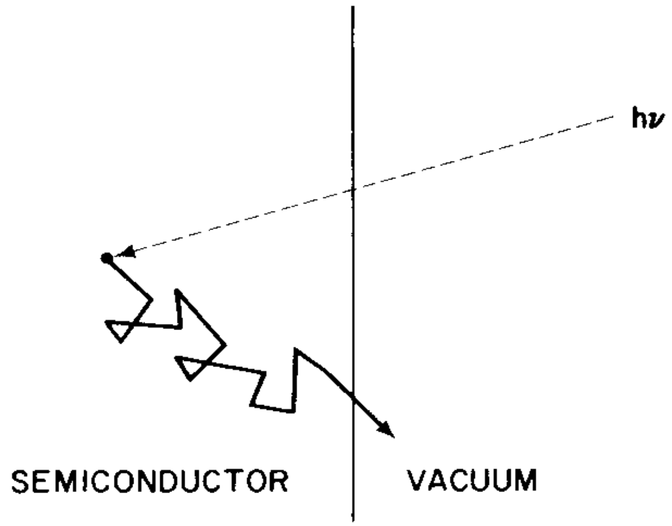
Metal: $n_e \sim 10^{23}$ cm^{-3}

Semiconductor: gap < 3 eV; n_e (n_h) < 10^{20} cm^{-3}

Insulator: gap > 3 eV; negligible n_e and n_h

Good quantum efficiency (10s %) available from semiconductor photocathodes (CsKSb, Cs2Te, GaAs) & lower photon energy due to a smaller gap

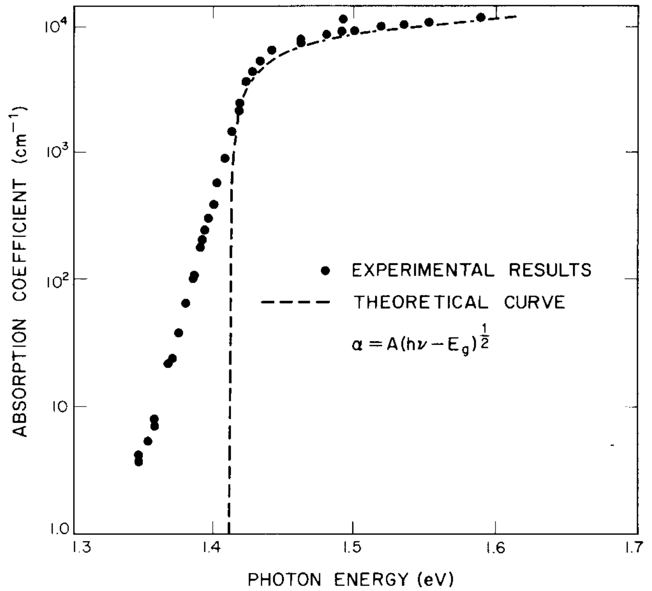
Photoemission process in semiconductors



(1) photon excites electron to a higher-energy state;

(2) electron-phonon scattering (~0.05 eV lost per collision);

(3) escape with kinetic energy in excess to E_{vac}



Absorption edge of GaAs at room temperature.

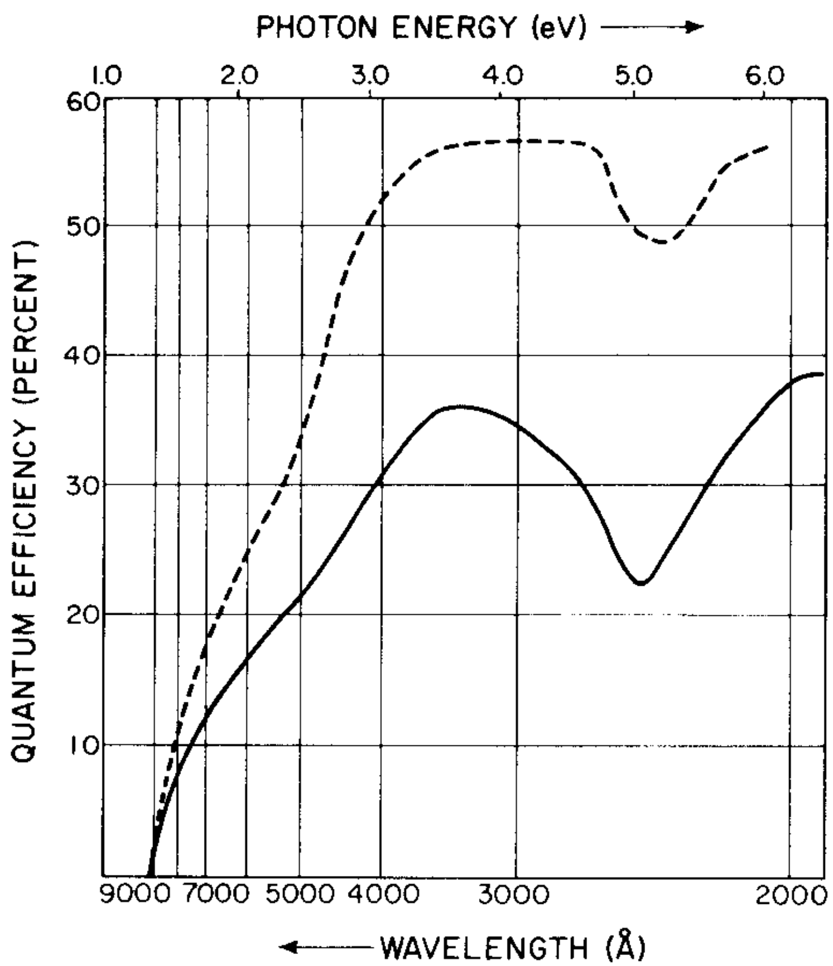
In GaAs the escape depth is sufficiently long so that photo-excited electrons are *thermalized* to the bottom of the conduction band before they escape.

