Supergroups for disordered Dirac fermions

Andre Leclair
Cornell University

Newton Institute for Mathematical Sciences

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Outline

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- Introduction
- Classification of universality
- Supersymmetric disorder averaging
- gl(1|1) supercurrent algebra as a critical point from super spin charge separation
- solution of the the gl(1|1) level k model.
- Critical points and logarithmic perturbations
- multi-fractal and localization length exponents
- Conclusions

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based on 0710.2906[hep-th] and 0710.3778[cond-math] (October)

Motivations from Mathematics and Physics

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- Anderson transitions in 2+1 dimensions
    - physics of metal-insulator transitions
    - the challenge: computing quenched disorder averages.
    - important physical examples: Quantum Hall Transition, Graphene
    - new universality classes beyond percolation

- Supergroups in Mathematical Physics

  - Anderson transitions: supergroups arise in Efetov's supersymmetric method of computing quenched disorder averages.
  - sigma models on Lie supergroups arise in string theory on AdS spaces, e.g. psl(2|2) sigma models.
  - Spin chains built on supergroups arise in the integrability approach to N=4 susy Yang-Mills.
  - various problems in statistical mechanics: percolation, self-avoiding walks, polymers, ...

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Berkovits, Vafa, Witten 1999

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Beisert and Staudacher 2005

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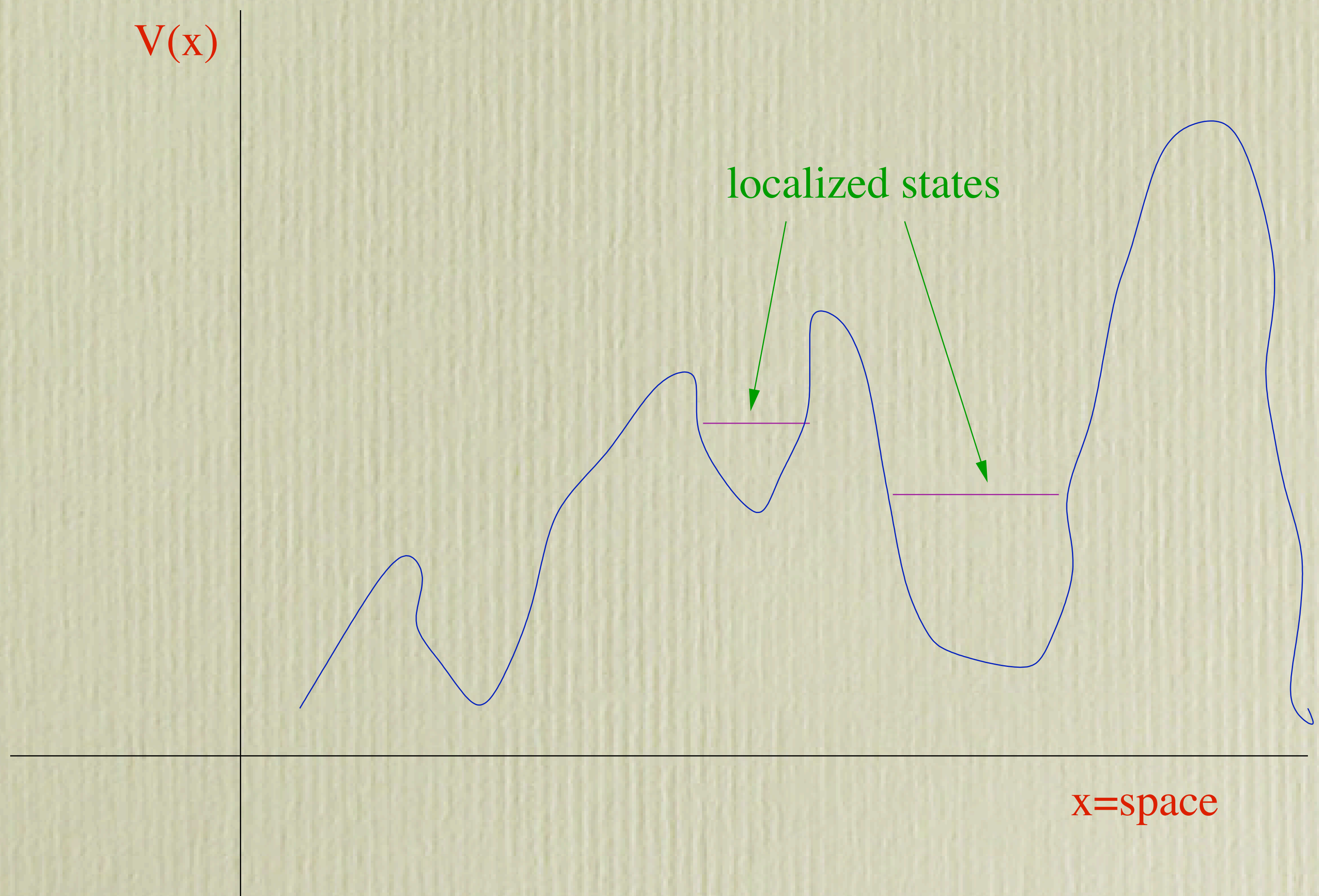

