Lecture 25: 03/25/09

- Photon Polarization States II

① Linear polarization analyzer and Projection amplitudes:

② Linear polarization analyzer loop/change of basis states:

Recap

III_6 Dirac Notation:

Dirac: chop bracket notation for inner product in two pieces:

bra: $\langle \alpha |$ $\leftrightarrow$ instruction to integrate / row vector

ket: $| \beta \rangle$ $\leftrightarrow$ state vector

state: $\boxed{|\alpha\rangle = \sum_n \underbrace{|e_n\rangle\langle e_n}|\alpha\rangle}$

call $|e_n\rangle\langle e_n|$ – projection operator

- Can expand state $|\alpha\rangle$ in terms of different basis states:

$$|\alpha\rangle = \sum_n |e_n\rangle\langle e_n|\alpha\rangle = \sum_n |e_n'\rangle\langle e_n'|\alpha\rangle$$

- Unity operator:

$$\sum_n |e_n\rangle\langle e_n| = 1 \quad (\text{if it acts on } |\alpha\rangle, \text{one gets } |\alpha\rangle)$$

Recap

III_8 Another example: Photon Polarization States

- Light

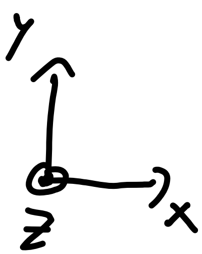
Classically	QM picture
electromagnetic wave	photons

In the following: all waves / photons have same frequency and direction of motion (+ z-direction) ←

light with arbitrary polarization:

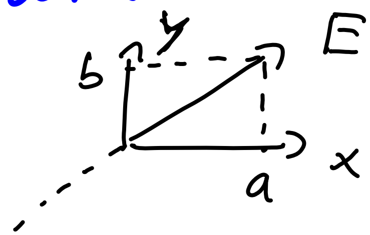
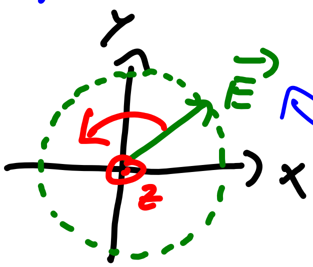
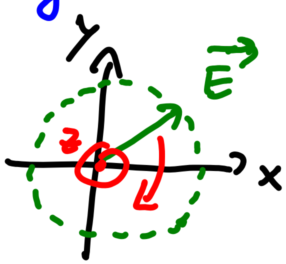
$$\vec{E}(z,t) = \mathrm{Re}\{(a\,e^{i\varphi_a}\hat{x} + b\,e^{i\varphi_b}\hat{y})\,e^{i(kz-\omega t)}$$

$$= a\cos(kz-\omega t+\varphi_a)\,\hat{x} + b\cos(kz-\omega t+\varphi_b)\,\hat{y}$$



Photon state:

$$|\underline{\psi}\rangle$$

Recap	Classically	photon
	1) linear polarized in x-direction: $\Rightarrow b=0 \quad \longleftrightarrow \vec{E}$	$\vert x\rangle$
	2) linear polarized in $\hat{y}$-direction $\Rightarrow a=0 \quad \updownarrow \vec{E}$	$\vert y\rangle$
	3) linear polarized in $a\hat{x}+b\hat{y}$ direction: $\Rightarrow a,b\neq 0 \quad \varphi_a=\varphi_b$ 	$\frac{1}{\sqrt{a^2+b^2}}\{a\vert x\rangle+b\vert y\rangle\}$ $\uparrow$ normalize
	4) left circular polarized:  $\vert\vec{E}\vert=$ const $\quad a=b \quad \varphi_a=\varphi_b-\underbrace{\frac{\pi}{2}}_{90^\circ}$	$\vert L\rangle=\frac{1}{\sqrt{2}}\{i\vert x\rangle+\vert y\rangle\}$
	5) right circular polarized:  $a=b \quad \varphi_a=\varphi_b+\frac{\pi}{2}$	$\vert R\rangle=\frac{1}{\sqrt{2}}\{(-i)\vert x\rangle+\vert y\rangle\}$

Recap

① Linear polarization analyzer and Projection amplitudes:

$E_x = a\cos(kz-\omega t+\varphi_a) = \hat{x}\cdot\vec{E}_i$ $\boxed{\text{detect.}}$ $|x\rangle, \langle x|\psi\rangle$