Response time ~ (10^{-4} cm)/(10^{7} cm/s) = 10 ps (wavelength dependant)

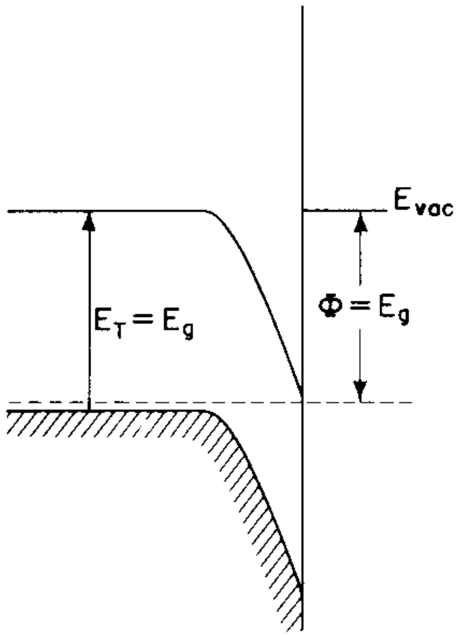
Photoemission: negative electron affinity

Surface condition induces a space charge, which may bend the bands either up or down (bottom of conduction band in the bulk relative to E_{vac} is called electron affinity).

If thickness of a low φ_W material << mean free path → e^- can traverse the surface material without much loss → better quantum efficiency / reduced threshold



Cs:GaAs. Dashed line – Q.E. per absorbed photon



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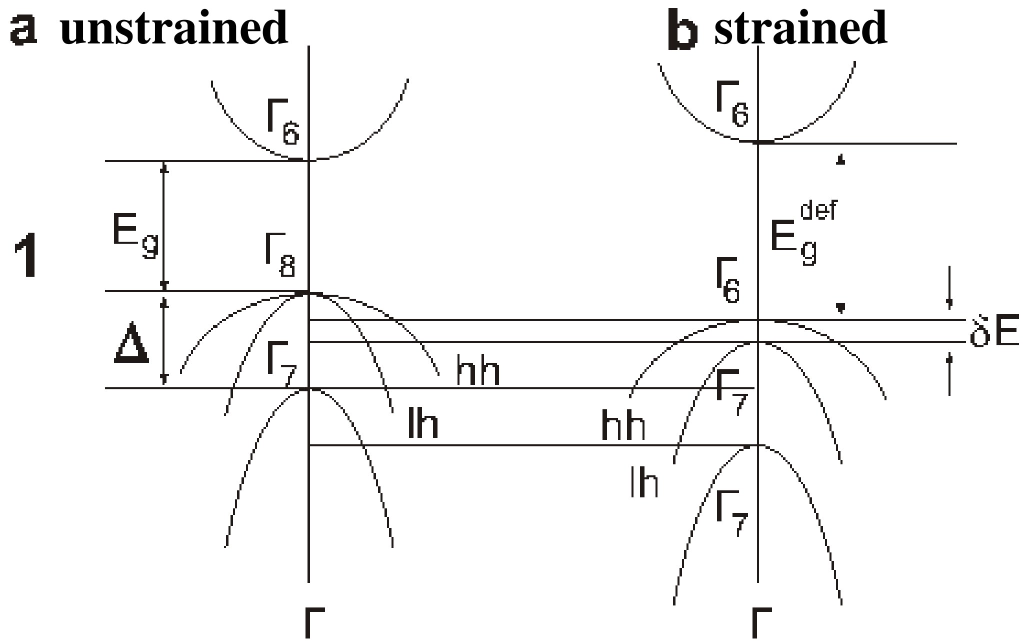
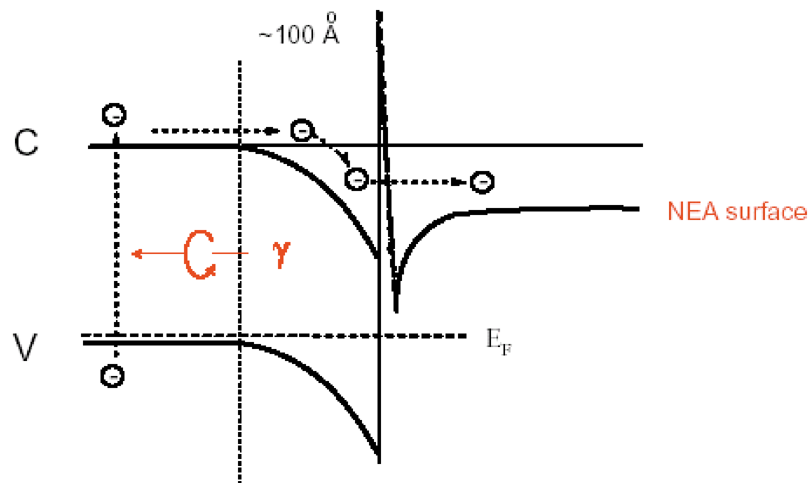
Photoemission: polarized electrons

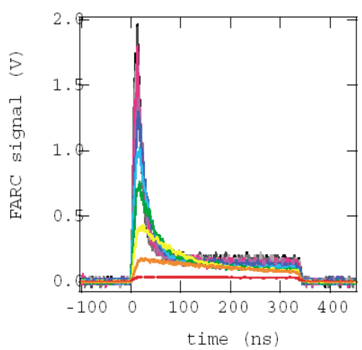


In strained GaAs, spin degeneracy of two states ($\Gamma_{6,7}$) is removed.

Circularly polarized light of the right wavelength produces polarized electrons (> 80% polarization measured). Low QE.



Doping is important to increase carrier density to avoid a so-called surface charge limit.