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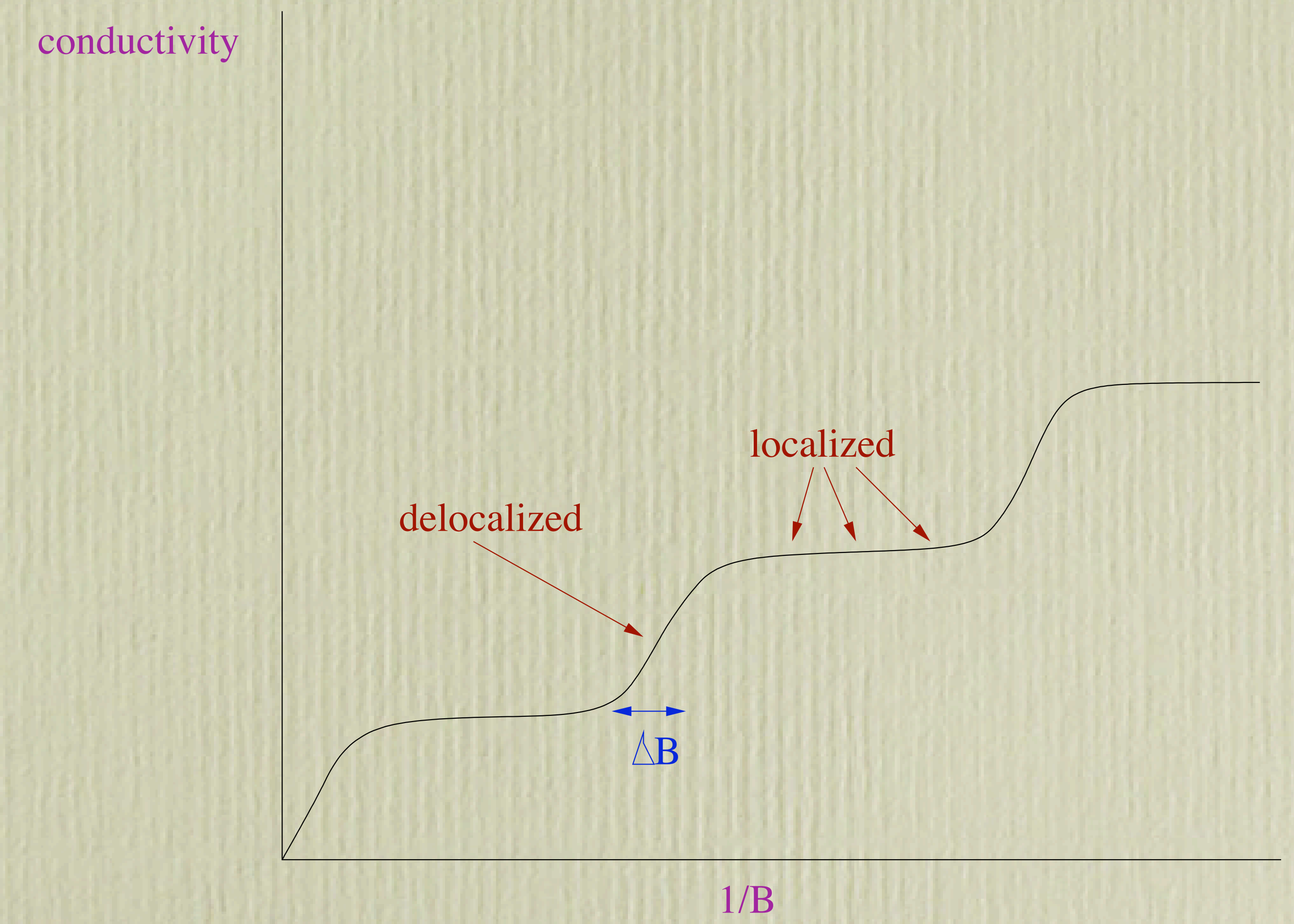

