• The quantized Atom
  - Evidence for quantized Energy Levels
  - Bohr Atom

Niels Bohr (1885 – 1962):
Nobel Prize 1922
Blackbody Radiation

Planck: Energy in each mode is quantized: $E_n = nh\nu$

$\Rightarrow \langle E \text{ per mode} \rangle \rightarrow 0 \text{ for } \nu \gg kT$

$\Rightarrow \rho_T(\nu)d\nu = \frac{8\pi \nu^2}{c^3} \frac{\nu}{e^{\nu/kT} - 1} d\nu$

The Quantized Atom

Evidence for quantized energy levels in atoms:
(a) Spectral Lines

Example: Hydrogen

$E_{\text{photon}} = h\nu = E_{n_i} - E_{n_f}$
Example: Hydrogen

Empirical result: \[ E_{\text{photon}} = E_i - E_f = \hbar c R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ = \text{Quantized energy levels:} \quad E_n = -\hbar c R_H \frac{1}{n^2} \]

\[ = \text{Energy level diagram:} \]

\[ \text{Energy level diagram:} \]

\[ \text{Photon emission:} \quad n_i \rightarrow n_f \quad \hbar \nu = E_{n_i} - E_{n_f} \]

\[ \text{Photon absorption:} \quad n_f \rightarrow n_i \quad \hbar \nu = E_{n_f} - E_{n_i} \]

\[ \text{Note: if } \hbar \nu \neq E_n - E_{n_f} \quad \text{nothing will happen?} \]
In a transition from which excited state will hydrogen atoms emit at the longest wavelength in the Balmer series \( (n_f = 2) \)?

A. \( n_i = 2 \)
B. \( n_i = 3 \) \( \text{Correct Answer} \)
C. \( n_i = 4 \)
D. \( n_i = \infty \)
E. Something else

\[ \text{Longest } \lambda \Rightarrow \text{Small } \textit{st} \text{ photon energy} \]
\[ E_{ph} = h \nu = h \frac{c}{\lambda} \]

\[ n_i = ? \quad \quad \quad n = 4 \]
\[ E \quad \quad \quad \-underbrace{n=3} \quad \quad \quad n=2 \]
More Evidence for Quantized Energy Levels in Atoms

(b) Franck-Hertz Experiment (1914):

Emission spectrum shows strong ultraviolet line of wavelength 2537 Å

⇒ $E_{\text{photon}} = 4.9 \text{ eV}$. 
(c) Electron Scattering on a Helium Gas Target

Inelastic scattering: He electrons are excited to higher energy levels

Use magnetic field to measure energy of electrons

Graph showing the number of scattered electrons versus scattered electron energy. The graph has peaks at 21.1 eV and 23.0 eV, labeled as He*(21.1 eV) and He*(23.0 eV) respectively. The energy range is from 160 to 200 eV.
(d) X-Ray Spectra: Bremsstrahlung, Lines

**Cut-off:**

\[ h\nu = KE \text{ of electron} \]

**Sharp spectral lines:** Electron transition between innermost "orbits":

\[ h\nu = E_m - E_n \]

**Continuous Bremsstrahlung spectrum**
Measured x-ray spectra from a rhodium-anode x-ray tube

Unfiltered Rh-anode Spectra

Photon Energy (keV)

Phontons/cm²·mAs

- 50 kVp
- 44 kVp
- 38 kVp
- 32 kVp
- 26 kVp
- 20 kVp
The Bohr Atom (1913)

Rutherford (1908): atoms consist of
small positive central core
\( r < 10^{-14} \) with light electrons
outside

- \( e^- \) in orbit around core?
  - Best: classical physics: unstable!
  - Orbiting \( e^- \) is accelerated \( \Rightarrow \) should radiate \( \Rightarrow \) lose energy \( \Rightarrow \) fall into nucleus

Bohr Postulates: "path work model"

I "stationary orbits" according to laws of Newtonian mechanics, \( e^- \) electron does not radiate while on orbit

II When electron passes from one orbit to another, a photon is emitted: \( E_{\text{photon}} = E_i - E_f = h\nu \)
1st: apply classical mechanics to electron in circular orbit around charge $Q = Z e$ nucleus.