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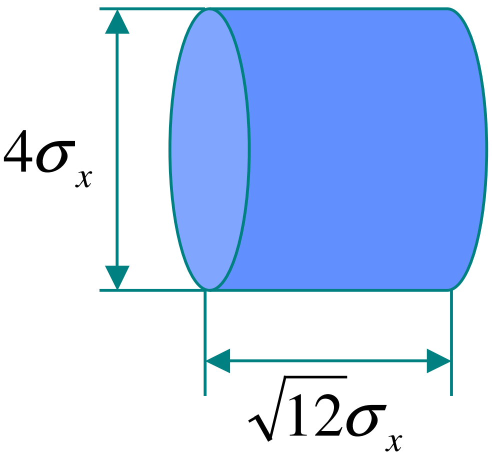
Bunch charge limit in guns

Let's estimate bunch charge limit of a short pulse in a gun.

Assume 'beer-can' with rms $\sigma_{x,y}$ σ_t



also that E_{cath} does not change much over the bunch duration (usually true for photoguns)

If $\dfrac{eE_{cath}\times(c\sigma_t)}{mc^2} << 1$ or $\dfrac{E_{cath}[\mathrm{MV/m}]\times(c\sigma_t)[\mathrm{mm}]}{511} << 1$

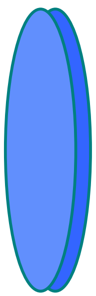
motion during emission stays nonrelativistic.

Aspect ratio of emitted electrons near the cathode after the laser pulse has expired:

$$A = \frac{\perp}{\parallel} = \frac{2\sigma_x}{3(c\sigma_t)}\frac{mc^2}{eE_{cath}(c\sigma_t)} = \frac{341}{E_{cath}[\mathrm{MV/m}]}\frac{\sigma_x[\mathrm{mm}]}{(c\sigma_t[\mathrm{mm}])^2}$$

Bunch charge limit in guns (contd.)

More often than not A >> 1 in photoinjectors, i.e. the bunch looks like a pancake near the cathode (!).



From PHYS101 (note a factor of 2 due to image charge)

$$E_{s.c.} = \frac{\sigma}{\varepsilon_0} \rightarrow \begin{aligned} q &= 4\pi\varepsilon_0 E_{cath}\sigma_x^2 \\ &= 0.11 \times E_{cath}[\mathrm{MV/m}]\sigma_x[\mathrm{mm}]^2 \ \mathrm{nC} \end{aligned}$$

Couple of points:

- shorter laser pulse does not harm (though the shortest one can go may be limited by photoemission response time)
- **if** emittance is dominated by thermal energy of emitted electrons, the following scaling applies (min possible emit.)

$$\varepsilon_n[\mathrm{mm\text{-}mrad}] \geq 4\sqrt{q[\mathrm{nC}]E_{th}[\mathrm{eV}]/E_{cath}[\mathrm{MV/m}]}$$

Beam dynamics

Beam dynamics without collective forces is simple.

$$\vec{\nabla}\cdot\vec{B}=0 \quad \vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}+\vec{J} \quad \vec{\nabla}\cdot\vec{D}=\rho \quad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