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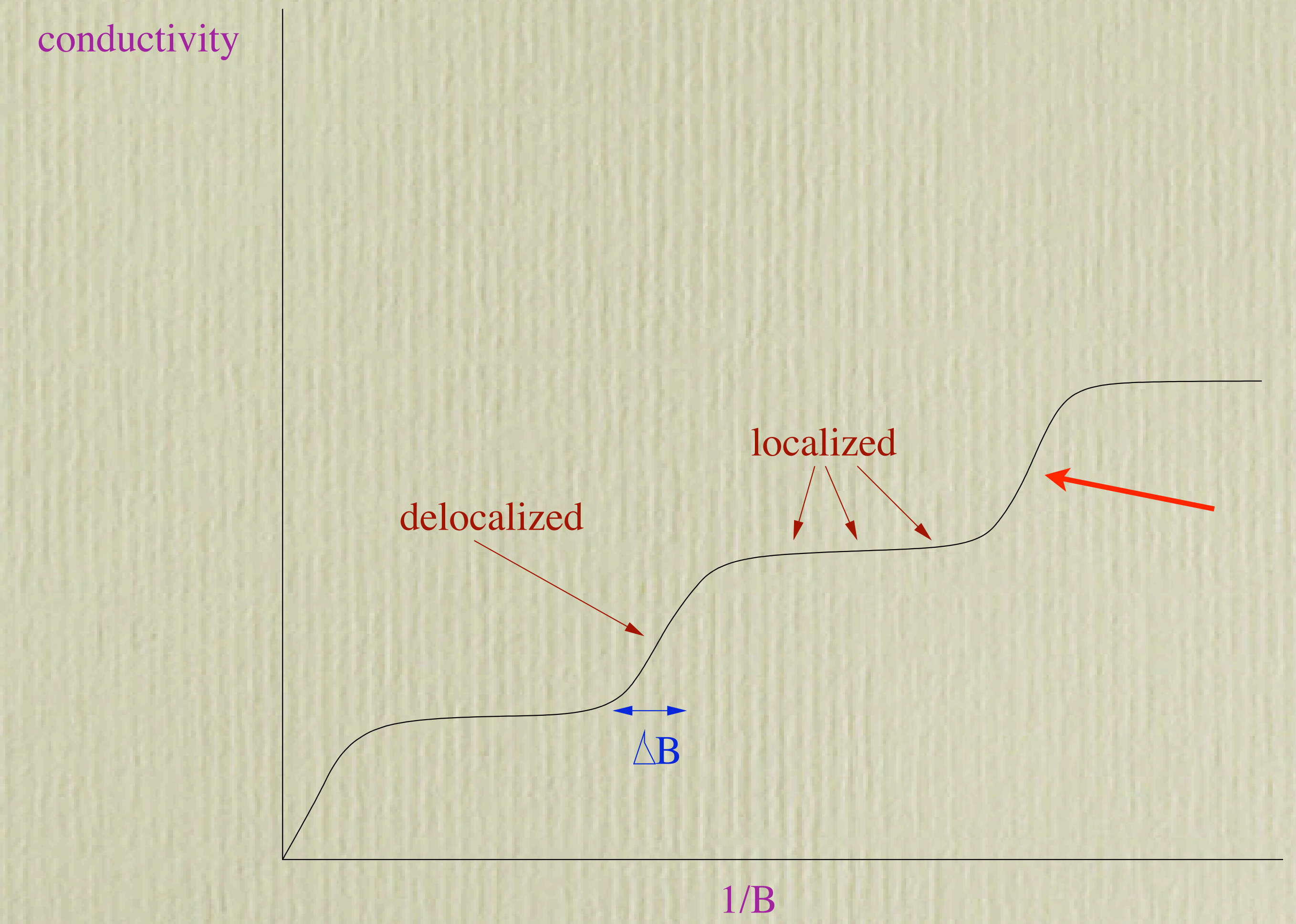

