Homework Problems I

1. Normalize, and compute the emittance of the following distributions:

   Gaussian $$f(x,x') = A\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{x'^2}{2\sigma_{x'}^2}\right)$$

   Waterbag $$f(x,x') = A\Theta\left(1 - \frac{x^2}{\Delta x^2} - \frac{x'^2}{\Delta x'^2}\right)$$

   K-V, or microcanonical $$f(x,x') = A\delta\left(1 - \frac{x^2}{\Delta x^2} - \frac{x'^2}{\Delta x'^2}\right)$$

   Klimontovich $$f(x,x') = A\sum_{i=1}^{N}\delta(x - x_i)\delta(x' - x'_i)$$

   Treat σ_x, $\sigma_{x'}$, Δx, $\Delta x'$, x_i, x'_i as parameters. Θ Unit step, δ Dirac's delta

   For distributions (1)-(3), what does the projected distribution,

   e.g., $p(x) = \int f(x,x')dx'$ look like?

2. Starting with the Lagrangian of a point particle with charge q and rest mass m in an electromagnetic field specified by the scaler potential Φ and the vector potential **A**

$$L = -mc^2\sqrt{1 - \vec{v}\cdot\vec{v}/c^2} - q\Phi + q\vec{v}\cdot\vec{A},$$

show the Euler-Langrange equations reduce to the well-known relativistic Lorentz Force Equation

$$\frac{d\left(\gamma m\vec{\mathrm{v}}\right)}{dt} = q\left(\vec{E} + \vec{\mathrm{v}}\times\vec{B}\right),$$

where **E** and **B** are the electric field and magnetic field given by the usual relations between the fields and potentials

$$\vec{E} = -\vec{\nabla}\Phi - \frac{\partial\vec{A}}{\partial t}$$

and

$$\vec{B} = \vec{\nabla}\times\vec{A}.$$

From the relativistic Lorentz Force Equation derive

$$\vec{\mathrm{v}}\cdot\frac{d\left(\gamma m\vec{\mathrm{v}}\right)}{dt} = q\vec{\mathrm{v}}\cdot\vec{E}.$$

From the usual expression

$$\gamma = \frac{1}{\sqrt{1-\vec{\mathrm{v}}\cdot\vec{\mathrm{v}}/c^2}},$$

show

$$\frac{d\left(\gamma mc^2\right)}{dt} = q\vec{E}\cdot\vec{\mathrm{v}}.$$

Therefore, even at relativistic energies, magnetic fields cannot change the particle energy when radiation reaction is neglected.