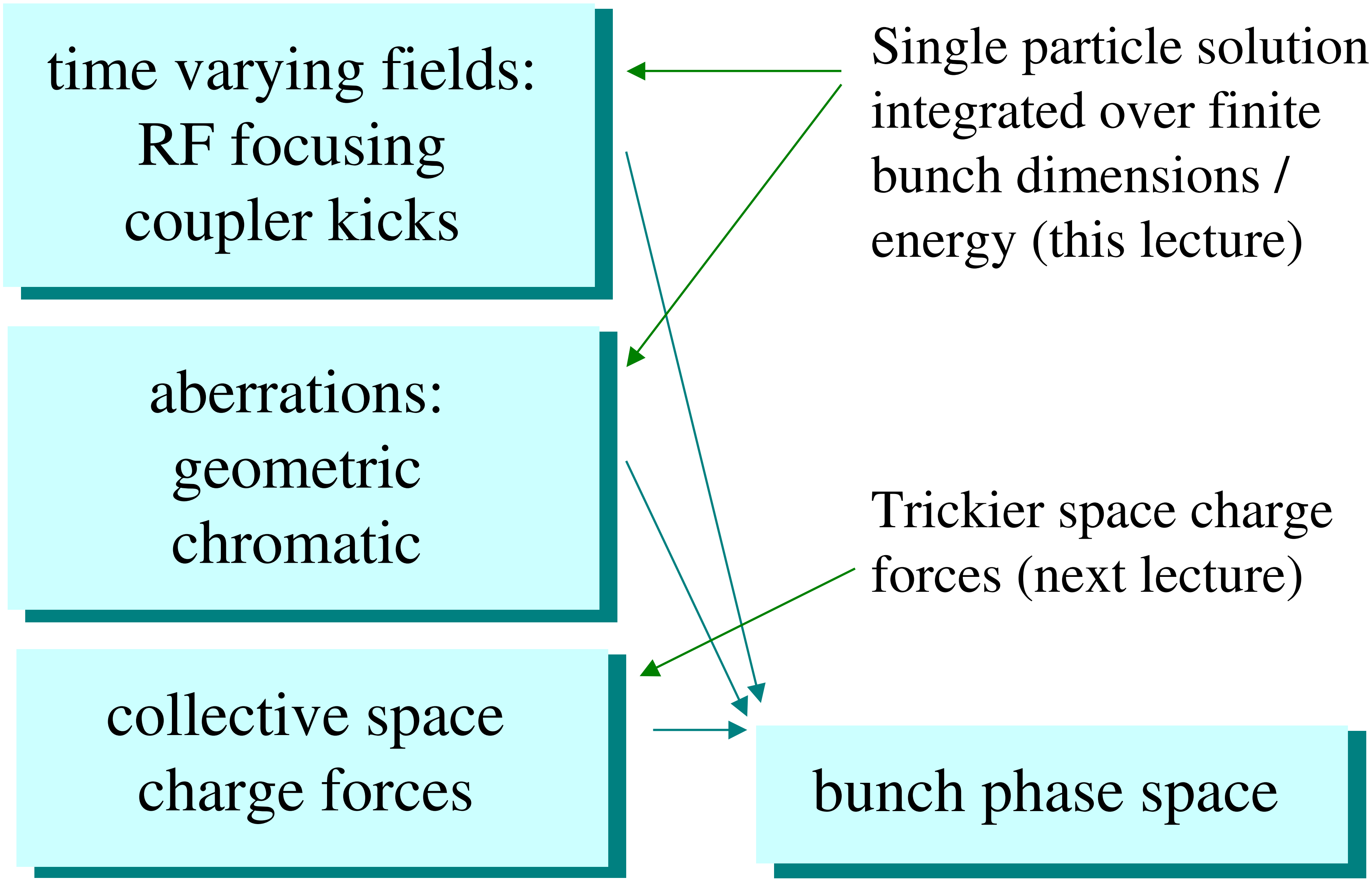
$$\vec{B}=\mu\vec{H}$$

$$\vec{D}=\varepsilon\vec{E} \qquad \frac{d}{dt}(\gamma m\vec{\beta}c)=e(\vec{E}+\vec{v}\times\vec{B})$$

$$\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$$

Calculating orbits in known fields is a single particle problem.

Effects on the phase space (emittance)



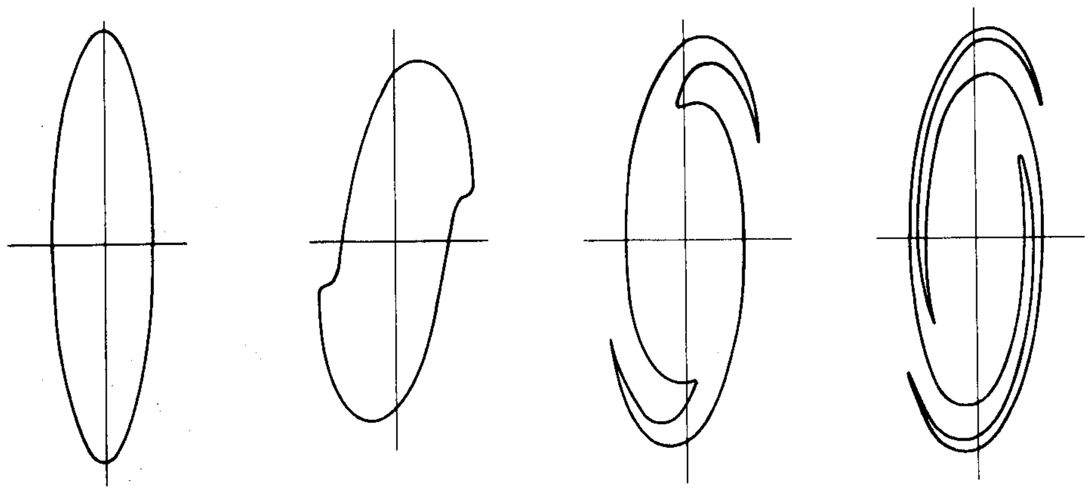
Emittance

Since emittance is such a central concept / parameter in the accelerator physics, it warrants few comments.

For Hamiltonian systems, the phase space density is conserved (a.k.a. Liouville's theorem). Rms (normalized) emittance most often quoted in accelerators' field is based on the same concept and defined as following [and similarly for (y, p_y) or (E, t)]

$$\varepsilon_{n,x} = \frac{1}{mc}\sqrt{\langle x^2\rangle\langle p_x^2\rangle - \langle xp_x\rangle^2} = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2} = \beta\gamma\varepsilon_x$$

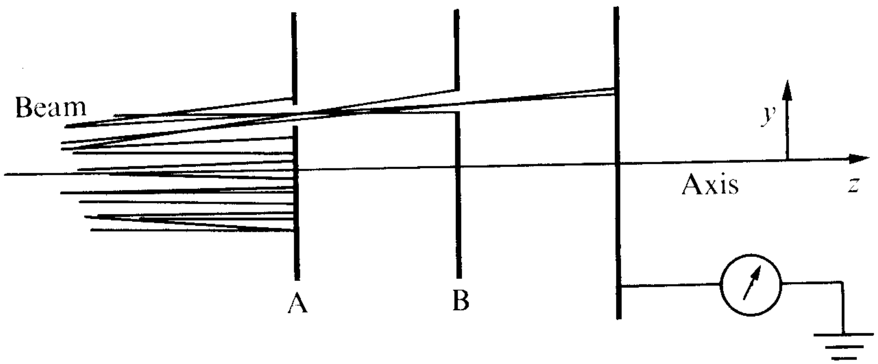
Strictly speaking, this quantity is not what Lioville's theorem refers to, i.e. it does not have to be conserved in Hamiltonian systems (e.g. geometric aberrations 'twist' phase space, increasing *effective* area, while *actual* phase space area remains constant). *Rms emittance is conserved for linear optics (and no coupling) only*.



Emittance measurement

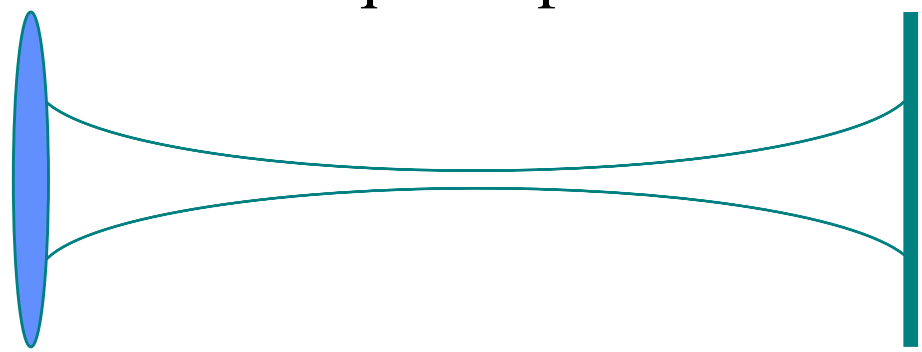
Usefulness of rms emittance: it enters envelope equations & can be readily measured, but it provides *limited* info about the beam.

slits / pinholes



The combination of two slits give position and divergence → direct emittance measurement. Applicable for space charge dominated beams (if slits are small enough).

lens / quadrupole scan



We'll see later that envelope equation in drift is

$$\sigma'' \approx \cancel{\frac{I}{2I_0}\frac{1}{\gamma^3\sigma}} + \frac{\varepsilon^2}{\sigma^3}$$

Vary lens strength and measure size to fit in eqn with 3 unknowns to find ε.