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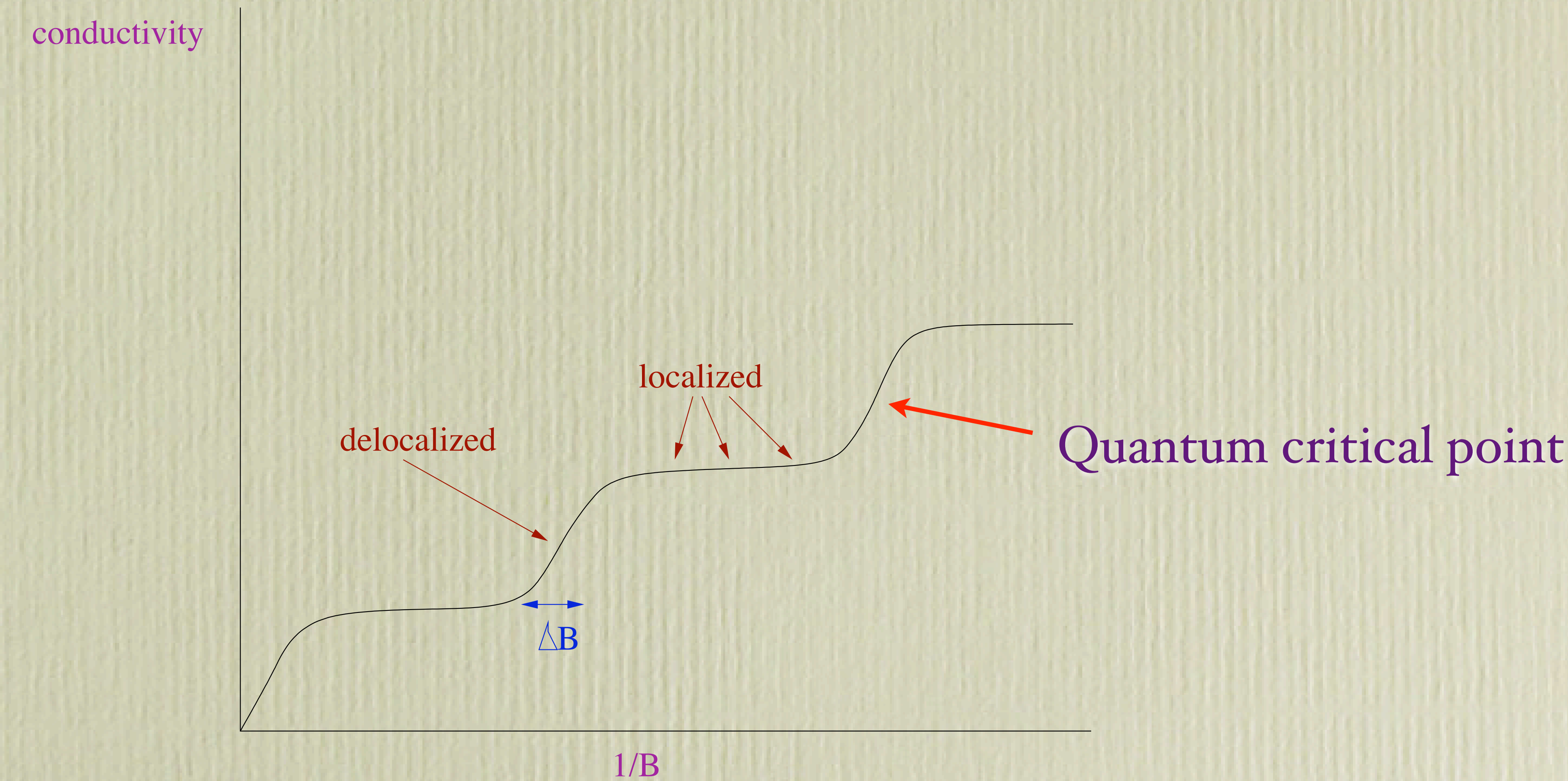