$I_x \propto \frac{1}{2} a^2$

$E_y = b\cos(kz-\omega t+\varphi_b) = \underbrace{\hat{y}\cdot\vec{E}_i}_{\text{projection!}}$ $\boxed{\text{detect.}}$ $|y\rangle, \langle y|\psi\rangle$

$I_y \propto \frac{1}{2} b^2$

$\hat{x}$ $\hat{y}$

x-y-analyzer

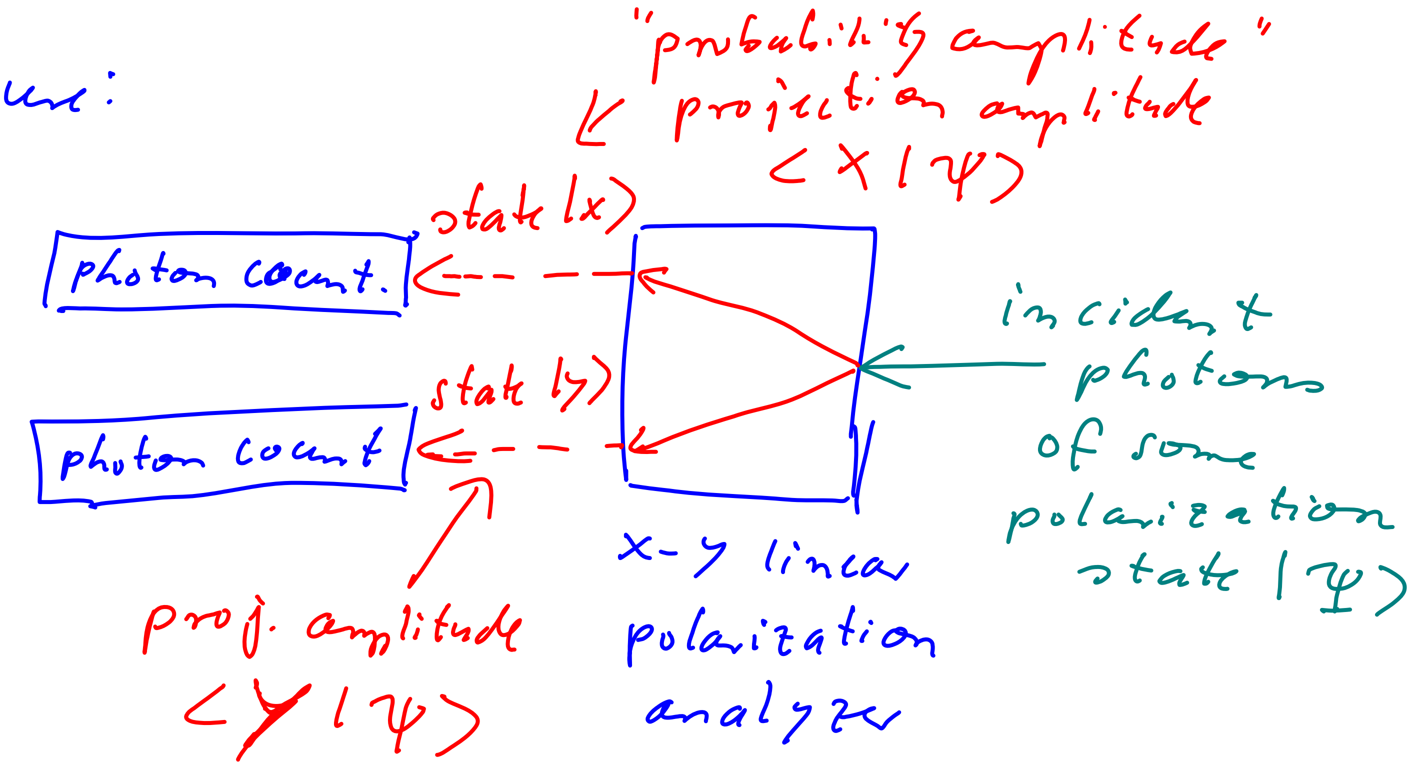
$\vec{E}_i = a\cos(kz-\omega t+\varphi_a)\hat{x} + b\cos(kz-\omega t+\varphi_b)\hat{y}$

$|\psi\rangle$

$I_{in} \propto \overline{|\vec{E}|^2} = \frac{1}{2} a^2 + \frac{1}{2} b^2$

separate light into linear $\hat{x}$-pol. and linear $\hat{y}$-pol. components

Photon Picture:



- Input intensity / # of photons = total output intensity / total # of photons

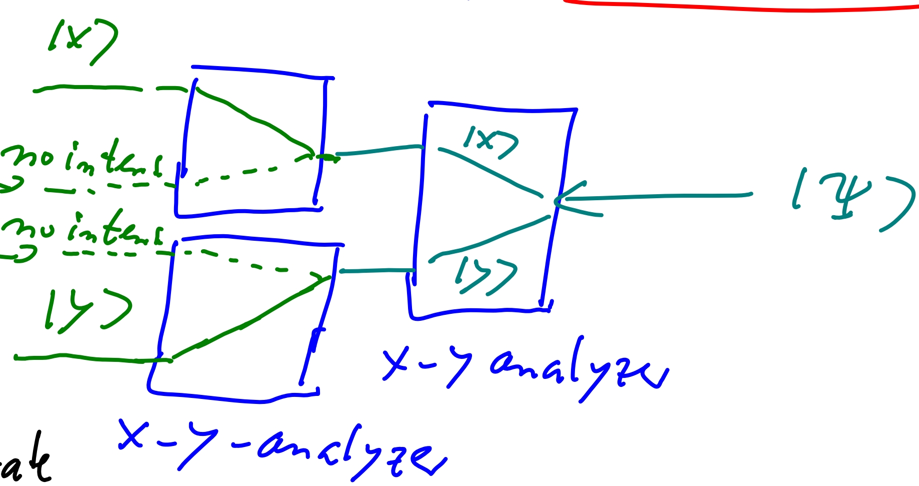
$\Rightarrow$ $|x\rangle$ and $|y\rangle$ linear polarization states give a complete set of basis states (only 2 linear indep. states)

$$\Rightarrow |\Psi\rangle = c_x|x\rangle + c_y|y\rangle = |x\rangle\langle x|\Psi\rangle + |y\rangle\langle y|\Psi\rangle$$
$$= \sum_n |\text{state}_n\rangle\langle \text{state}_n|\Psi\rangle$$

- $|x\rangle$ and $|y\rangle$ polarization states are orthonormal to each other:

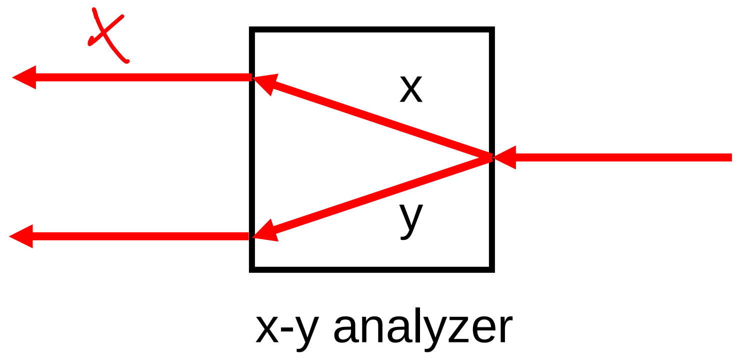
ortho-normal!
$$\left\{\begin{array}{l} \langle x|x\rangle = 1 \\ \langle y|x\rangle = 0 \\ \langle x|y\rangle = 0 \\ \langle y|y\rangle = 1 \end{array}\right.$$

onto state (bra), from state (ket)



Suppose a single photon is send through an x-y analyzer, and ends up in the x-channel.

Does that mean that the original photon was polarized in the x-direction?



A. yes

B. no

example:

$$|\psi\rangle_{in} = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle$$

- not all photons behave the same way!
  (some end up in the $\hat{x}$-detector, some in the $\hat{y}$-detector, even though they all start with the <u>same</u> polarization prior to the measurement!)

$\Rightarrow$ statistical behavior in QM, probabilities only!