OK to use if $\sigma \ll \varepsilon_n \sqrt{2\gamma I_0 / I}$

Busch's theorem

Consider axially symmetric magnetic field

$$F_\theta = -e(\dot{r}B_z - \dot{z}B_r) = \frac{1}{r}\frac{d}{dt}(\gamma m r^2 \dot{\theta})$$

Flux through a circle centered on the axis and passing through *e*

$$\Psi = \int_0^r 2\pi r B_z dr$$

When particle moves from (r,z) to (r+dr, z+dz) from $\vec{\nabla}\cdot\vec{B} = 0$

$$\dot{\theta} = 0$$

$$\frac{d\Psi}{dt} = 2\pi r(\dot{r}B_z - \dot{z}B_r) \Rightarrow \boxed{\dot{\theta} = \frac{-e}{2\pi\gamma m r^2}(\Psi - \Psi_0)}$$

Busch's theorem simply states that canonical angular momentum is conserved

$$P_\theta = erA_\theta + \gamma m r^2 \dot{\theta} \quad (\Psi \to 2\pi r A_\theta, \Psi_0 \to 2\pi P_\theta / e \to \text{get Busch's formula})$$

Magnetized beam (immersed cathodes)

If magnetic field $B_z \neq 0$ at the cathode, the bunch acquires angular velocity

$$\dot{\theta} = -\frac{eB_z}{2\gamma m} \quad \rightarrow \quad \sigma_{p_\perp} = \gamma m \sigma_{x,y} \dot{\theta}$$

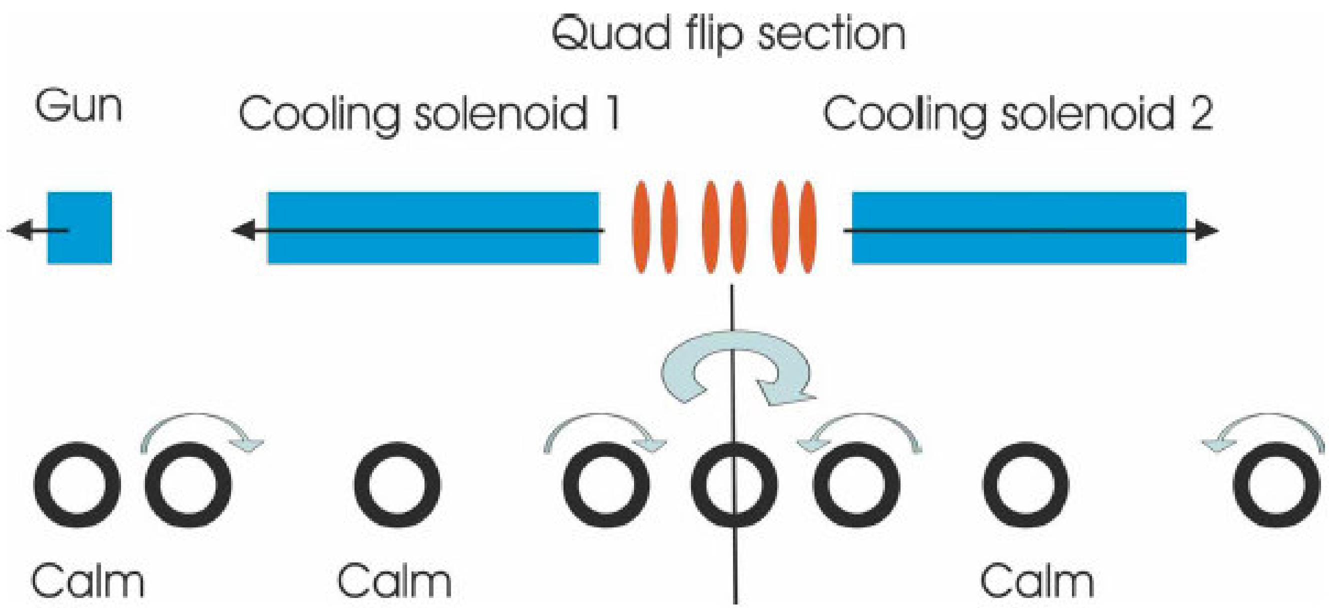
$$\varepsilon_{n,mag} \sim \frac{\sigma_{p_\perp}}{mc} \sigma_x \sim \frac{eB_0}{2mc} \sigma_x^2$$

$$\varepsilon_{n,mag} [\mathrm{mm\text{-}mrad}] \sim 0.3 B[\mathrm{mT}] \sigma_x [\mathrm{mm}]^2$$

Normally, magnetic field at the cathode is a nuisance. However, it is useful for a) magnetized beams; b) round to flat beam transformation.

Magnetized beam (example: RHIC cooler)

Similarly, rms emittance inside a solenoid is increased due to Busch's theorem. This usually does not pose a problem (it goes down again) except when the beam is used in the sections with non-zero longitudinal magnetic field. In the latter case, producing magnetized beam from the gun becomes important.