- **Total energy:** \( E = \frac{1}{2} m v^2 - \frac{1}{4 \pi \varepsilon_0} \frac{2 e^2}{r} \)  \[\text{(1)}\]

- **Newton's law:**
  \[ F = m a = \frac{1}{4 \pi \varepsilon_0} \frac{2 e^2}{r^2} = \frac{1}{2} m \frac{v^2}{r} \]

  \( \Rightarrow \) \( m v^2 = \frac{1}{4 \pi \varepsilon_0} \frac{2 e^2}{r} \)  \[\text{(2)}\]

From (1), (2)

\[ E = \frac{1}{2} \frac{1}{4 \pi \varepsilon_0} \frac{2 e^2}{r} - \frac{1}{4 \pi \varepsilon_0} \frac{2 e^2}{r} = -\frac{1}{2} \frac{1}{4 \pi \varepsilon_0} \frac{2 e^2}{r} = \frac{1}{2} U \]

\( < 0 \) take energy to remove e\((-\infty)\) bound!
Consider a quantum jump from an initial orbit to (low energy) final orbit:

- Have: $E_{\text{photon}} = h\nu = E_i - E_f = \frac{1}{8\pi\varepsilon_0} \frac{Z^2 e^2}{r_f} - \frac{1}{r_i}$

- Need to be consistent with empirical Balmer series:
  
  $E_{\text{photon}} = h\nu = E_i - E_f = \hbar c \frac{R_H}{n^2} \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$

(→) Introduce quantized energy levels and quantized electron orbits:

Energy levels: $E_n = -\hbar c \frac{R_H}{n^2} \frac{1}{\hbar^2} = -\frac{1}{8\pi\varepsilon_0} \frac{Z^2 e^2}{r_n}$

(→) Allowed, quantized electron orbits: $r_n = \frac{Z^2 e^2}{8\pi\varepsilon_0 \hbar c R_H} n^2 = a_0 n^2$

with Bohr radius $a_0 = \frac{Z^2 e^2}{8\pi\varepsilon_0 \hbar c R_H} = 0.5 \text{Å}$ for $Z=1$

Radius of $\text{e}^-$ in ground state ($n=1$) about right size.
allowed, quantized electron energies (energy levels/states)

\[ E_n = -\frac{1}{8\pi \varepsilon_0} \frac{Z e^2}{a_0 n^2} = -13.6 \text{eV} \frac{Z e^2}{n^2} \]

\[ n = 1, 2, 3, \ldots \]

\( n = 1 \): ground state energy \( E_1 = -13.6 \text{eV} \)

\( n > 1 \): excited states of hydrogen

\( n \to \infty \Rightarrow E_n \to 0 \)

\( r_n \to \infty \)
Can the electron in the ground state of the hydrogen atom absorb a photon of energy less than 13.6 eV?

A. Yes
B. No
C. It depends

Only if excited state $E_f$ exist such that

$$E_f - E_i = h \nu = E_{\text{photon}}$$
Can the electron in the ground state of the hydrogen atom absorb a photon of energy more than 13.6 eV?

A. Yes  
B. No  
C. It depends

\[ E_{\text{photon}} > 1 \ E_1 \]

\[ \Rightarrow \text{ionizes hydrogen atom} \]

\[ e^- \text{ is no longer bound} \]
3rd: Derive value for \( a_0 \):

- Use "correspondence principle":
  Predictions of quantum theory should agree with predictions of classical physics for large quantum numbers.

\[
\text{Bohr: } E_{\text{photon}} = h \nu = \frac{2e^2}{8\pi \varepsilon_0} \left( \frac{1}{a_0 n^2} - \frac{1}{a_0 (n+1)^2} \right) \frac{1/\nu_f}{1/\nu_i}
\]

\[
= \frac{2e^2}{8\pi \varepsilon_0 a_0} \left\{ \frac{1+2n}{n^2(n+1)^2} \right\} \rightarrow \frac{2e^2}{8\pi \varepsilon_0 a_0} \frac{2}{n^3} \frac{1}{\log n}
\]

\( \Rightarrow \) for large \( n \): \( V^2 \approx \frac{2^2 e^4}{(4\pi \varepsilon_0)^2 a_0^2 n^6 \hbar^2} \) (a)
clausically: frequency of radiation = orbital frequency of e⁻ = \frac{1}{T_{\text{orbit}}}

\Rightarrow \nu = \left(\frac{1}{T}\right)^2 = \left(\frac{\nu}{2\pi r_n}\right)^2 = \frac{1}{4\pi \varepsilon_0 m r_n} \left(\frac{2e^2}{2\pi r_n}\right)^2

\text{from (2)}

\Rightarrow \frac{2e^2}{4\pi \varepsilon_0 m (2\pi)^2 a_0^3 n^6} \leq r_n = a_0 n^2 \tag{6}

\Rightarrow (a) = (6) \text{ if:}

\frac{2^2 e^4}{(4\pi \varepsilon_0)^2 h^2 a_0^2 n^6} = \frac{2e^2}{4\pi \varepsilon_0 m (2\pi)^2 a_0^3 n^6}

\Rightarrow a_0 = \frac{h^2 (4\pi \varepsilon_0)}{(2\pi)^2 m 2e^2} \Rightarrow R_H, \text{ theoretical} = 105700 a_n^{-1}

R_H, measured = 105700 a_n^{-1}