M. Planck (1858 –1947) won Nobel price in Physics for his work on blackbody radiation (1918)

Blackbody Spectrum:

1) Classical Theory

   ⇒ Rayleigh–Jeans formula
   ⇒ "ultraviolet catastrophe"

2) Planck’s Theory

   ⇒ Quantized energy
Recap:

**I1,3 The Compton Effect**

⇒ Demonstrates particle like nature of radiation

Elastic collision between photon and free e-

**Photon**:
- Energy: \( E = h\nu \)
- Momentum: \( p = \frac{h}{\lambda} \)

Compton shift: \( \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta) \)
$I_{1,4}$ Blackbody Radiation

⇒ Demonstrates quantization

- Blackbody: 100% Absorbs at all wavelengths - none reflection
  ⇒ emit a universal spectrum that only depends on temperature $T$

- Why?

Two black bodies at same $T$.
Radiation emitted by ① is absorbed by ② and vice versa.

notch filter: transmission in one narrow frequency range

⇒ Energy emitted at given frequency must be same, or one would heat up and other cool down
   - violates thermodynamics

⇒ whole spectrum must be same! ⇒ universal
Does a blackbody always appear black?

A. Yes
B. No
C. Maybe

B. No
- **How to make a black body with real materials?**
  
  ![Diagram of a cavity with a small hole](image)
  
  - cavity with small hole, walls at $T$
  - absorbs everything (hole)
  - radiation from hole in cavity has black body spectrum

- **Why interesting?**
  
  - calculate spectrum from classical physics and thermodynamics $\Rightarrow$ Result can not be correct! (predicts infinite energy...)

- **Solution:**
  
  Planck (1900) $\Rightarrow$ quantized chunks of light energy (photons)
  
  allowed energies: $E_n = n \cdot h \cdot \nu$  $n : \text{integer}$
- **Black body spectrum:**

\[ P_T (\nu) d\nu = \text{energy of radiation per unit volume in a cavity at temperature } T \text{ in the frequency interval } \nu \text{ to } \nu + d\nu \]

\[ [P_T (\nu)] = \frac{J}{m^3 \text{ Hz}} \]

\[ R_T (\nu) d\nu = \text{spectral radiance = energy emitted by hole in cavity at temperature } T \text{ per unit time per unit area of the hole in the frequency interval } \nu \text{ to } \nu + d\nu \]

\[ [R_T (\nu)] = \frac{W}{m^2 \text{ Hz}} \]

\[ P_T (\nu) \propto R_T (\nu) \]
Experimental Results:

- Frequency of maximum radiance \( \propto T \)
- Total power emitted per area \( \propto T^4 \)
Blackbody Radiation Spectrum

Graph showing the blackbody radiation spectrum for different temperatures: T=3500K, T=4000K, T=4500K, T=5000K, and T=5500K. The y-axis represents the energy per wavelength in [kJ/nm], and the x-axis represents the wavelength in [nm]. The visible light region is indicated by an arrow.
The Cosmic Microwave Background

The cosmic microwave background is a perfect blackbody emission and corresponds to a temperature of 2.725 Kelvin with an emission peak of 160.4 GHz.

WMAP image of the cosmic microwave background radiation anisotropy.
- towards $P(\nu)$:

1. Count the number of modes for standing waves of electromagnetic radiation inside a metal box in the frequency range $\nu$ to $\nu + d\nu$

2. Use statistical mechanics to compute the average energy per mode:
   - 1st classically with continuous energies
   - 2nd with quantized energy

3. Calculate:

$$P_T(\nu) = \left( \frac{\text{# of modes between } \nu \text{ and } \nu + d\nu}{\text{volume of cavity}} \right) \left( \frac{\text{average } E}{\text{per mode}} \right)$$
Step 1: 
- \( E_{\text{trans}} = 0 \) at metal walls 
- EBM wave are transversal 
- Standing waves have electric field modes at walls