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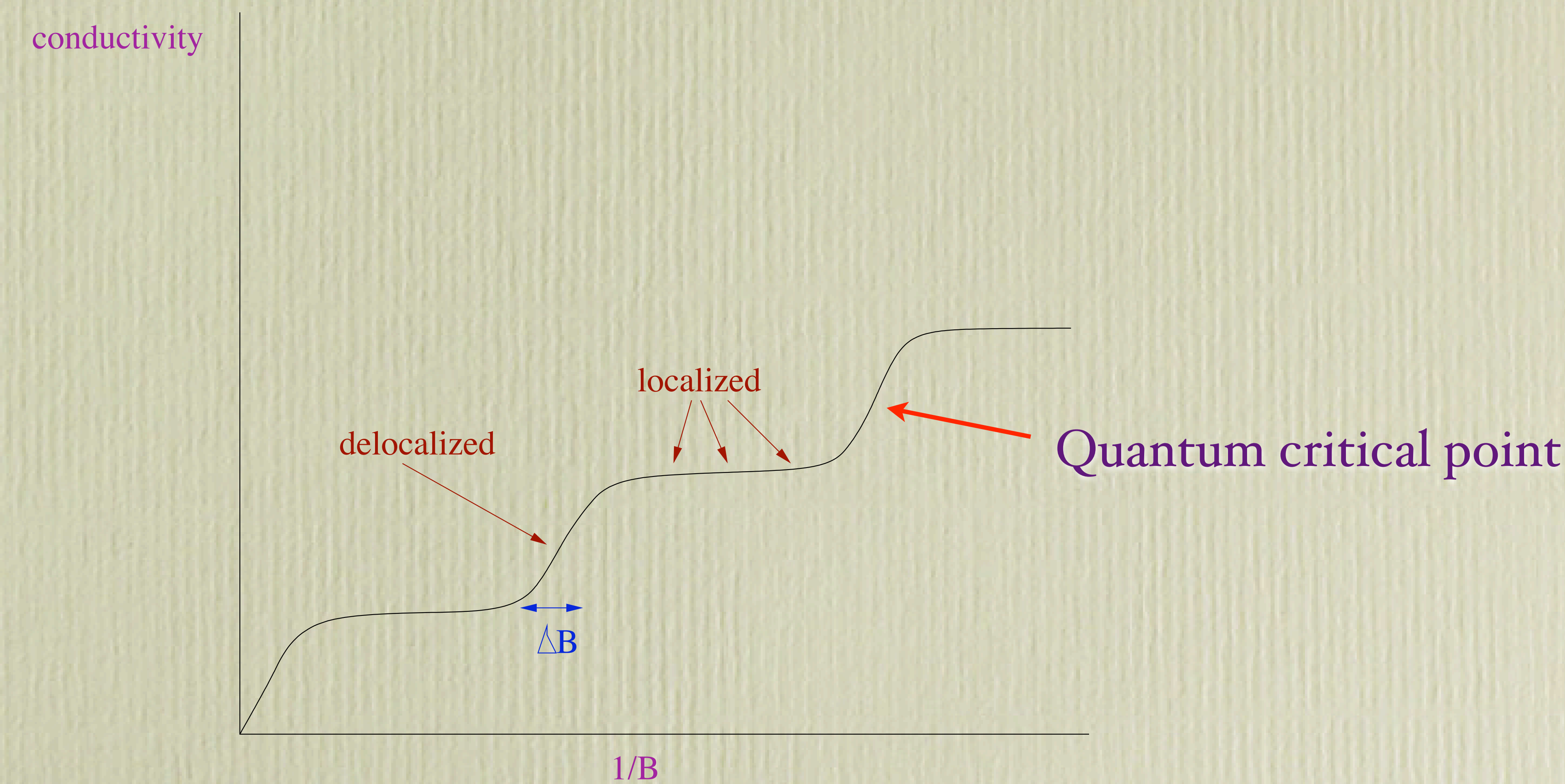

* important open problem: critical properties of the transition, exponents, etc. E.g:

$$\xi_c \sim (E - E_c)^\nu \quad , \qquad \Delta B \propto T^{1/\nu}, \qquad \nu \approx 7/3$$

Universality Classes

Why Dirac fermions? Nearly all interesting cases have 1-st order actions.