<u>Example</u>: $|\psi\rangle_{in} = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle \Rightarrow$
- 50% chance result is $|x\rangle \rightarrow$ photon is in $|x\rangle$ state <u>after</u> measurem.
- 50% chance result is $|y\rangle \rightarrow$ photon is in $|y\rangle$ state <u>after</u> measurement

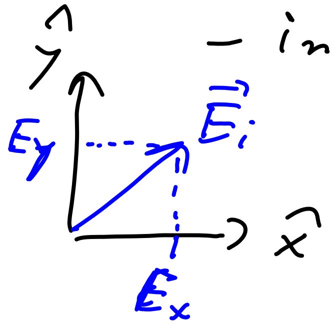
$\Rightarrow$ Measurement can change the quantum state!
(wave function "collapses")

$\Rightarrow$ Measurements give only discrete results
(x-y analyzer: only $\hat{x}$ or $\hat{y}$ polarized, nothing in between)

$\Rightarrow$ Measurements = projection of initial state onto some basis state

Projection amplitudes:

→ classically:



- initial: $\vec{E}_i = a\cos(kz-\omega t+\varphi_A)\,\hat{x} + b\cos(kz-\omega t+\varphi_B)\,\hat{y}$
- after $\hat{x}$-$\hat{y}$ analyzer:

  $\hat{x}$-path: $E_x = \underbrace{\hat{x}\cdot\vec{E}_i}$ $= a\cos(kz-\omega t+\varphi_A)$

  measurement = projection onto x-axis

  Intensity along x $\propto |E_x|^2 = |\hat{x}\cdot\vec{E}_i|^2$

  $\hat{y}$-path: $E_y = \hat{y}\cdot\vec{E}_i = b\cos(kz-\omega t+\varphi_B)$

  Intensity $\propto |E_y|^2 = |\hat{y}\cdot\vec{E}_i|$

→ Quantum picture:

- initial: $|\Psi\rangle$
- after $\hat{x}$-$\hat{y}$-analyzer:

  $\hat{x}$-path: $|\text{photon}\rangle_{\text{after}} = |x\rangle$

  define $\langle x|\Psi\rangle$ = projection amplitude of incomming state $|\Psi\rangle$ onto final pol. state $|x\rangle$

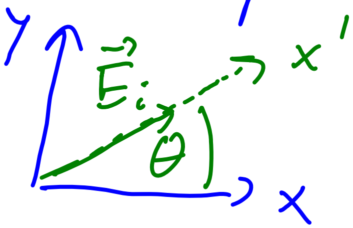
  $\Rightarrow |c_x|^2 = |\langle x|\Psi\rangle|^2$ = probability of finding photon in $|x\rangle$ pol. state

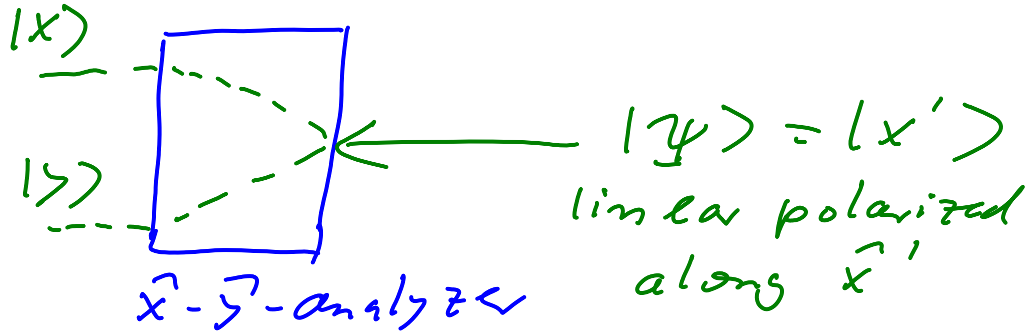
similar for $\hat{y}$-path

$c_y = \langle y|\Psi\rangle$

$|c_y|^2 = |\langle y|\Psi\rangle|^2$

Example: $|\psi\rangle = |x'\rangle$: linear pol. along $\hat{x}'$





→ <u>classically:</u>

– initial:

$$\vec{E}_i = E_0 \cos\theta \cos(kz - \omega t)\,\hat{x} + E_0 \sin\theta \cos(kz-\omega t)\,\hat{y}$$

$$= E_0 \cos(kz - \omega t)\,\hat{x}'$$

$$\text{Intensity} = I_i \propto \tfrac{1}{2}|E_0|^2$$

– after $\hat{x}$-$\hat{y}$ analyzer:

$\hat{x}$-path:

$$E_x = \hat{x}\cdot\vec{E}_i = E_0 \cos\theta \cos(kz - \omega t)$$

$$I_x \propto \overline{|E_x|^2} = \tfrac{1}{2} E_0^2 \cos^2\theta \rightarrow I_x = I_i \cos^2\theta$$