- Start with 1-D:
- Standing waves have integer number of half wave lengths 
  \[ L = n_x \left( \frac{\lambda}{2} \right) \Rightarrow \lambda_x = \frac{2L}{n_x} \quad n_x = 1, 3, 5, \ldots \]
- \( E(x,t) = \overrightarrow{E_0} \sin \left( \frac{2\pi \nu t}{\lambda_x} \right) \sin (2\pi \nu t) \)
- \( \infty \) number of modes!
now in 3D: standing waves in x, y, and z directions

\[
E(x, y, z, t) = E_0 \sin \left( \frac{2\pi}{\lambda_x} x \right) \sin \left( \frac{2\pi}{\lambda_y} y \right) \sin \left( \frac{2\pi}{\lambda_z} z \right) \sin(2\pi vt)
\]

with \( \lambda_x = \frac{2\pi}{n_x} \), \( \lambda_y = \frac{2\pi}{n_y} \), \( \lambda_z = \frac{2\pi}{n_z} \), \( n_x, n_y, n_z > 0 \)

Note: 1) \( \infty \) number of modes

2) Two polarizations \( \rightarrow \) 2 standing waves for every set of integers \((n_x, n_y, n_z)\)

wave equation: \( \nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \) (free space) \( \rightarrow \) \( \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \)

\( \Rightarrow \) \( - \left( \frac{2\pi}{\lambda_x} \right)^2 - \left( \frac{2\pi}{\lambda_y} \right)^2 - \left( \frac{2\pi}{\lambda_z} \right)^2 = - \left( \frac{2\pi v}{c} \right)^2 / c^2 \)

\( \Rightarrow \) \( v = c \sqrt{1/\lambda_x^2 + 1/\lambda_y^2 + 1/\lambda_z^2} = \frac{c}{2\pi} \sqrt{n_x^2 + n_y^2 + n_z^2} \)

\( \Rightarrow \) \# of modes in \( \Delta v, v + dv \) \( = 2 \left( \text{combinations of } n_x, n_y, n_z \right) \)
\[ \frac{\sqrt{V}}{C} 2L = \sqrt{n_x^2 + n_y^2 + n_z^2} = r = \text{radius of sphere} \]

In 2-D:
- \((n_x, n_y, n_z)\) define a grid of points in octant \((n > 0)\)
- density of points: 1 point per unit volume

In 3D for large \(n\):
- \(2\times\) # of points with radii \(r = \frac{\sqrt{V}}{C} 2L\) to \(r + dr = \frac{\sqrt{V + dr}}{C} 2L\)
- volume of shell \(A\) \(= 4\pi r^2 dr \)

In 3D for cavity:
- \(#\) of modes between \(r\) and \(r + dr\) \(\approx\) \(\frac{4\pi r^2 dr}{8\pi}\) octant only

\(=\) \(2\times\frac{3L}{8} r^2 dr = 2\frac{3L}{2} \left( \frac{V}{C} 2L \right)^2 \frac{2L}{C} dv\)

\(= 8\pi \frac{L^3}{C^3} V^2 dv\)
The number of standing wave nodes between \( n \) and \( n + d\nu \) is
\[
\frac{\text{volume of cavity}}{8\pi c} \frac{\nu^2}{c^3} d\nu
\]

Note: 1) for \( \nu = 5.1 \cdot 10^{14} \text{ Hz} \) (yellow):
\[
= 2.9 \cdot 10^5 \text{ Hz}^{-1}
\]
2) \# of modes \( \propto \nu^2 \rightarrow \infty \) number of modes
(powers of \( \nu = \# \) of space dimension - 1)

Step 2: average energy per standing wave at wall temp. \( T \)

1st classically:

**Boltzmann Principle**: for a large number of physical entities of the same kind in thermal equilibrium, the relative probability that a system at temp. \( T \) will be found in a given state with energy \( E \) is

\[
P(E) \propto e^{-E/kT}
\]

Here: \( P(E) \) = probability that a standing wave in cavity will have energy \( E \)