Most general Dirac hamiltonian in 2d:

$$H = \begin{pmatrix} V_+ + V_- & -i\partial_{\bar{z}} + A_{\bar{z}} \\ -i\partial_z + A_z & V_+ - V_- \end{pmatrix}$$

V, A = V(x), A(x) = random potentials

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Classification according to discrete symmetries:

- Chirality: $H = -PHP^{-1}, \quad P^2 = 1$
- Particle-hole: $H = -CH^TC^{-1}, \quad C^T = \pm C$
- Time-reversal: $H = KH^*K^{-1}, \quad K^T = \pm K$

Universality Classes

*with D. Bernard, J.Phys. A35 (2002)

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* Guruswamy, AL,Ludwig (1999)

Supersymmetric Disorder Averaging

Consider a free hamiltonian in a random potential V(x):

e.g. Schrodinger for simplicity: $$H = -\frac{\vec{\nabla}^2}{2m} + V(x)$$

We are interested in disorder averaged Green functions:

$$\overline{< \psi(x)\psi^\dagger(x') >} = \int DV P[V] < \psi(x)\psi^\dagger(x') >_V$$

The problem: properly normalize the Green function at fixed V by Z(V):
The trick: represent Z with bosonic ghosts:

$$\frac{1}{Z(V)} = \int D\beta \; e^{-S(\psi \to \beta, V)}$$

We can now perform the functional integral over the random potential V:

$$\overline{\langle \psi(x)\psi^\dagger(y)\rangle} = \int D\psi D\beta e^{-S_{\rm eff}} \; \psi(x)\psi^\dagger(y)$$

$S_{\rm eff}$ is an interacting quantum field theory of fermions and ghosts.

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- Other approaches:

  - Replica sigma models (Pruisken 1984)

  - Supergroup sigma models (Zirnbauer 1999)

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- OUR NEW APPROACH: Resolve the RG flow in 2 stages; use super spin charge separation; new results for gl(1|1) current algebra; explicit form of logarithmic operators in terms of symplectic fermions.

Supergroup symmetries in the N-copy theory

$= \int dx \;\; \Psi^* \;\; H \;\; \Psi$

For any realization of the disorder the action has a $gl(N|N)$ symmetry.

The important super subgroup symmetry which commutes with permutations of the copies is:

$$gl(1|1)_N$$

Supergroup symmetries in the N-copy theory

Introduce N-copies of the theory in order to compute multiple moments:

fields: $\Psi^\alpha_\pm = (\psi^\alpha_\pm, \beta^\alpha_\pm), \qquad \alpha = 1, .., N$

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The action at fixed realization of disorder:

$$S_{\rm susy} = \int \frac{d^2x}{2\pi} [\, \overline{\Psi}_-(\partial_z - iA_z(x))\overline{\Psi}_+ + \Psi_-(\partial_{\overline{z}} - iA_{\overline{z}}(x))\Psi_+ - iV(x)\left(\overline{\Psi}_-\Psi_+ + \Psi_-\overline{\Psi}_+\right) - iM(x)\left(\overline{\Psi}_-\Psi_+ - \Psi_-\overline{\Psi}_+\right) \,]$$

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The $gl(1|1)_N$ affine Lie algebra symmetry is generated by the chiral currents:

$$H = \sum_\alpha \psi_+^\alpha \psi_-^\alpha, \qquad J = \sum_\alpha \beta_+^\alpha \beta_-^\alpha, \qquad S_\pm = \pm \sum_\alpha \psi_\pm^\alpha \beta_\mp^\alpha$$

which satisfy the operator product expansion: (k=N = level)

$$\begin{aligned} H(z)H(0) &\sim \frac{k}{z^2}, \qquad J(z)J(0) \sim -\frac{k}{z^2} \\ H(z)S_\pm(0) &\sim J(z)S_\pm(0) \sim \pm\frac{1}{z}\; S_\pm \\ S_+(z)S_-(0) &\sim \frac{k}{z^2} + \frac{1}{z}\,(H - J) \end{aligned}$$

Additional symmetries that commute with the above: su(N) at level k=0

currents: $L_\psi^a = \psi_-^\alpha t_{\alpha\alpha'}^a \psi_+^{\alpha'}, \qquad L_\beta^a = \beta_-^\alpha t_{\alpha\alpha'}^a \beta_+^{\alpha'}, \qquad L^a = L_\psi^a + L_\beta^a$

Important symmetry: $gl(1|1)_N \oplus su(N)_0$

Critical points from Super Spin-Charge Separation

* First separate the theory into two commuting sets of degrees of freedom. This involves a remarkable identity for the Sugawara stress-tensors:

$$T_{\text{free}}^{\text{N}-\text{copy}} = -\frac{1}{2}\sum_{\alpha=1}^{N}(\psi_-^\alpha \partial_z \psi_+^\alpha + \beta_-^\alpha \partial_z \beta_+^\alpha) = T_{gl(1|1)_{k=N}} + T_{su(N)_0}$$

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Strategy for resolving the renormalization group (RG) flow: Based on the idea that the RG flow to low energies decouples massive degrees of freedom.