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Paraxial ray equation

Paraxial ray equation is equation of 'about'-axis motion (angle with the axis small & only first terms in off-axis field expansion are included).

$$\frac{d}{dt}(\gamma m\dot{r})-\gamma m r\dot{\theta}^2=e(E_r+r\dot{\theta}B_z)$$

with $-\dot{\theta}=\frac{q}{2\gamma m}\left(B_z-\frac{\Psi_0}{\pi r^2}\right)$ and $\dot{\gamma}=\beta e E_z/mc$

$$\ddot{r}+\frac{\beta e E_z}{\gamma mc}\dot{r}+\frac{e^2B_z^2}{4\gamma^2m^2}r-\frac{e^2\Psi_0^2}{4\pi^2\gamma^2m^2}\frac{1}{r^3}-\frac{eE_r}{\gamma m}=0$$

eliminating time and using $E_r\approx-\frac{1}{2}rE_z'=-\frac{1}{2}r\gamma'' mc^2/e$

$$\boxed{r''+\frac{\gamma' r'}{\beta^2\gamma}+\left(\frac{\gamma''}{2\beta^2\gamma}+\frac{\Omega_L^2}{\beta^2c^2}\right)r-\left(\frac{P_\theta}{\beta\gamma mc}\right)^2\frac{1}{r^3}=0}$$

$$P_\theta\equiv erA_\theta+\gamma m r^2\dot{\theta}$$

$$\Omega_L\equiv -eB_z/2\gamma m$$

$$\boxed{\theta_L=\int_0^z\Omega_L\frac{dz}{\beta c}}$$

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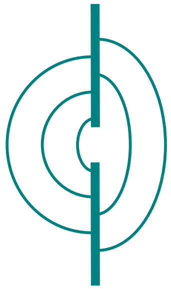
Focusing: electrostatic aperture and solenoid

With paraxial ray equation, the focal length can be determined

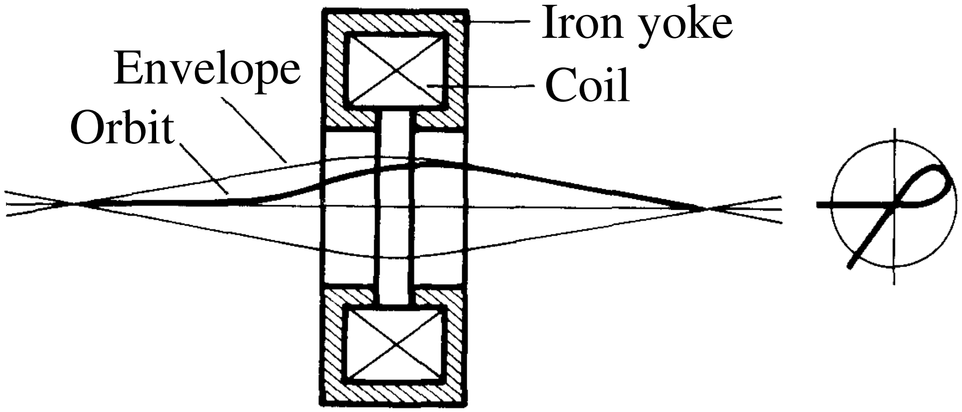
electrostatic aperture



$$F = 4V \frac{1+\frac{1}{2}eV/mc^2}{1+eV/mc^2}\frac{1}{E_2 - E_1}$$

eV is equal to beam K.E., E_1 and E_2 are electric fields before and after the aperture

solenoid



$$\frac{1}{F} = \int \left(\frac{\Omega_L}{\beta c}\right)^2 dz$$

$$= \int \left(\frac{eB_z}{2\beta\gamma mc}\right)^2 dz \approx \frac{1}{4}\left(\frac{e}{cp}\right)^2 B_z^2 L$$

$$cp/e\,[1\,\mathrm{MeV/c}] \rightarrow (B\rho)[33.4\,\mathrm{G\text{-}m}]$$

RF effect

SW longitudinal field in RF cavities requires transverse components from Maxwell's equations → cavity can impart tranverse momentum to the beam

Chambers (1965) and Rosenzweig & Serafini (1994) provide a fairly accurate (≥ 5 MeV) matrix for RF cavities (Phys. Rev. E **49** (1994) 1599 – beware, formula (13) has a mistypo)

Edges of the cavities do most of the focusing. For $\gamma >> 1$

$$\frac{1}{F} = -\frac{\gamma'}{\gamma_2}\left[\frac{\cos^2\varphi}{\sqrt{2}} + \frac{1}{\sqrt{8}}\right]\sin\alpha \qquad \text{with } \alpha = \frac{\ln(\gamma_2/\gamma_1)}{\sqrt{8}\cos\varphi}$$

γ_1, γ_2, γ', φ Lorentz factor before, after the cavity, cavity gradient and off-crest phase

On crest, and when $\Delta\gamma = \gamma' L << \gamma$:

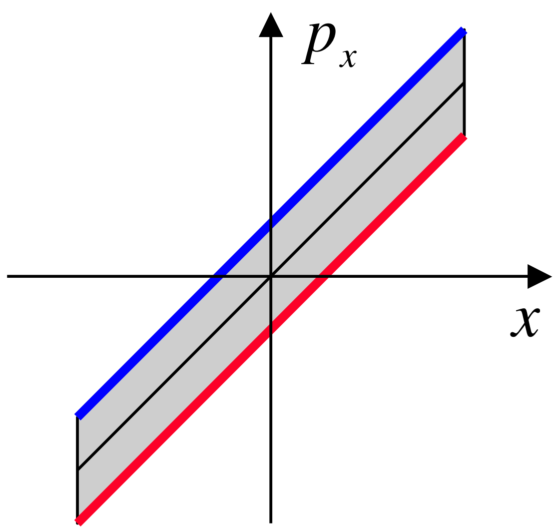
$$\frac{1}{F} \approx -\frac{3}{8}\frac{\gamma'^2 L}{\gamma^2}$$

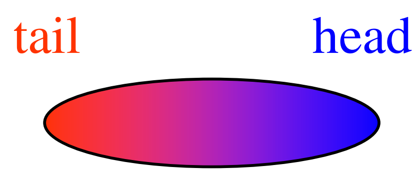
Emittance growth from RF focusing and kick

$$\varepsilon_n = \frac{1}{mc}\sqrt{\langle x^2\rangle\langle p_x^2\rangle - \langle xp_x\rangle^2}$$