$\hat{y}$-path:

$$E_y = \hat{y}\cdot\vec{E}_i = E_0 \sin\theta \cos(kz - \omega t)$$

$$I_y \propto \overline{|E_y|^2} = \tfrac{1}{2} E_0^2 \sin^2\theta \rightarrow I_y = I_i \sin^2\theta$$

note: $I_x + I_y = I_i$

$\rightarrow$ Quantum picture:

+ sign $\langle x | x \rangle = 1$

$\hat{x}$ - path: projection amplitude: $\langle x | x' \rangle = \cos\theta$

$\hat{y}$ - path: " " $\langle y | x' \rangle = \sin\theta$

$\Rightarrow$ probability of measuring photon in x-pol. state $|x\rangle$

$$|C_x|^2 = |\langle x | x' \rangle|^2 = \cos^2\theta \underbrace{\propto I_x}_{\text{classically}}$$

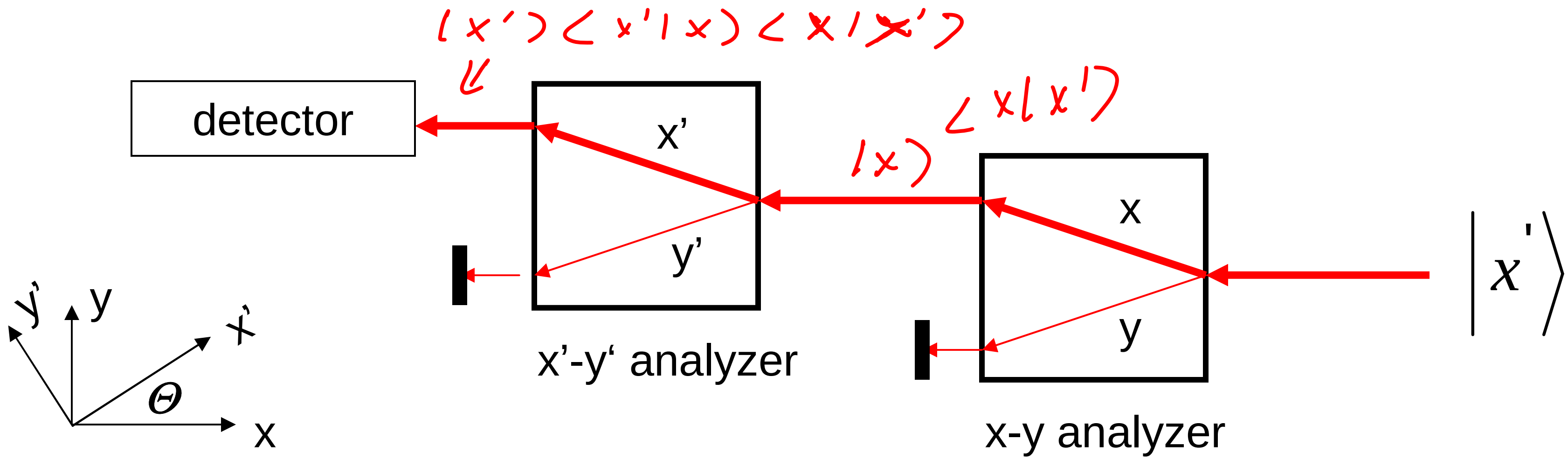
$\Rightarrow$ probability of measuring photon in y-pol. state $|y\rangle$

$$|C_y|^2 = |\langle y | x' \rangle|^2 = \sin^2\theta \propto I_y$$

Note: $|C_x|^2 + |C_y|^2 = 1 = \sum_n |C_n|^2 = 1$

$\Rightarrow$ general: if $|a\rangle$, $|b\rangle$ are linear polarization states, then

$$\langle a | b \rangle = \cos(\text{angle between polarization})$$



What is the probability that one of the initial photons in linear polarization state $|x'\rangle$ reaches the detector?

A. 0

B. $\cos(\Theta)^2$

C. $\cos(\Theta)^4$

D. $\cos(\Theta)^2 \sin(\Theta)^2$

E. 1

$$\left\{ \text{total prob. of getting to detector} \right\} = |\langle x|x'\rangle|^2 \cdot |\langle x'|x\rangle|^2$$

$$= ((\cos\theta)^2 \cdot (\cos\theta)^2$$

$$= \cos^4\theta$$