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* In the first stage of the RG flow, carry out the flow for the couplings in $S_{\rm eff}$ corresponding to these two sets of degrees of freedom:

$$S = S_{\rm cft} + \int \frac{d^2x}{2\pi}\left(g_A\, J_A \cdot \overline{J}_A + g_B\, J_B \cdot \overline{J}_B\right)$$

where J_A = gl(1|1) currents, J_B = su(N) currents. The 1-loop beta functions are:

$$\frac{dg_A}{d\ell} = -g_A^2, \qquad \frac{dg_B}{d\ell} = +g_B^2$$

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* Introduce additional forms of disorder as relevant perturbations of $gl(1|1)_N$

Solution of the $gl(1|1)_k$ theory

AL 0710.2906 [hep-th], builds on Schomerus and Saleur 2006

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Free field representation: two scalar field and a symplectic fermion:

Action:

$$S = \frac{1}{8\pi}\int d^2x \sum_{a,b=1}^{2}\left(\eta_{ab}\partial_\mu\boldsymbol{\phi}^a\partial_\mu\boldsymbol{\phi}^b + \epsilon_{ab}\partial_\mu\boldsymbol{\chi}^a\partial_\mu\boldsymbol{\chi}^b\right)$$

$$\eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Representation of the current algebra:

$$\begin{aligned} H &= i\sqrt{k}\,\partial_z\phi^1, \qquad J = i\sqrt{k}\,\partial_z\phi^2 \\ S_+ &= \sqrt{k}\,\partial_z\chi^1\,\mathrm{e}^{i(\phi^1-\phi^2)/\sqrt{k}}, \qquad S_- = -\sqrt{k}\,\partial_z\chi^2\,\mathrm{e}^{-i(\phi^1-\phi^2)/\sqrt{k}} \end{aligned}$$

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Twist fields:

$$\chi^1(\mathrm{e}^{2\pi i}z)\mu_\lambda(0) = \mathrm{e}^{-2\pi i\lambda}\chi^1(z)\mu_\lambda(0)$$

$$\chi^2(\mathrm{e}^{2\pi i}z)\mu_\lambda(0) = \mathrm{e}^{2\pi i\lambda}\chi^2(z)\mu_\lambda(0)$$

$$\Delta(\mu_\lambda) = \frac{\lambda(\lambda-1)}{2} \equiv \Delta_\lambda^{(\chi)}$$

VERTEX OPERATORS: fields transforming in finite dimensional reps of $gl(1|1)_k$

The corresponding vertex operator:

$$V_{\langle h,j\rangle} = (h-j)^{1/4} \begin{pmatrix} -\mu_\lambda \; e^{i(h\phi^1 - j\phi^2)/\sqrt{k}} \\ \sigma_\lambda^2 \; e^{i((h-1)\phi^1-(j-1)\phi^2)/\sqrt{k}} \end{pmatrix}, \qquad \lambda = \frac{h-j}{k}$$

Conformal scaling dimension:

$$\Delta_{\langle h,j\rangle} = \frac{(h-j)^2}{2k^2} + \frac{(h-j)(h+j-1)}{2k}$$

Closed operator algebra: $-k < h-j < k$ h,j,k = integers

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2-dimensional reps <h,j>:

$$H = \begin{pmatrix} h & 0 \\ 0 & h-1 \end{pmatrix}, \quad J = \begin{pmatrix} j & 0 \\ 0 & j-1 \end{pmatrix}$$

$$S_+ = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}, \quad S_- = \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix}$$

(bc = h-j)

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Logarithmic vertex operators for indecomposable representations.