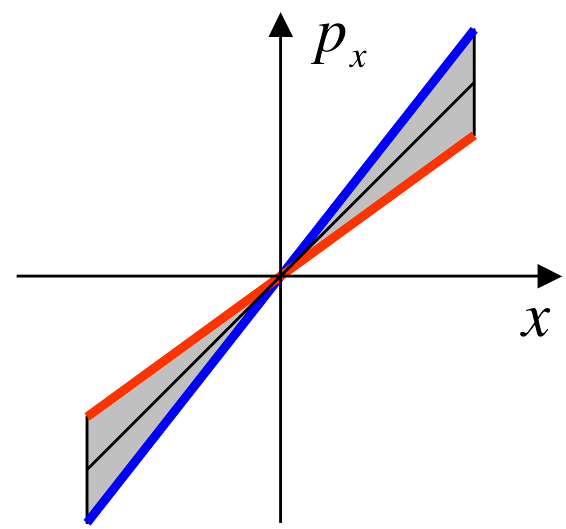
$$p_x(x,z) = p_x(0,0) + \frac{\partial p_x}{\partial x}x + \underbrace{\frac{\partial p_x}{\partial z}z}_{kick} + \underbrace{\frac{\partial^2 p_x}{\partial x \partial z}xz}_{focusing} + \ldots$$

kick





focusing



RF focusing and kick

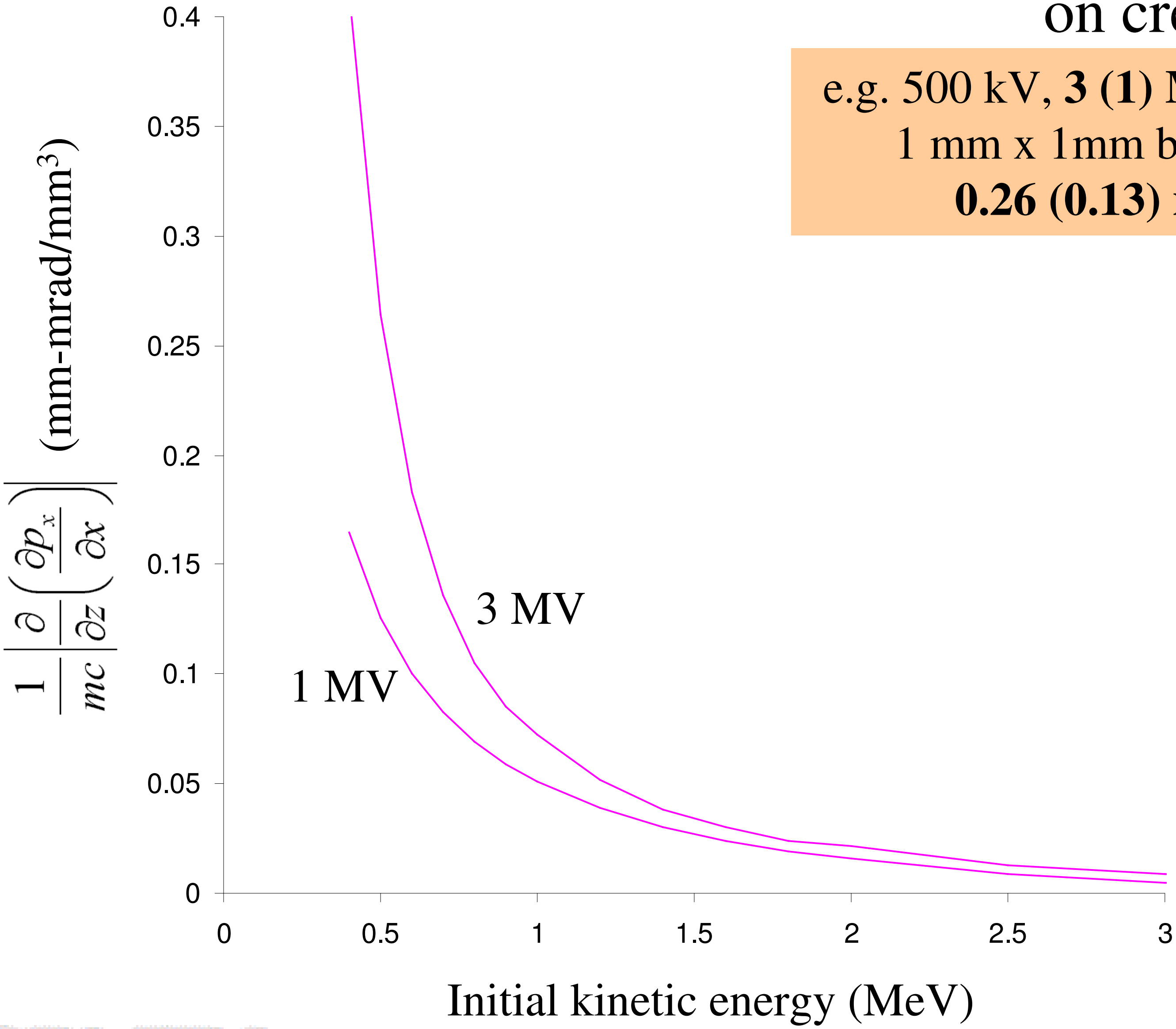
$$\varepsilon_n^2 = \varepsilon_0^2 + \varepsilon_{kick}^2 + \varepsilon_{focus}^2$$

- Kick effect on emittance is energy independent (modulo beam size) and can be cancelled downstream
- RF focusing effect scales $\propto \dfrac{1}{\gamma}$ (in terms of p_x) and generally is not cancelled

$$\varepsilon_{kick} = \frac{1}{mc}\left|\frac{\partial p_x}{\partial z}\right|\sigma_x \sigma_z$$

$$\varepsilon_{focus} = \frac{1}{mc}\left|\frac{\partial^2 p_x}{\partial z \partial x}\right|\sigma_x{}^2 \sigma_z$$