4-dimensional indecomposable reps <0>$_4$:

$$\langle 1,0\rangle \otimes \langle 0,1\rangle = \langle 0\rangle_{(4)}$$

Corresponding vertex operator (Δ=0):

$$V_{\langle 0\rangle_{(4)}} = \begin{pmatrix} \chi^1 \ \mathrm{e}^{i(\phi^1-\phi^2)/\sqrt{k}} \\ \sqrt{k} \\ \chi^1\chi^2/\sqrt{k} \\ \chi^2 \ \mathrm{e}^{-i(\phi^1-\phi^2)/\sqrt{k}} \end{pmatrix}$$

Logarithmic property: *Virasoro zero mode is not diagonal (Jordan block form)*

$$L_0 = -\frac{1}{k}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(similar properties found for osp(2|2) by Maassarani and Serban 1997)

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(due to the log pair (1, $\chi^1\chi^2$))

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Logarithmic perturbations

Quantum numbers:

* under the gl(1|1) x su(N) symmetries:

$$\psi_\pm,\ \beta_\pm \qquad \Longleftrightarrow \qquad (\langle 1,0\rangle \oplus \langle 0,1\rangle)\otimes[\text{vec}]$$

* currents= bilinears in these fields. Examining the quantum numbers:
  For N<2 the most relevant operator corresponds to $\langle 0\rangle_{(4)}$. Leads to:

$$\begin{aligned} S &= S_{gl(1|1)_N} + g\int \frac{d^2x}{8\pi}\ \mathbf{\Phi}_{\langle 0\rangle_{(4)}} \\ &= \int \frac{d^2x}{8\pi}\left(\sum_{a,b=1}^{2} \eta_{ab}\,\partial_\mu\phi^a\partial_\mu\phi^b + \epsilon_{ab}\,\partial_\mu\chi^a\partial_\mu\chi^b \ + g\,\chi^1\chi^2\ \cos\left((\phi^1-\phi^2)/\sqrt{N}\right)\right) \end{aligned}$$

Logarithmic perturbations

Additional disorder as perturbations of the gl(1|1) cft:

* in the original theory they correspond to left/right current interactions.

* after gapping out the $su(N)_0$ degrees of freedom, additional disorder corresponds to relevant perturbations consistent with quantum numbers.

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$$= \int \frac{d^2x}{8\pi}\left(\sum_{a,b=1}^{2} \eta_{ab}\,\partial_\mu\phi^a\partial_\mu\phi^b + \epsilon_{ab}\,\partial_\mu\chi^a\partial_\mu\chi^b \;+ g\,\chi^1\chi^2\,\cos\left((\phi^1-\phi^2)/\sqrt{N}\right)\right)$$

* The above action defines a gl(1|1) version of sine-Gordon theory.

* The logarithmic perturbations do not drive the theory to a new fixed point:

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Thus: The critical exponents should be in the gl $(1|1)_N$ conformal field theory

Multi-fractal exponents

* a probe of disorder averaged higher moments; must be computed in the N-copy theory

density of states operator: $\rho(x) = \overline{\Psi}_- \Psi_+ + \Psi_- \overline{\Psi}_+$

q-th moment: $$P^{(q)} = \frac{\int d^2x \overline{\rho(x)^q}}{(\int d^2x \overline{\rho(x)})^q}$$

scaling at the critical point: $P^{(q)} \sim L^{-\tau_q}$ (L = size)

Relation to scaling dimension of operators: $\tau_q = \widehat{\Gamma}_q + 2(q-1)$

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$\widehat{\Gamma}_q$ ⇔ scaling dimension of ρ^q

We compute $\widehat{\Gamma}_q$ in the N=2 copy theory since for $q>q_c$ the multi-fractal spectrum is known to cross over to a non-parabolic spectrum and $2 < q_c < 3$.

The most revelant operator in ρ^q corresponds to the <0,q> gl(1|1) rep.

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agrees to 1-2% with numerical results of Klesse&Metzer (1995); Evers, Mildenberger and Mirlin (2001)

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What is the operator $\mathcal{O}_\nu$?

no simple quantum number arguments to identify it

Hint from spin quantum Hall: here $gl(1|1)_N$ becomes $osp(2|2)_{-2N}$

Use the exact embedding: $gl(1|1)_2 \subset osp(2|2)_{-2}$

By comparing conformal dimensions: $gl(1|1)_2$ = percolation

In the N=2 theory, the localization length exponent for percolation ~ <2,1> field.

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Real experiments: 2.3 ± 0.1 , S. Koch et. al. (1991)

Numerical simulations: 2.33-2.35 ± 0.03 , Huckestein (1995); D.-H. Lee and Wang (1996)

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The End