RF focusing in 2-cell Cornell injector cavity



on crest

e.g. 500 kV, **3 (1) MV** in the 1st cell,
1 mm x 1mm bunch receives
0.26 (0.13) mm-mrad

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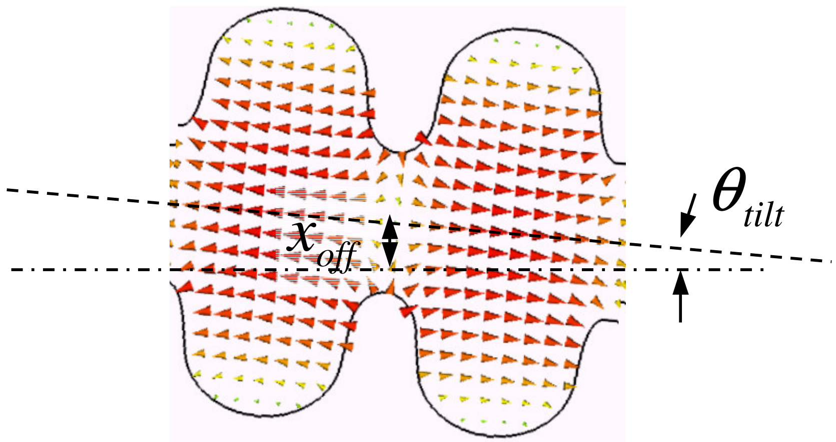
RF tilt and offset

$$\varepsilon_{kick} \approx \sigma_x \sigma_z \left[\underbrace{\theta_{tilt} \frac{\Delta E}{mc^2} k_{RF} \sin\varphi}_{\text{tilt}} + \underbrace{x_{off} \frac{1}{mc} \frac{\partial}{\partial z}\left(\frac{\partial p_x}{\partial x}\right)}_{\text{offset}} \right]$$

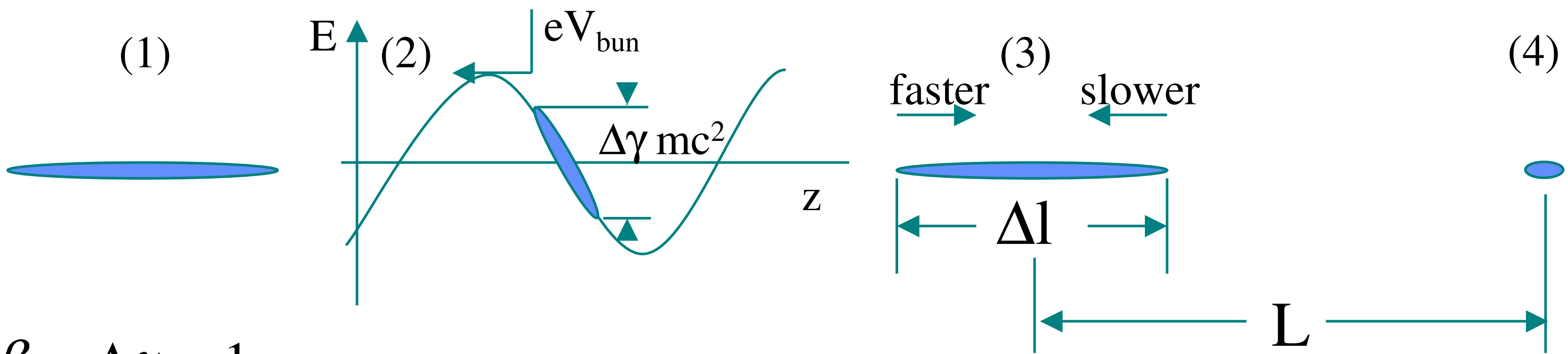
e.g. **3 MeV** energy gain
for 1 mm x 1mm yields
0.16 sin φ mm-mrad
per mrad of **tilt**



- One would prefer on-crest running in the injector (and elsewhere!) from tolerances' point of view

Drift bunching

For bunch compression, two approaches are used: magnetic compression (with lattice) and drift bunching. Magnetic compression relies on path vs. beam energy dependence, while drift bunching relies on velocity vs. energy dependence (i.e. it works only near the gun when $\gamma \geq 1$).



$$\frac{\Delta\beta}{\beta} \approx \frac{\Delta\gamma}{\gamma}\frac{1}{\gamma^2 - 1}$$

$$L = c\beta\frac{\Delta l}{c\Delta\beta} = \frac{\Delta l}{(\Delta\gamma/\gamma)}(\gamma^2 - 1) = \frac{\lambda_{RF}}{2\pi}\frac{E}{eV_{bun}}\left(\frac{E^2}{(mc^2)^2} - 1\right)$$

Problems

1) Thermal emission is a process in which the thermal energy provides non-zero density of electrons at energies larger than the potential barrier allowing them to escape. The current density associated with this process can be written as

$$J_z = \int_{E \geq E_{\min}} e n(E) v_z(E) dE$$

n(E)dE is density of electrons per unit volume, $v_z(E)$ velocity distribution along z-component (perpendicular to the surface). The integral is evaluated for energies sufficient to escape the barrier, i.e. $E \geq E_{min} = e(\varphi_W + \varphi_F)$. Derive Richardson-Dushman equation recalling that

$$n(E)dE = \frac{8\pi m^3}{h^3} \frac{v^2 dv}{1 + \exp\left(\frac{E - e\varphi_F}{kT}\right)} \cong \frac{2m^3}{h^3} \exp\left(-\frac{E - e\varphi_F}{kT}\right) 4\pi v^2 dv$$

Problems

2) a) Derive Child-Langmuir formula. b) For initial tests with Cornell ERL gun it is planned to use a CW laser to investigate the photocathode lifetime issues. Gun power supply will be limited to 300 kV for these tests. Estimate illuminated laser spot size required to produce 100 mA average current. Assume a planar diode geometry and 5 cm cathode-anode gap.

3) Provide the expression for beam size used to fit measurements to in quad scan to determine rms beam emittance (use Twiss parameters and assume thin-lens scenario).